Michael Hartmann
Unit for Demography
Statistics Sweden
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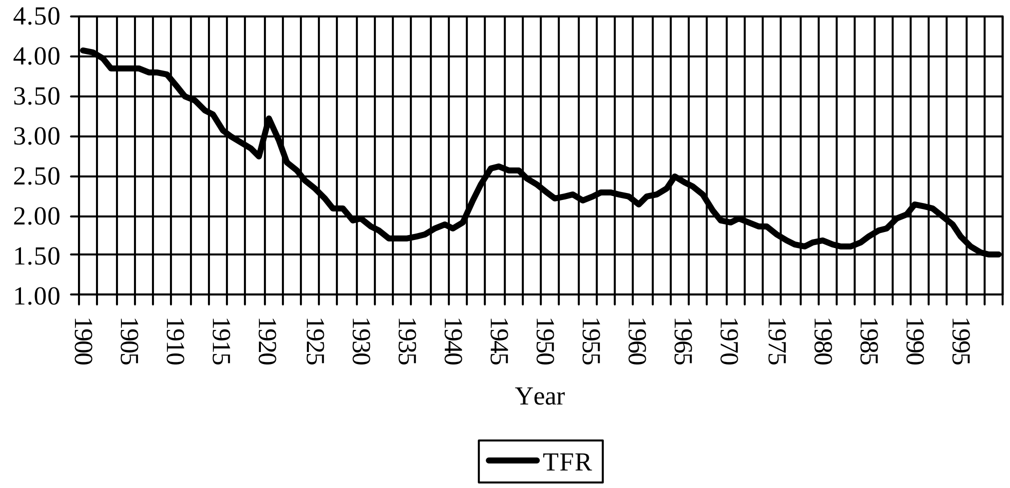
Abstract: This paper discusses modeling fertility and migration in the context of stochastic population projections. The focus is on methodological rather than substantive demographic issues. For illustrative purposes, the Swedish 2000 mid-year both sexes population is projected 50 years into the future allowing fertility and migration to vary stochastically in accordance with different models. Fertility is modeled in terms of the total fertility rate, which is assumed to change over time in accordance with a random walk with reflecting barriers. Net-migration is modeled as a stationary stochastic process partly by means of finite Fourier series, partly by means of the first-order autoregressive model. It is shown that in the context of a projection these two techniques lead to similar results.

1.0 STOCHASTIC PROJECTIONS OF THE SWEDISH POPULATION
1.1 Introduction

The traditional component method of projecting a population involves making piecewise constant definitions of mortality, fertility and migration during the projection period. The simplest assumption concerning the future population is that mortality, fertility and migration remain constant during the projection period.

The drawback of this approach is that it overlooks that mortality, fertility and migration are stochastic processes the realizations of which never draw themselves as straight lines. In the case of fertility, this is illustrated by fig. 1.1 which shows total fertility rates (TFR) for Sweden during the 20th century.

Fig. 1.1. Time-pattern of total fertility rates for Sweden: 1900-1999



There are two main reasons why in the past demographic components of change were seen as “static” configurations rather than as stochastic processes. First, historically demographic analysis has mainly involved the stance that mortality, fertility and other demographic variables unfold over time in compliance with typical time invariant data configurations and regularities (see e.g., Benjamin, 1963, pp. 38-65 for an interesting historical discussion). This view is generally upheld in standard literature on demographic methodologies where random variation, due to sampling or other circumstances, usually is not discussed. Second, before the arrival of electronic computers, it was a computationally arduous task to make a population projection. In the absence of an electronic computer, the additional burden of incorporating stochastic features made it unrealistic to perform the calculations manually. It was with the arrival of the mainframe computer during the 1950s that it became possible, within the limits of a working day, to make a large number of population projections corresponding to different assumptions. It was however not until the mid-1970s that mainframe population projection software became widely available (see e.g., Shorter, Pasta and Sandek, 1990).

With the arrival of the personal computer during the 1980s, several software packages were made available to demographers and other social researchers. These packages involved that the analyst interactively could select model life tables (see e.g., Coale and Demeny, 1966) and model fertility schedules (see e.g., Coale and Trussell, 1974) relevant for making the projections. Indeed, considerable effort was devoted during the 1960s and 1970s to development of model life tables and model fertility schedules for use with population projection packages. During the past twenty-five years or so, these easy-to-handle standard software packages have been widely used for making population projections for the majority of nations, especially developing nations.

Nonetheless, the traditional population projection method, albeit it is universally applied, has some shortcomings in that it overrides the stochastic nature of demographic processes. As a result, it is usually difficult, if not impossible, to ascertain the likely range within which the future population is likely to fluctuate. Indeed, the traditional component projection method does not enable estimation of fiduciary limits for the future population size or other projected demographic characteristics. In contrast, the purpose of making a stochastic population projection is to imbed into the projection *modus operandi* natural features of temporal demographic variability. This approach makes it possible (by means of simple simulation techniques) to attach statistical measures of confidence concerning the projected characteristics of the population (see e.g., Keilman, Pham and Hetland, 2002 for a discussion with several references).

