Uppgift 1

(i)

Pop.: $N = 4569$
Usv.: antal barn, y
Param.: medelvärde, $\mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i$
Urv.: OSU, $n = 538$
Punktsk.:

$$\hat{\mu}_y = \bar{y}_s = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Varsk.:

$$\hat{V}\left(\hat{\mu}_y\right) \underset{\substack{\uparrow \\ \text{från form-bl.}}}{=} \frac{N-n}{N}\frac{s_y^2}{n}$$

Data:

antal barn	antal kvinnor	antal barn*antal kvinnor	antal kvinnor*(antal barn - medelv)^2	
0	238	0	157.7468	
1	189	189	6.5298	
2	87	174	122.3478	
3	21	63	100.3389	
4	3	12	30.4494	
	538	438	417.4126	
	medelv.=	0.8141	0.7773	=s2y

Svar:

$$\begin{aligned}
\hat{\mu}_y &= \bar{y}_s = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{538} \times 438 = .81413 \\
\hat{V}\left(\hat{\mu}_y\right) &= \frac{N-n}{N}\frac{s_y^2}{n} = \frac{4569-538}{4569} \times \frac{.7773}{538} = 1.2747 \times 10^{-3} \\
B &= 2\sqrt{\hat{V}\left(\hat{\mu}_y\right)} = 2 \times \sqrt{1.2747 \times 10^{-3}} = .0714
\end{aligned}$$

(ii)

Pop.: $N = 4569$
Usv.: huruvida kvinnan har minst två barn, y - en 0/1-variabel
Param.: medelvärde (dvs andel), $\mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i$
Urv.: OSU, $n = 538$
Punktsk.:

$$\hat{\mu}_y = \hat{p} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Varsk.:

$$\hat{V}(\hat{p}) \underset{\substack{\uparrow \\ \text{från form-bl.}}}{=} \frac{N-n}{N}\frac{\hat{p}(1-\hat{p})}{n}$$

Data:

(se föregående tabell)

Svar:

$$\begin{aligned}
\hat{p} &= \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{538} \times 111 = .20632 \\
\hat{V}(\hat{p}) &= \frac{N-n}{N}\frac{\hat{p}(1-\hat{p})}{n} = \frac{4569-538}{4569} \times \frac{.20632 \times (1-.20632)}{538} = 2.6853 \times 10^{-4} \\
B &= 2\sqrt{\hat{V}(\hat{p})} = 2 \times \sqrt{2.6853 \times 10^{-4}} = .0328
\end{aligned}$$

(iii)

Pop.: $N = 4569$
Usv.: antal barn, y
Param.: total i undergrupp B, $\tau_2 = \sum_{i=1}^{N_2} y_{2i}$
Urv.: OSU, $n = 538$; $n_2 = 234$
Punktsk.:

$$\hat{\tau}_2 = N\frac{\sum_{i=1}^{n_2} y_{2i}}{n}$$

Varsk.:

$$\hat{V}(\hat{\tau}_2) \underset{\text{från form-bl.}}{\uparrow} = N^2\frac{N-n}{N}\frac{\sum_{i=1}^{n_2} y_{2i}^2 - \frac{\left(\sum_{i=1}^{n_2} y_{2i}\right)^2}{n}}{n(n-1)}$$

Data

B	antal barn	(antal barn)^2	antal kvinnor	antal kvinnor * antal barn	antal kvinnor * (antal barn)^2
	0	0	88	0	0
	1	1	95	95	95
	2	4	41	82	164
	3	9	9	27	81
	4	16	1	4	16
		30	234	208	356

Svar:

$$\begin{aligned}
\hat{\tau}_2 &= N\frac{\sum_{i=1}^{n_B} y_{2i}}{n} = 4569 \times \frac{208}{538} = 1766 \\
\hat{V}(\hat{\tau}_2) &= N^2\frac{N-n}{N}\frac{\sum_{i=1}^{n_2} y_{2i}^2 - \frac{\left(\sum_{i=1}^{n_2} y_{2i}\right)^2}{n}}{n(n-1)} = 4569^2 \times \frac{4569-538}{4569} \times \frac{356 - \frac{208^2}{538}}{538 \times (538-1)} = 17568 \\
B &= 2\sqrt{\hat{V}(\hat{\tau}_2)} = 2 \times \sqrt{17568} = 265 \\
KI &= \hat{\tau}_2 \pm B = [1501,\ 2031]
\end{aligned}$$

