Uppgift 1

I.

Se FR, kap. 5, i synnerhet s. 30.

II.

(Egen reflektion.)

Uppgift 2

I.

Se Häftet F4-7, s. 16, samt KD, avsn. 9.9.2.

II.

$$\begin{aligned}
MSE\left(\hat{\theta}\right) &= E\left\{\left[\hat{\theta}-\theta\right]^2\right\} \\
&= E\left\{\left[\hat{\theta}-E\left(\hat{\theta}\right)+E\left(\hat{\theta}\right)-\theta\right]^2\right\} \\
&= E\left\{\left[\hat{\theta}-E\left(\hat{\theta}\right)\right]^2+\left[E\left(\hat{\theta}\right)-\theta\right]^2+2\left[\hat{\theta}-E\left(\hat{\theta}\right)\right]\left[E\left(\hat{\theta}\right)-\theta\right]\right\} \\
&= E\left[\left(\hat{\theta}-E\left(\hat{\theta}\right)\right)^2\right]+E\left[E\left(\hat{\theta}\right)-\theta\right]^2+2E\left[\hat{\theta}E\left(\hat{\theta}\right)-\hat{\theta}\theta-E\left(\hat{\theta}\right)E\left(\hat{\theta}\right)+E\left(\hat{\theta}\right)\theta\right] \\
&= Var\left(\hat{\theta}\right)+\left[E\left(\hat{\theta}\right)-\theta\right]^2+2\left[E\left(\hat{\theta}\right)E\left(\hat{\theta}\right)-E\left(\hat{\theta}\right)\theta-E\left(\hat{\theta}\right)E\left(\hat{\theta}\right)+E\left(\hat{\theta}\right)\theta\right] \\
&= Var\left(\hat{\theta}\right)+Bias^2\left(\hat{\theta}\right)
\end{aligned}$$

Uppgift 3

I.

Vi avänder oss av s k Neymanallokering:

$$n_\ell = n\frac{N_\ell\sigma_\ell}{\sum_{\ell=1}^{L}N_\ell\sigma_\ell}$$

Stratum	I	II	III	$\sum$
N_ℓ	1500	500	1000	
σ_ℓ^2	9	25	16	
σ_ℓ	3	5	4	
$N_\ell\sigma_\ell$	4500	2500	4000	11000
$n\frac{N_\ell\sigma_\ell}{\sum_{\ell=1}^{L}N_\ell\sigma_\ell}$	41	23	36	

II.

$$SE\left(\hat{\tau}_{\pi,STOSU}\right) = 1.96 \times \sqrt{\hat{V}\left(\hat{\tau}_{\pi,STOSU}\right)}$$

$$\hat{V}\left(\hat{\tau}_{\pi,STOSU}\right) = \sum_{\ell=1}^{L} \hat{V}\left(\hat{\tau}_{\pi,\ell}\right) = \sum_{\ell=1}^{L} N_\ell^2 \frac{N_\ell - n_\ell}{N_\ell} \frac{s_\ell^2}{n_\ell} = \sum_{\ell=1}^{L} N_\ell^2 \frac{N_\ell - n_\ell}{N_\ell} \frac{\sigma_\ell^2}{n_\ell}$$

Stratum	I	II	III	$\sum$
N_ℓ	1500	500	1000	
σ_ℓ^2	9	25	16	
n_ℓ	41	23	36	
σ_ℓ^2/n_ℓ	$9/41$	$25/23$	$16/36$	11000
$N_\ell^2 \frac{N_\ell - n_\ell}{N_\ell}$	$1500 \times \frac{1500-41}{1500}$	$500 \times \frac{500-23}{500}$	$1000 \times \frac{1000-36}{1000}$	

,

vilket ger

$$\begin{aligned}\hat{V}\left(\hat{\tau}_{\pi,STOSU}\right) &= \sum_{\ell=1}^{L} N_\ell^2 \frac{N_\ell - n_\ell}{N_\ell} \frac{\sigma_\ell^2}{n_\ell} = 1500^2 \times \frac{1500-41}{1500} \times \frac{9}{41} + 500^2 \times \frac{500-23}{500} \times \frac{25}{23} \\ &\quad + 1000^2 \times \frac{1000-36}{1000} \times \frac{16}{36} \\ &= 1.1681 \times 10^6\end{aligned}$$

från vilket vi får

$$SE\left(\hat{\tau}_{\pi,STOSU}\right) = 1.96 \times \sqrt{\hat{V}\left(\hat{\tau}_{\pi,STOSU}\right)} = 1.96 \times \sqrt{1.1681 \times 10^6} \approx 2118.$$