2.0 MIGRATION AS A STOCHASTIC PROCESS

2.1 In and out-migration for both sexes

The obvious stochastic nature of the time-pattern of in-migration to Sweden between 1875 and 2002 is illustrated by fig. 2.1. What perhaps first meets the eye is the relatively steady state of the process before the 1940s and the volatility of the ensuing process after the 1940s. Interestingly enough, it can be shown that both before and after the early 1940s, in-migration performs nearly as a first-order autoregressive process (Hartmann, 2004 and appendix). Stated otherwise, while in both cases in-migration nearly types as a first-order autoregressive process, in fact nearly a random walk, what separates them is a regime shift in variability around the early 1940s. Fig. 2.1 shows out-migration from Sweden between 1851 and 2002. The time-pattern of out-migration also nearly types as a first-order autoregressive process (almost a random walk) (Hartmann, 2004).

Both sexes net-migration for the period 1875-2002 is shown in fig. 3.1. Here it will be seen that the early 1940s marks a period when net-migration shifted from being negative to mainly being positive. As in the case of in and out-migration, net-migration performs nearly as a first-order autoregressive process (Hartmann, 2004).

Fig. 2.1. Time-pattern of in-migration to Sweden between 1875 and 2002

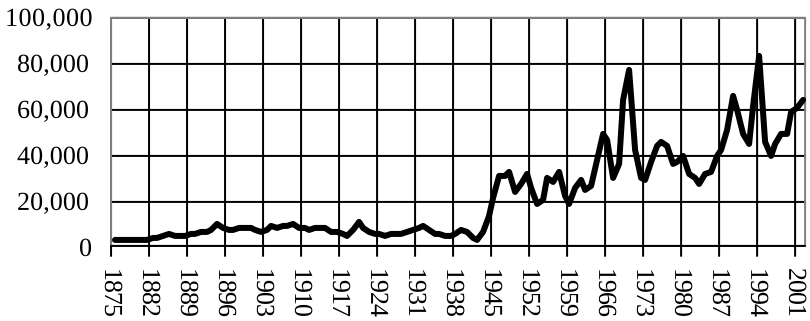
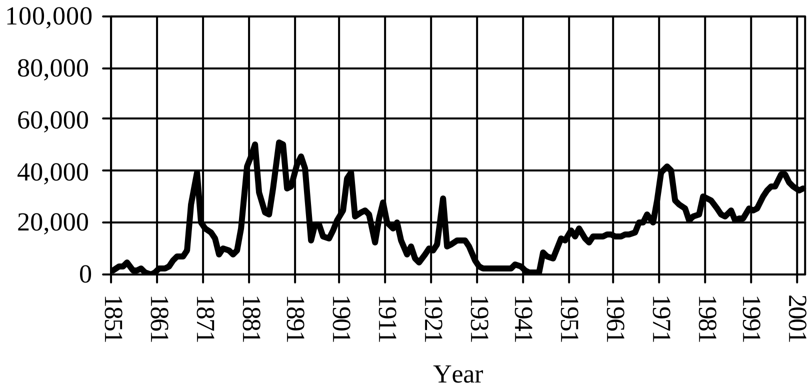


Fig. 2.2. Time-pattern of out-migration from Sweden between 1851 and 2002



3.0 STOCHASTIC MODELING OF MIGRATION AND FERTILITY
3.1 Temporal features of migration

The purpose of enmeshing stochastic features of migration into the making of a population projection rests with the desire to ascertain the limits within which the population and it demographic variables fluctuate in any given year during the period of projection. This is illustrated by means of an example.

Suppose the Swedish population in the year 2000 is projected 50 years into the future with the assumption that mortality and fertility are constant while it is assumed that net-migration is that of the period 1953-2002 and, moreover, that each new projection involves a new pseudo-realization of this net-migration process[1]. Given a large number of projections carried out in this fashion, it is possible to study the variation of the population size 50 years into the future due to the stochastic nature of such a net-migration process. The time-pattern of net-migration for both sexes during the fifty-year period 1953-2002 is shown in fig. 3.1.

Fig. 3.1. Time-pattern of net-migration for both sexes,
Sweden, 1953-2002

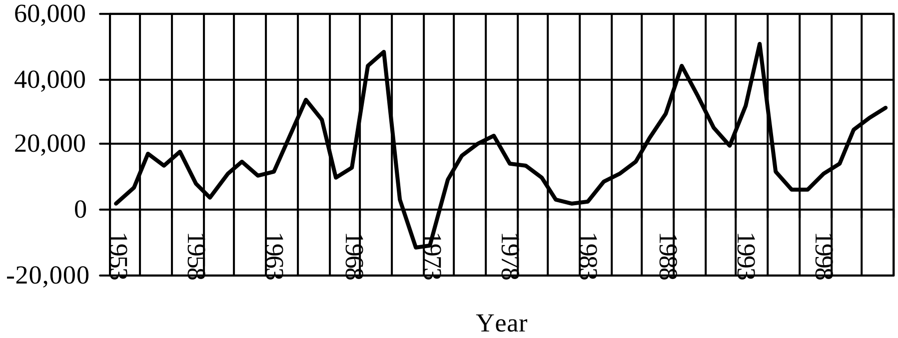


Table 3.1. Characteristics of net-migration for Sweden, 1953-2002

Mean	16,707
Standard deviation	13,611
Minimum	-11,685
Maximum	50,937

Mean net-migration for both sexes during 1953-2002 was 16,707 persons with a standard deviation of 13,611 (table 3.1). It is desirable, of course, that any new realization of net-migration included in the projection process should share these fundamental characteristics.