(iv)

Pop.: $N = 4569$, $N_1 = 1153$, $N_2 = 1838$, $N_3 = 1578$; $L = 3$
Usv.: antal barn, y
Param.: medelvärde, $\mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{N}\sum_{\ell=1}^{L}\sum_{i=1}^{N_\ell} y_{\ell,i} = \sum_{\ell=1}^{L}\frac{N_\ell}{N}\mu_\ell$
Urv.: OSU, $n = 538$; $n_1 = 123$, $n_2 = 234$, $n_3 = 181$
Punktsk.:

$$\hat{\mu}_{y,post} = \sum_{\ell=1}^{L}\frac{N_\ell}{N}\bar{y}_\ell$$

Varsk.:

$$\hat{V}\left(\hat{\mu}_{y,post}\right) \underset{\text{från form-bl.}}{\approx} \sum_{\ell=1}^{L}\left(\frac{N_\ell}{N}\right)^2 \frac{N_\ell - n_\ell}{N_\ell}\frac{s_{y,\ell}^2}{n_\ell}$$

Data:

A	antal barn	antal kvinnor	antal barn*antal kvinnor	antal kvinnor*(antal barn - medelv)^2	
	0	94	0	7.6112	
	1	23	23	11.7729	
	2	6	12	17.6566	
	3	0	0	0.0000	
	4	0	0	0.0000	
		123	35	37.0407	
		medelv.=	0.2846	0.3036	=s2y_A
B	antal barn	antal kvinnor	antal barn*antal kvinnor	antal kvinnor*(antal barn - medelv)^2	
	0	88	0	69.5309	
	1	95	95	1.1728	
	2	41	82	50.6173	
	3	9	27	40.1111	
	4	1	4	9.6790	
		234	208	171.1111	
		medelv.=	0.8889	0.7344	=s2y_B
C	antal barn	antal kvinnor	antal barn*antal kvinnor	antal kvinnor*(antal barn - medelv)^2	
	0	56	0	64.9980	
	1	71	71	0.4248	
	2	40	80	34.0515	
	3	12	36	44.3591	
	4	2	8	17.0838	
		181	195	160.9171	
		medelv.=	1.0773	0.8940	=s2y_C

Stratum:	A	B	C	summa
N_l	1153	1838	1578	4569
n_l	123	234	181	
N_l/N	0.2524	0.4023	0.3454	1.0000
(N_l/N)^2 [A]	0.0637	0.1618	0.1193	
fpc [B]	0.8933	0.8727	0.8853	
y_bar_l	0.2846	0.8889	1.0773	
s2_l	0.3036	0.7344	0.8940	
s2_l/n_l [C]	0.0025	0.0031	0.0049	
N_l/N*y_bar_l	0.0718	0.3576	0.3721	0.8015
A*B*C	0.0001	0.0004	0.0005	0.001105

Svar:

$$\begin{aligned}
\hat{\mu}_{y,post} &= \sum_{\ell=1}^{L} \frac{N_\ell}{N} \bar{y}_\ell = \frac{1153}{4569} \times .2846 + \frac{1838}{4569} \times .8889 + \frac{1578}{4569} \times 1.0773 = .8015 \\
\hat{V}\left(\hat{\mu}_{y,post}\right) &= \sum_{\ell=1}^{L} \left(\frac{N_\ell}{N}\right)^2 \frac{N_\ell - n_\ell}{N_\ell} \frac{s_{y,\ell}^2}{n_\ell} \underset{\text{se bif. tabell}}{=} .001105 \\
B &= 2\sqrt{\hat{V}\left(\hat{\mu}_{y,post}\right)} = 2 \times \sqrt{.001105} = .0665
\end{aligned}$$

(v)