Altså, punktskattningen kommer att ha konfidensintervall

$$KI = \hat{\tau}_{\pi,STOSU} \pm 2118$$

1 *Uppgift 4*

$$\begin{aligned} M &= 1010 \\ m_{c1} &= 1000 \\ m_{c2} &= 10 \\ N &= 2 \\ n &= 1 \\ \pi_{c1} &= \pi_{c2} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} y_{c1} &= 2000 \\ y_{c2} &= 20 \end{aligned}$$

I.

$$\begin{aligned} s &= \{c1\} \\ \hat{\mu}_{yr,CL} &= \frac{\hat{\tau}_{y\pi,CL}}{\hat{M}_{\pi,CL}} = \frac{\sum_{i\in s}\frac{y_i}{\pi_i}}{\sum_{i\in s}\frac{m_i}{\pi_i}} = \frac{2\times 2000}{2\times 1000} = 2 \\ \hat{\mu}_{y\pi,CL} &= \frac{1}{M}\sum_{i\in s}\frac{y_i}{\pi_i} = \frac{1}{1010}\times 2\times 2000 = \frac{4000}{1010} = 3.9604 \end{aligned}$$

II.

$$\begin{aligned} s &= \{c2\} \\ \hat{\mu}_{yr,CL} &= \frac{\hat{\tau}_{y\pi,CL}}{\hat{M}_{\pi,CL}} = \frac{\sum_{i\in s}\frac{y_i}{\pi_i}}{\sum_{i\in s}\frac{m_i}{\pi_i}} = \frac{2\times 20}{2\times 10} = 2 \\ \hat{\mu}_{y\pi,CL} &= \frac{1}{M}\sum_{i\in s}\frac{y_i}{\pi_i} = \frac{1}{M}\times 2\times 20 = \frac{40}{1010} = 3.9604\times 10^{-2} \end{aligned}$$

III.

För π-estimatorn,

$$E_s\left[\hat{\mu}_{y\pi,CL}\right] = \sum_{\Omega} p_s \hat{\mu}^{(s)}_{y\pi,CL} = \frac{1}{2}\times 3.9604 + \frac{1}{2}\times 3.9604\times 10^{-2} = 2;$$

att kvotestimatorn är vvr är uppenbart.

Uppgift 5

I.

Pop.: elever, indelade i $N = 34$ klasser med varierande antal elever per klass.

Usv.: antal timmar en elev deltagit i undervisningen under refrensveckan; totala antalet timmar samtliga elever i en klass deltagit i undervisningen betecknar vi med y_i.

Param.: Total

$$\tau_{CL} = \sum_{i\in U} y_i$$

Urv.: SY (slumpmässigt ordnad ram), $n = 8$.

Estim.:

$$\begin{aligned} \hat{\tau}_{\pi,CL} &= \sum_{i\in s}\frac{y_i}{\pi_i} = N\bar{y}_t \\ \hat{V}\left(\hat{\tau}_{\pi,CL}\right) &= N^2\frac{N-n}{N}\frac{s_t^2}{n} \end{aligned}$$

Svar:

$$\begin{aligned}
\hat{\tau}_{\pi,CL} &= N\bar{y}_t = 34 \times \frac{5867}{8} = 24935 \\
\hat{V}\left(\hat{\tau}_{\pi,CL}\right) &= N^2 \frac{N-n}{N} \frac{s_t^2}{n} = 34^2 \times \frac{34-8}{34} \times \frac{88241.125}{8} = 9.7506 \times 10^6 \\
KI &= \hat{\tau}_{\pi,CL} \pm 1.96 \times \sqrt{\hat{V}\left(\hat{\tau}_{\pi,CL}\right)} = 24935 \pm 1.96 \times \sqrt{9.7506 \times 10^6} \\
&= [18815,\ 31055]
\end{aligned}$$

II.

Se utdelat föreläsningsmaterial, F 8.