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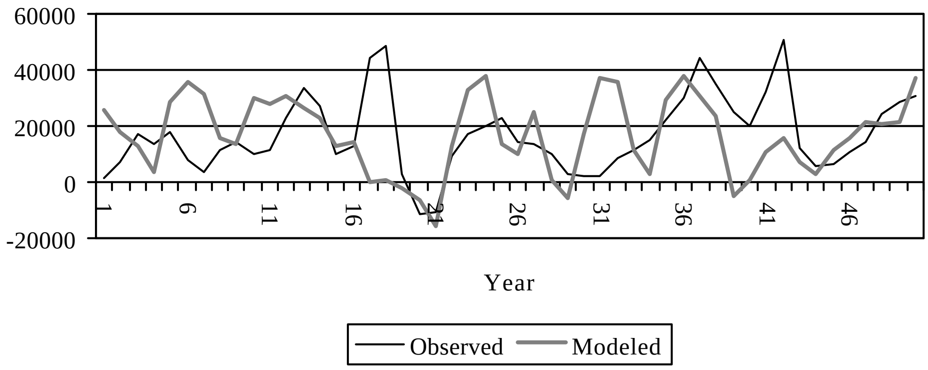
We distinguish between the abstract notion of a stochastic process x(t), $-\infty < t < \infty$, and its observed or instantiated values $x(t_1),...x(t_n)$ (at discrete unit spaced times $t_1,...t_n$) which are known as a realization of the process x(t). In most situations, we can only observe one realization of a process. In this paper, when we attempt to mimic what might have been yet another realization of the process, this is referenced as a pseudo-realization.

Fig. 3.2. Observed and Fourier modeled net-migration,
Sweden, 1953-2002



Basically, this would reflect the understanding that the time series segment in fig. 3.1 is a sample drawn from a stationary stochastic process with these underlying characteristics. Although this may seem a rather stringent assumption, it paves the way for making new (pseudo) realizations of the process. For example, by fitting a finite Fourier series to the data, and by allowing the phases to vary randomly on the interval [0, 2π], new pseudo-realizations (Fourier-simulations) are accomplished that have the same mean and variance as the observed time segment (see Hartmann, 2004 and the appendix). Fig. 3.2 shows observed net-migration and a Fourier simulation. At a later stage, we discuss and illustrate how to create pseudo-realizations of net-migration based on the first-order autoregressive model.

For each projection, it is necessary to distribute net-migration onto age distributions for males and females. Fig. 3.3 and fig. 3.4 show age distributions of in and out-migrants for males and females, respectively, during 2000-2002 (these distributions sum to unity). Fig. 3.5 shows the corresponding age distributions of net-migration for males and females per 10,000. It will be noticed that age distributions of net-migration are virtually the same for males and females and that they peak at reproductive ages. As a result, net-migration contributes significantly to the yearly number of births. It is assumed that these (deterministic) age-patterns persist during the period of projection.

In addition, it will be assumed that for each year of projection the number of net-migrants is evenly divided between males and females. With respect to fertility, time invariant normalized age-specific fertility rates for the year 2000 are scaled by the chosen level of TFR.

Fig. 3.3. Age distribution for male in and out-migrants, Sweden, 2000-2002

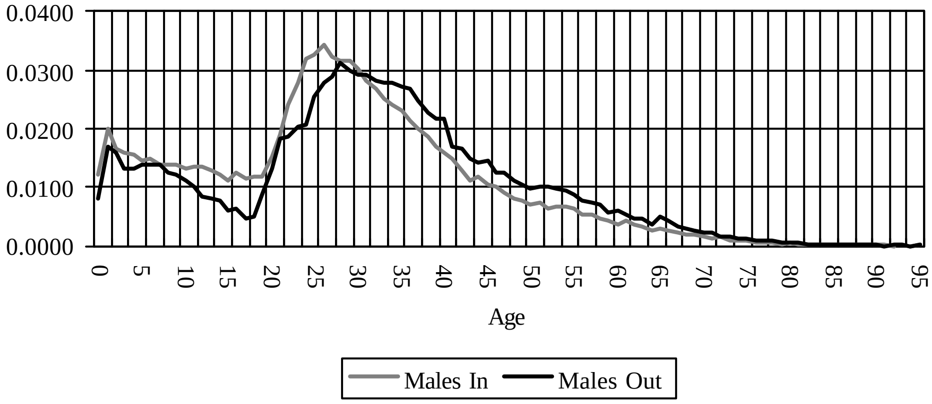


Fig. 3.4. Age distribution for female in and out-migrants, Sweden, 2000-2002

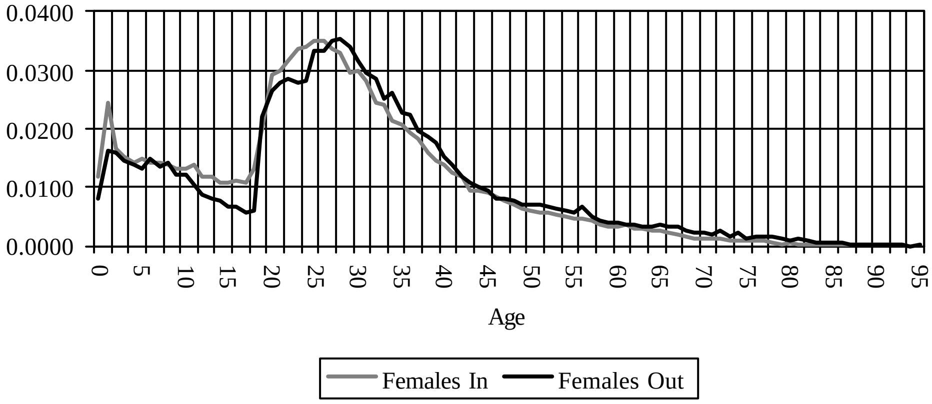


Fig. 3.5. Age distributions of net-migration for males and females, per 10,000 population, Sweden, 2000-2002