Pop.: $N = 4569$, $N_1 = 1153$, $N_2 = 1838$, $N_3 = 1578$; $L = 3$
Usv.: huruvida kvinnan har minst två barn, y - en 0/1-variabel
Param.: medelvärde (dvs andel), $\mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{N}\sum_{\ell=1}^{L}\sum_{i=1}^{N_\ell} y_{\ell,i} = \sum_{\ell=1}^{L}\frac{N_\ell}{N}\mu_\ell$
Urv.: OSU, $n = 538$; $n_1 = 123$, $n_2 = 234$, $n_3 = 181$
Punktsk.:

$$\hat{\mu}_{y,post} = \hat{p}_{post} = \sum_{\ell=1}^{L}\frac{N_\ell}{N}\hat{p}_\ell$$

Varsk.:

$$\hat{V}\left(\hat{p}_{post}\right) \underset{\substack{\uparrow \\ \text{från form-bl.}}}{\approx} \sum_{\ell=1}^{L}\left(\frac{N_\ell}{N}\right)^2 \frac{N_\ell - n_\ell}{N_\ell}\frac{\hat{p}_\ell\left(1-\hat{p}_\ell\right)}{n_\ell - 1}$$

Data:

(se föregående tabell)

Svar:

$$\begin{aligned}
\hat{p}_{post} &= \sum_{\ell=1}^{L}\frac{N_\ell}{N}\hat{p}_\ell = \frac{1153}{4569}\times\frac{6}{123} + \frac{1838}{4569}\times\frac{51}{234} + \frac{1578}{4569}\times\frac{54}{181} \\
&= \frac{1153}{4569}\times .0488 + \frac{1838}{4569}\times .2180 + \frac{1578}{4569}\times .2983 = .20304 \\
\hat{V}\left(\hat{p}_{post}\right) &= \sum_{\ell=1}^{L}\left(\frac{N_\ell}{N}\right)^2 \frac{N_\ell - n_\ell}{N_\ell}\frac{\hat{p}_\ell\left(1-\hat{p}_\ell\right)}{n_\ell - 1} \\
&= \left(\frac{1153}{4569}\right)^2 \times \frac{1153-123}{1153}\times\frac{.0488\times(1-.0488)}{123-1} \\
&\quad + \left(\frac{1838}{4569}\right)^2 \times \frac{1838-234}{1838}\times\frac{.2180\times(1-.2180)}{234-1} \\
&\quad + \left(\frac{1578}{4569}\right)^2 \times \frac{1578-181}{1578}\times\frac{.2983\times(1-.2983)}{181-1} \\
&= 2.1645\times 10^{-5} + 1.0333\times 10^{-4} + 1.228\times 10^{-4} = 2.477\,8\times 10^{-4} \\
B &= 2\sqrt{\hat{V}\left(\hat{p}_{post}\right)} = 2\times\sqrt{2.4778\times 10^{-4}} = .0315
\end{aligned}$$

(vi)

Pop.: $N = 4569$, $N_2 = 1838$
Usv.: antal barn, y
Param.: total i undergrupp B, $\tau_2 = \sum_{i=1}^{N_2} y_{2i}$
Urv.: OSU, $n = 538$; $n_2 = 234$
Punktsk.:

$$N_2\bar{y}_2 = N_2\frac{\sum_{i=1}^{n_2} y_{2i}}{n_2}$$

Varsk.:

$$\hat{V}\left(N_2\bar{y}_2\right) \underset{\substack{\uparrow \\ \text{från form-bl.}}}{=} N_2^2\frac{N_2 - n_2}{N_2}\frac{s_2^2}{n_2}$$

Data:

(se tidigare tabell)

Svar:

$$\begin{aligned}
N_2\bar{y}_2 &= N_2\frac{\sum_{i=1}^{n_2} y_{2i}}{n_2} = 1838 \times \frac{208}{234} = 1634 \\
\hat{V}\left(N_2\bar{y}_2\right) &= N_2^2\frac{N_2 - n_2}{N_2}\frac{s_2^2}{n_2} = 1838^2 \times \frac{1838 - 234}{1838} \times \frac{.7344}{234} = 9252.7 \\
B &= 2\sqrt{\hat{V}\left(N_2\bar{y}_2\right)} = 2 \times \sqrt{9252.7} = 192 \\
KI &= N_2\bar{y}_2 \pm B = [1442,\ 1826]
\end{aligned}$$

(Anm.: mer precist än i *1.iii*)