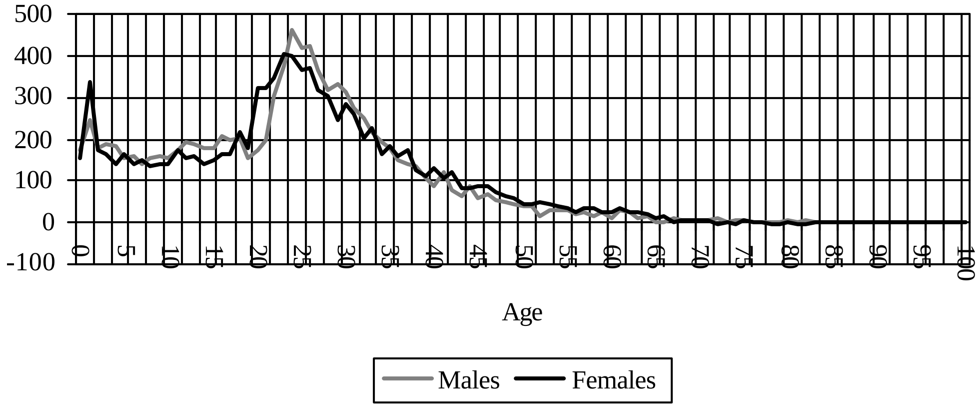


Table 3.2. Traditional projection of the Swedish 2000 population 50 years into the future with the assumption of zero net-migration and that mortality and fertility remain constant.

		Assumption I	
Characteristics	TFR	Mortality	Zero net-migration
	2.1	Life table for 2000	
Population	Both sexes	Males	Females
Population in 2000	8,872,294	4,386,522	4,485,772
Population in 2050	9,006,916	4,473,305	4,533,611
Gain per year	2,692	1,735	957
		Assumption II	
	TFR	Mortality	Zero net-migration
	1.8	Life table for 2000	
	Both sexes	Males	Females
Population in 2050	7,932,444	3,924,356	4,008,088
Gain per year	-18,797	-9,243	-9,554

In order to ascertain the effects of net-migration, table 3.2 shows two traditional projections of the Swedish 2000 population 50 years into the future. In projection I it is assumed that there is no migration in and out of the population, that the total fertility rate remains constant at TFR = 2.1 and that mortality is that of the Swedish 2000 life table. Projection II builds on the same assumptions except that TFR = 1.8 during the projection period. These projections will serve as yardsticks against which to measure the influence of migration. It will be noticed that while TFR = 2.1 sustains the population size, TFR = 1.8 leads to a slump.

Table 3.3 shows the effect of imbedding stochastic net-migration into projections I and II shown in table 3.2. The chosen experience is that of Sweden 1953-2002. The average number of annual net-migrants is 16,707 with a standard deviation of 13,611 (table 3.1). Projections A and B corresponding to I and II, respectively, are made a hundred times and each time net-migration is a Fourier simulation of the above-mentioned net-migration experience (see fig. 3.2).

For projection A (TFR = 2.1), the mean of the 100 simulations of the 2050 population is 10,234,730 for both sexes, 5,087,583 for males and 5,147,147 for females (table 3.3). The standard deviations of the 100 simulations are 38,489 for both sexes, 19,136 for males and 19,355 for females. The corresponding 95 percent confidence limits for the projected populations are also shown in table 3.3. Fig. 3.6 shows the sequence of “2050 both sexes populations” for the 100 simulations of projection A.

For projection B (TFR = 1.8), the mean of the 100 simulations of the 2050 population is 9,080,508 for both sexes, 4,497,832 for males and 4,582,676 for females (table 3.3). The standard deviations of the 100 simulations are 30,860 for both sexes, 15,305 for males and 15,558 for females. The corresponding 95 percent confidence limits for the projected populations are also shown in table 3.3. Fig. 3.7 shows the sequence of “2050 both sexes populations” for the 100 simulations of projection B.

Table 3.3. Characteristics for 100 repetitions of projections A and B.

Characteristics	Assumption A TFR 2.1	Mortality Life table for 2000	Net-migration Yearly mean is 16,707
	Both sexes	Males	Females
Population in 2000	8,872,294	4,386,522	4,485,772
Mean population in 2050	10,234,730	5,087,583	5,147,147
Gain per year	27,249	14,021	13,228
Standard deviation	38,489	19,136	19,355
Approximate 95 percent confidence interval for 2050 population	10,234,730 ± 76,978	5,087,583± 38,272	5,147,147± 38,710
	Assumption B TFR 1.8	Mortality Life table for 2000	Net-migration Yearly mean is 16,707
Mean population in 2050	9,080,508	4,497,832	4,582,676
Gain per year	4,164	2,226	1,938
Standard deviation	30,860	15,305	15,558
Approximate 95 percent confidence interval for 2050 population	9,080,508 ± 61,720	4,497,832 ± 30,610	4,582,676 ± 31,116

Fig. 3.6. Projection A repeated a hundred times
(the Swedish 2050 both sexes populaton).

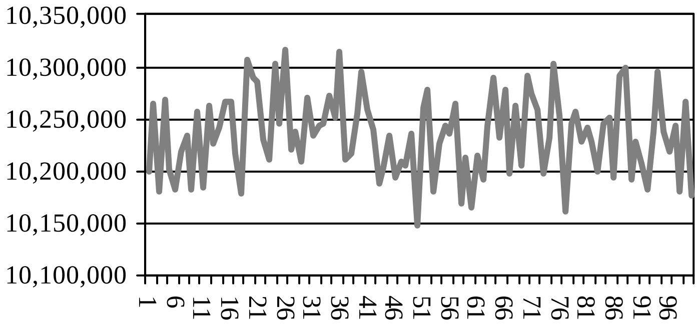


Fig. 3.7. Projection B repeated a hundred times
(the Swedish 2050 both sexes population).

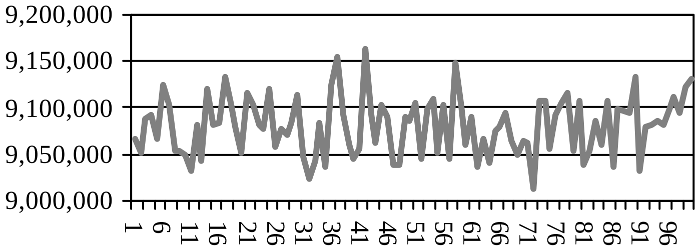


Table 3.3 suggests that if TFR is close to 2.1 during the period of projection, the stochastic variability in net-migration would involve that the both sexes population in year 2050 would be about 10,234,730 ± 76,978; a relatively small error margin. If TFR were to be about 1.8 during the period of projection, the error margin would be even smaller (table 3.3).

Table 3.4 gives results similar to those of table 3.3 except that now the yearly average net-migration is 30,000. Projection C corresponds to TFR = 2.1 and projection D to TFR = 1.8. Here, too, it will be noted that for TFR = 2.1, the estimated both sexes population in the year 2050 would be about 11,218,607 ± 73,416 (projection C). For TFR = 1.8 the similar estimate would be about 10,000,000 ± 63,400 (projection D in table 3.4). To visualize the variability in population size in the case of projection D, fig. 3.8 shows the results of the 100 repetitions in ascending order.

Table 3.3 and table 3.4 give results that are somewhat artificial in that it is assumed that the total fertility rate remains constant during the period of projection. While this is a classic approach in the making of population projections, it overrides the stochastic behavior that "inevitably" is associated with the temporal unfolding of the total fertility rate. For this reason, we now turn to a stochastic representation of fertility.

Fig. 3.8. The results of 100 repetitions of projection D sorted in ascending order

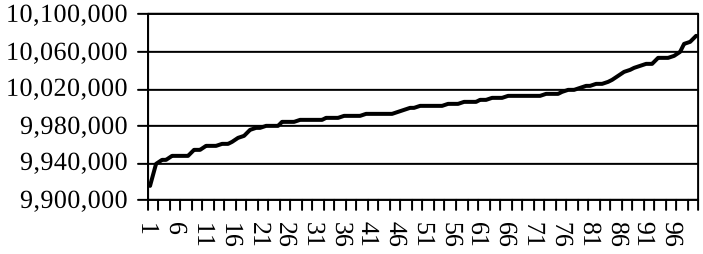


Table 3.4. Characteristics for 100 repetitions of projections C and D.

Characteristics			
		Assumption C	
	TFR 2.1	Mortality Life table for 2000	Net-migration Yearly mean is 30,000
	Both sexes	Males	Females
Population in 2000	8,872,294	4,386,522	4,485,772
Mean population in 2050	11,218,607	5,579,774	5,638,833
Gain per year	46,927	23,865	23,061
Standard deviation	36,708	18,256	18,454
Approximate 95 percent confidence interval for 2050 population	11,218,607 ± 73,416	5,579,774 ± 36,512	5,638,833 ± 36,908
		Assumption D	
	TFR 1.8	Mortality Life table for 2000	Net-migration Yearly mean is 30,000
Mean population in 2050	9,999,928	4,957,069	5,042,859
Gain per year	4,164	2,226	1,938
Standard deviation	31,716	15,698	16,022
Approximate 95 percent confidence interval for 2050 population	9,999,928 ± 63,432	4,957,069 ± 31,359	5,042,859 ± 32,044

3.2 Temporal features of fertility

To ascertain the relative importance of net-migration and fertility, we now turn to a projection for which net-migration is zero and where the total fertility rate varies stochastically between 2.4 and 1.6 (a post World War II reproductive range). This is accomplished by means of a random walk with reflecting barriers and starting value TFR = 2.1. Denoting the total fertility rate in year t by R_t, the process is

$$R_t = R_{t-1} + e_t \quad (1)$$

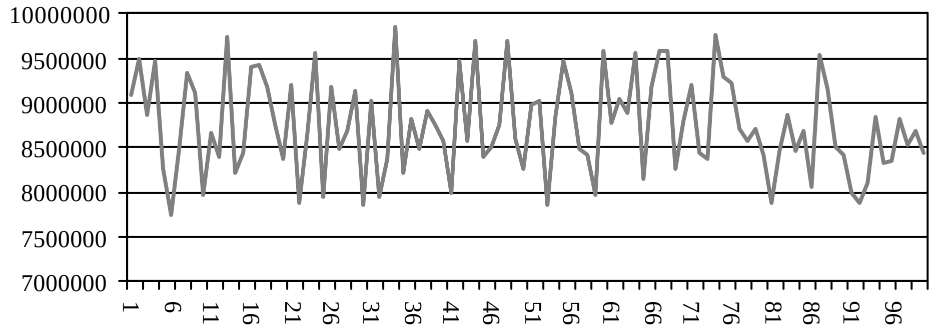
if $R_t > 2.4$ then $R_t = 2.4 + e_t$

else if $R_t < 1.6$ then $R_t = 1.6 + e_t$

Error terms e_t in (1) are independently normally distributed with $E[e_t] = 0$ and standard deviation $\sigma[e_t] = 0.1$ as determined empirically from previous studies (Hartmann, 2003).