Uppgift 2

a

$$
\begin{aligned}
E\left[\left(\hat{\theta}-\theta\right)^2\right] &= E\left[\left(\hat{\theta}-E\left(\hat{\theta}\right)+E\left(\hat{\theta}\right)-\theta\right)^2\right] \\
&= E\left[\left(\hat{\theta}-E\left(\hat{\theta}\right)\right)^2\right]+\left(E\left(\hat{\theta}\right)-\theta\right)^2+2E\left(\hat{\theta}E\left(\hat{\theta}\right)-\theta\hat{\theta}-E\left(\hat{\theta}\right)E\left(\hat{\theta}\right)+E\left(\hat{\theta}\right)\theta\right) \\
&= Var\left(\hat{\theta}\right)+\left[Bias\left(\hat{\theta}\right)\right]^2+2\left(\left[E\left(\hat{\theta}\right)\right]^2-\theta E\left(\hat{\theta}\right)-\left[E\left(\hat{\theta}\right)\right]^2+\theta E\left(\hat{\theta}\right)\right) \\
&= Var\left(\hat{\theta}\right)+\left[Bias\left(\hat{\theta}\right)\right]^2
\end{aligned}
$$

b

Se kursboken eller material utdelat vid F7.

c

$$\pi_{\ell,i} = \frac{n_\ell}{N_\ell} \text{ - stratumspecifik inklusionssannolikhet,}$$
$$\text{samma för alla objekt inom ett stratum}$$

$$\pi_{\ell,i} = \frac{n_\ell}{N_\ell} \underset{\substack{\uparrow \\ \text{vid proportionell} \\ \text{allokering}}}{=} \frac{n\frac{N_\ell}{N}}{N_\ell} = \frac{n}{N} \text{ oavsett stratum}$$

Anm.: Det var underförstått i uppgiften – med borde även ha stått explicit där – att det åsyftades de urvalsdragningsmekanismer som tilldelar lika inklusionssannolikheter, $\frac{n_\ell}{N_\ell}$, till alla element i samma stratum, till vilka hör de två stratifierade urvalsmekanismer som tas upp under kursen: STOSU och STSY. Om inte inklusionssannolikheter för alla element i ett stratum är $\frac{n_\ell}{N_\ell}$ så gäller resultatet inte.

Uppgift 3

Pop.: samhällen, $N = 105$
Usv.: odlad areal, y
Hjv.: folkmängd, x; $\tau_x = 415149$
Param.:

$$\tau_y = \sum_{i=1}^{N} y_i$$

Urv.:OSU, $n = 10$
Punktsk.:

$$\hat{\tau}_{y,r} = r\tau_x, \text{ där } r = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}$$

Varsk.:

$$\hat{V}\left(\hat{\tau}_{y,r}\right) \underset{\substack{\uparrow \\ \text{från form-bl.}}}{=} N^2 \frac{N-n}{N}\frac{s_r^2}{n}, \text{ där } s_r^2 = \frac{\sum_{i=1}^{n}\left(y_i - rx_i\right)^2}{n-1}$$

Data:

Samhälle	Folkmängd	Odlad areal	(areal-folkmängd*r)^2	
1	5511	4824	37304.8185	
2	865	924	38866.7194	
3	2535	1948	33176.1044	
4	3523	3013	2771.7380	
5	8368	7678	417867.9664	
6	7357	5506	457025.4398	
7	5131	4051	67883.2054	
8	4654	4060	22283.2546	
9	1146	809	23708.5865	
10	1165	1013	1159.9816	
	40255	33826	1102047.8147	
r=	0.8403		122449.7572	=s2r

Svar:

$$\begin{aligned}
r &= \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} = \frac{33826}{40255} = .8403 \\
\hat{\tau}_{y,r} &= r\tau_x = .8403 \times 415149 = 348850 \\
\hat{V}\left(\hat{\tau}_{y,r}\right) &= N^2\frac{N-n}{N}\frac{s_r^2}{n} = 105^2 \times \frac{105-10}{105} \times \frac{122450}{10} = 1.22144 \times 10^8 \\
B &= 2\sqrt{\hat{V}\left(\hat{\tau}_{y,r}\right)} = 2 \times \sqrt{1.22144 \times 10^8} = 22104
\end{aligned}$$