Fig. 3.9 shows the results of the one hundred repetitions of the projection. Survival is that of the 2000 life table. The corresponding characteristics are shown in table 3.5.

Fig. 3.9. The Swedish 2000 both sexes population projected 50 years into the future a hundred times with zero net-migration and TFR varying stochastically between 1.6 and 2.4



It will be seen that now the margin of error or "prediction interval" is much larger than in the case of assuming constant fertility (tables 3.3 and 3.4). We would now expect the 2050 population to be about 8,738,815 with a standard deviation of about 535,945. For the one hundred repetitions of the projection, the difference (range) between the lowest and highest both sexes population is 2.1 million (table 3.5).

The reproductive range in (1) might be deemed somewhat on the high side. Possibly a more realistic future range is 1.8 < TFR < 2.2. Table 3.6 shows the result of one hundred repetitions of a projection with this fertility range. This projection points to a population in 2050 of about 8.5 million with a standard deviation of about 200,000.

Table 3.7 shows a projection similar to that of table 3.6 except that now net-migration is assumed to be 20,000 persons per year. For each of the one hundred repetitions of the projection, the time-pattern of net-migration is a Fourier-simulation of the 1953-2002 experience. The projected population in 2050 is 9,976,492 with a standard deviation of 216,128. In other words, when net-migration is about 20,000 persons per year, this raises the estimated 2050 population by about one million relative to zero net-migration. This estimate, of course, builds on the assumption that net-migrants share the fertility experiences of the resident population. When the yearly amount of net-migration is raised to 30,000 persons per year, the estimated 2050 population is about 10,703,681 with a standard deviation of about 213,046. The range for the one hundred repetitions of the projection is 1,185,196 (table 3.8).

Table 3.5. Projecting the Swedish 2000 population 50 years into the future with zero net-migration, constant mortality and TFR varying stochastically between 1.6 and 2.4. Projection repeated a hundred times.

Characteristics	Both sexes	Males	Females
Mean	8,738,815	4,336,238	4,402,577
Standard deviation	535,945	273,904	262,040
Maximum	9,854,024	4,906,182	4,947,842
Minimum	7,753,863	3,832,961	3,920,902
Range	2,100,161	1,073,221	1,026,940

Fig. 3.10. The Swedish 2000 both sexes population projected 50 years into the future a hundred times with zero net-migration and TFR varying stochastically between 1.8 and 2.2

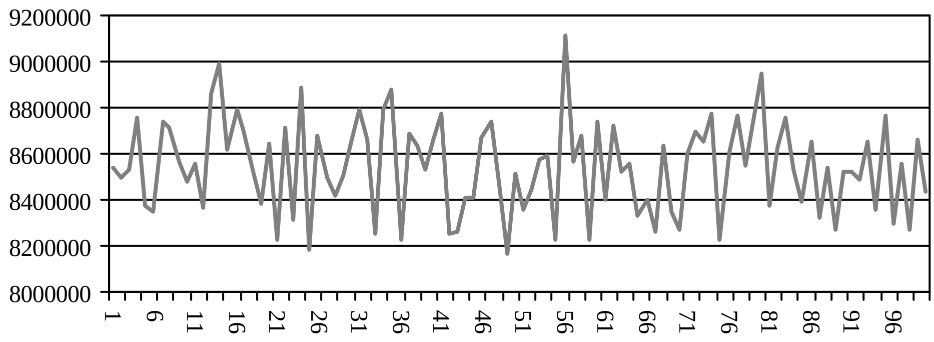


Table 3.6. Projecting the Swedish 2000 population 50 years into the future with zero net-migration, constant mortality and TFR varying stochastically between 1.8 and 2.2. Projection repeated a hundred times.

Characteristics	Both sexes	Males	Females
Mean	8,544,889	4,237,340	4307,549
Standard deviation	200,810	102,610	98,198
Maximum	9,116,917	4,529,628	4,587,289
Minimum	8,166,782	4,044,089	4,122,693
Range	950,135	485,539	464,596

Table 3.7. Projecting the Swedish 2000 population 50 years into the future with yearly average net-migration 20,000, constant mortality and TFR varying stochastically between 1.8 and 2.2. Projection repeated a hundred times.

Characteristics	Both sexes	Males	Females
Mean	9,976,492	4,953,131	5,023,361
Standard deviation	216,128	110,325	105,806
Maximum	10,482,426	5,210,877	5,271,549
Minimum	9,460,860	4,690,722	4,770,138
Range	1,021,566	520,155	501,411

Table 3.8. Projecting the Swedish 2000 population 50 years into the future with yearly average net-migration 30,000, constant mortality and TFR varying stochastically between 1.8 and 2.2. Projection repeated a hundred times.

Characteristics	Both sexes	Males	Females
Mean	10,703,681	5,316,844	5,386,837
Standard deviation	213,046	108,693	104,355
Maximum	11,372,002	5,657,680	5,714,322
Minimum	10,186,806	5,052,336	5,134,470
Range	1,185,196	605,344	579,852

3.3 Autoregressive simulation of migration

The time-pattern of net-migration during 1953-2002 shown in fig. 3.1 (repeated below for easy reference) is very nearly a first-order autoregressive process (Hartmann, 2004). For this time-segment, estimation of the first-order autoregressive model

$$z_t = \lambda z_{t-1} + (1-\lambda)\mu + e_t \qquad (2)$$

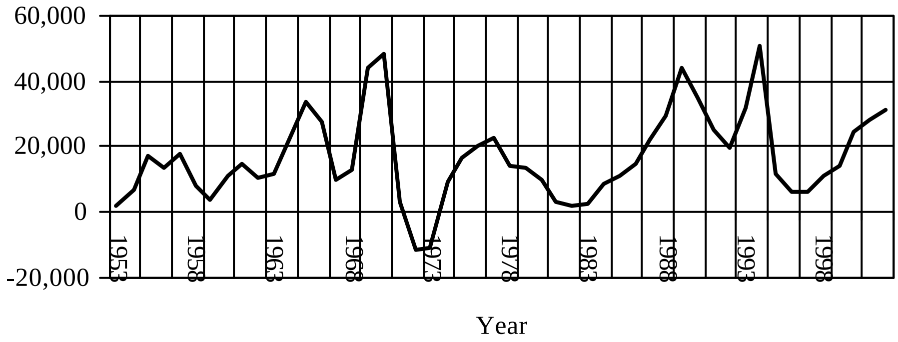
yields $\hat{\lambda} = 0.57$, $\hat{\sigma}(e_t) \approx 11{,}000$ and mean $\hat{\mu} = 16{,}707$. This means that

$$\hat{z}_t = 0.57\, z_{t-1} + 7{,}184 + \hat{e}_t \qquad (3)$$

is an AR(1)-simulation of the net-migration experience. In (2), e_t is a purely random process with zero mean and fixed standard deviation $\sigma = \sigma(e_t)$. In the estimated model (3), $\hat{e}_t$ are independently normally distributed random numbers with zero mean and standard deviation $\hat{\sigma}(e_t)$.

The material difference between a Fourier-simulation and an AR(1)-simulation, as provided by (2), is that the former conserves means and variances whereas the latter does not. Moreover, if the time series observations contain a regime shift, it is easy to misestimate the variance of e_t (see the appendix for a discussion). Here it is in place to add that it may be exceedingly difficult to ascertain if an observed time series segment contains a regime shift, a difficulty that is exacerbated when only a short time interval is covered.

Fig. 3.1. Time-pattern of net-migration for both sexes,
Sweden, 1953-2002



Projections E and F compare applications of Fourier and AR(1)-simulations of the net-migration experience. Projection E is such that for each year during the period of projection, the total fertility rate varies in accordance with (1). For each repetition of the projection, net-migration is a Fourier realization of net-migration during 1953-2002 with a yearly mean of 20,000 persons. Projection F is such that for each year during the period of projection, the total fertility rate varies in agreement with (1) and net-migration for the period of projection is an AR(1)-realization determined by (3). The assumptions are detailed in table 3.9.

Table 3.9. Assumptions and results for projections E and F

		Statistics for 100 repetitions of projections		
Assumptions	Characteristics	Both sexes	Males	Females
Projection E	Mean	10,245,020	5,090,140	5,154,881
Fourier-simulation	Standard deviation	546,363	279,135	267,229
1.6 < TFR < 2.4	Minimum	9,040,268	4,474,874	4,565,394
Mean net-migration is 20,000	Maximum	11,272,736	5,615,944	5,656,792
	Range	2,232,468	1,141,070	1,091,398
Projection F	Mean	10,325,785	5,132,301	5,193,484
AR(1)-simulation	Standard deviation	563,907	287593	276,320
1.6 < TFR < 2.4	Minimum	8,912,983	4,411,555	4,501,428
Mean net-migration is 19,564	Maximum	11,359,059	5,658,523	5,700,536
	Range	2,446,076	1,246,968	1,199,108

Fig. 3.11 shows 100 repetitions of projections E and F. It will be appreciated that the two models of simulation, for practical purposes, lead to identical results.

Fig. 3.11. Results of one hundred repetitions of projections E and F

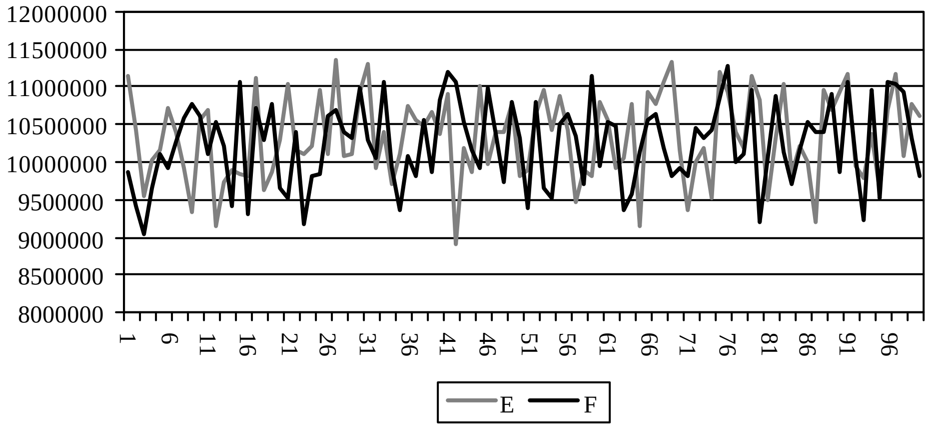
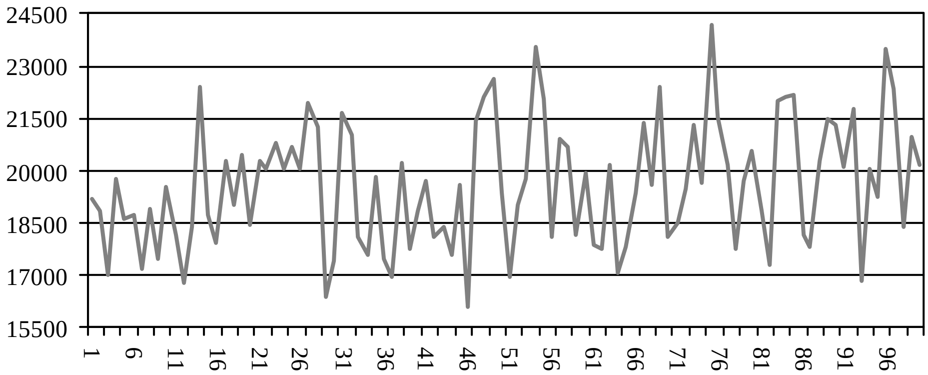


Fig. 3.12. Net-migration as simulated by AR(1)-model for each of 100 repetitons of projection F



As noted, the material difference between a Fourier simulation and an AR(1)-simulation of net-migration is that while the Fourier simulation preserves the mean and variance of the base time-pattern of net-migration, the AR(1)-simulation does not. Nevertheless, for each AR(1)-simulation, the corresponding mean and variance are numerically close to those of the base time-pattern (see appendix for numerical examples). Fig. 3.12 shows net-migration as simulated by (3) for the above-mentioned 100 repetitions of projection F. Mean net-migration across the 100 repetitions of projection D is 19,564 with standard deviation s = 1,785.

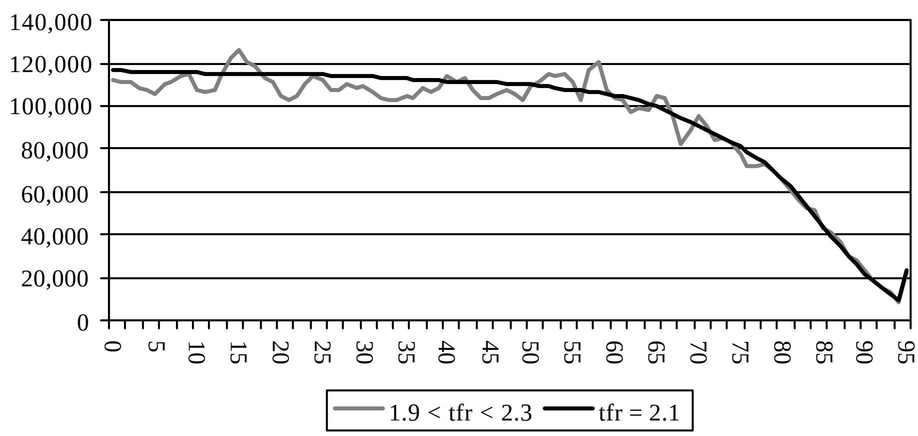
4.0 DISCUSSION

It is in place to make a few comments about the above-mentioned examples. These comments will focus on techniques of simulation rather than on the substantive nature of the demographic characteristics.

It seems reasonable that when modeling fertility and migration, the chosen models should portray, as closely as possible, the corresponding observed processes. In this paper, which has only been written for brief illustrative purposes, fertility has been modeled as a random walk with reflecting barriers. This model has the intuitive appeal that it limits the total fertility rate to varying within a range deemed reasonable in the light of empirical observations (see e.g., Hartmann, 2003). Migration has been treated as net-migration, which is the common approach to dealing with it in the light of making population projections. The paper illustrates two different models for simulation of net-migration; one that is based on finite Fourier series and another that draws on the autoregressive nature of the data. The paper does not deal with stochastic models of mortality. This issue will be discussed in a forthcoming paper.

The confidence limits that can be estimated hinge on the validity of the chosen stochastic models. They are not, so to speak, confidence limits in an absolute sense. To this must be added that there is the question of how many simulations need to be carried out in order to estimate reasonably accurately the confidence limits imputed by the applied models. In this paper, a hundred simulations have been carried out as a means of illustrating the technique. Clearly, one could make an even larger number of simulations. Here it must be realized however that when carrying out a very large number of simulations that the invoked sequences of pseudo-random numbers may repeat themselves so that results attain a spurious degree of precision. While there are ways around this problem, it must be noted that there is an upper limit as to how many simulations are required for reasonable estimation of confidence or prediction limits.

Fig. 4.1. Comparison between stable and stochastic population



If a population closed to migration is projected many years into the future with constant mortality and fertility, its age distribution will become stable. However, because in reality, mortality and fertility are stochastic processes, such a perfect age distribution remains a mathematical abstraction.

Fig. 4.1 shows the results of projecting the 2000 Swedish population one hundred years into the future assuming (i) constant fertility TFR = 2.1 along with constant mortality and zero-net-migration, and (ii) a total fertility rate that varies as a random walk with reflecting barriers 1.9 < TFR < 2.3 and mean value TFR = 2.1, constant mortality and zero net-migration. Projection (ii) brings forth a portrayal of the age distribution that is more “seesaw-real” than that of projection (1), which shall always remain uninstantiated.

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