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Estimating interviewer variance under a measurement error model for continuous survey data – Stratified sampling

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Abstract

This is the second paper dealing with a simple measurement error model for continuous data collected by interviewers. The model makes a clear distinction between three different sources of randomness, namely, sample selection, interviewer assignment, and interviewing. The concept of interviewer variance is defined in the context of this measurement error model, and the problem of estimating the interviewer variance is considered, assuming stratified simple random sampling. Two different methods to estimate interviewer effects are formulated and compared through a simulation study.

Key words: Response variance; survey nonsampling error; interviewer effects; interpenetration.

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1. Introduction

Sample survey data is usually more or less affected by measurement errors. This means that each selected element is affected during the data collection stage (by an error) and the recorded values on the study variables differ from the true values. We will in this paper focus on how to study interviewer errors in surveys of individuals or households. The data is collected via telephone, which makes it possible to assign the sampled elements completely at random to the interviewers.

To be able to discuss the statistical aspects of measurement errors, we need a statistical model describing how measurement errors arise. We will use the model described in Biemer and Trewin (1997). Historically the chosen measurement error model is based on the analysis-of-variance (ANOVA) type of model used by Kish (1962) and further developed by Hartley and Rao (1978) and others. The terminology will closely follow Wolter (1985), and Särndal, Swensson and Wretman (1992).

In Lundquist and Wretman (2002) interviewer effects are studied for simple random sampling. The present paper expands the theory to be valid for stratified simple random sampling as well. As in Lundquist and Wretman we make the simplifying assumptions that there is no nonresponse.

The main purpose of this paper is to introduce new estimators of the interviewer effect. In Section 2, the measurement error model is specified, and the concept of interviewer variance is introduced. Basic assumptions about the interviewer assignment are also made. In Section 3, we look at the problem of estimating a population mean under the assumed measurement error model. In Section 4, we discuss how to estimate the variance of an estimator of the population mean. In Section 5, we suggest two methods to estimate the interviewer variance and the intra-interviewer correlation. In Section 6, the estimators are examined in a simulation study. (The simulation results are given in the Appendices.) A short discussion based on the simulation results is given in Section 7.

2. Sampling design, interviewer assignment, and measurement error model

In this section we will describe how a sample of elements is selected from the population, how the sampled elements are assigned to interviewers, and how measurement errors arise. Some necessary notation for the stratified sampling is to be introduced. The finite population U consisting of N elements is now partitioned into H nonoverlapping subpopulations, called strata where subpopulation U_h has N_h elements for $h = 1,\ldots,H$ and the number of elements in the finite population is $N = \sum_{h=1}^{H} N_h$. Let μ_k be the unknown true value for element k, with respect to the actual study variable. The purpose of the survey is to estimate the true population mean

$$\bar{\mu}_U = \frac{1}{N}\sum_{h=1}^{H}\sum\nolimits_{k\in U_h} \mu_k$$

Let s be a sample (that is, a subset of U) consisting of n elements, drawn from U by *stratified simple random sampling without replacement*. This means that we within every U_h (for $h = 1,\ldots,H$) select a simple random sample, s_h, of the size n_h. (Thus, we have that $s = s_1 \cup \ldots \cup s_h \cup \ldots \cup s_H$ and $n = \sum_{h} n_h$.) Ideally, we would like to observe the true value μ_k for each element $k \in s$, but what we will really observe is a value y_k affected by measurement error, that is,

$$y_k = \mu_k + d_k = \text{true value} + \text{measurement error}$$

The problem now is to estimate $\bar{\mu}_U$ using observed data y_k for $k \in s$.

We will in this paper give a brief description of the interviewer allocation and the model specification. A more complete description of the definitions is given in Lundquist and Wretman (2002).

We assume, as in Lundquist and Wretman, that there is a set of I interviewers available for the survey. Sampled elements are assigned to these interviewers in the following way: Each interviewer is given a randomly chosen subset of elements from the sample s, under the restriction that the subsets should be non-overlapping and of equal size. This technique, the random division of the initial sample into I subsamples, is called interpenetrated subsampling and for each interviewer it will be possible (at least theoretically) to produce population estimates. In this paper this interviewer allocation is generalized in a symmetric way to stratified sampling. The following notation will be used. Let the interviewers be labeled $i = 1, 2, \ldots, I$. Let the sample s_h (for $h = 1,\ldots,H$) be partitioned at random into I nonoverlapping groups of equal size $m_h = n_h/I$. (We assume that m_h is an integer for $h = 1,\ldots,H$.) These groups are denoted $s_{h1}, \ldots, s_{hi}, \ldots, s_{hI}$. Now, the rule is that interviewer i is to make all the interviews in group $s_i = s_{1i} \cup \ldots \cup s_{hi} \cup \ldots \cup s_{Hi}$, $i = 1, 2, \ldots, I$. In particular this means that all interviewers are assigned elements from every stratum.

The measurement error model, denoted M, is specified conditionally on a given sample s and given interviewer assignments $s_1, s_2, \ldots, s_I$. Following Biemer and Trewin (1997) we assume that the measurement error is the sum of two components, b_i an "interviewer error" due to the interviewer, and ε_k a "response error" which depends on the respondent (and possibly other remaining sources of error). Thus, the measurement error model says that when element $k \in s_i$ is interviewed by interviewer i, the observed value y_k can be written as

$$y_k = \mu_k + b_i + \varepsilon_k$$

In the present set-up, the survey is thus viewed as a three-stage process, where randomness is involved in each stage:

Stage 1: A sample s is drawn from the population U.
Stage 2: The sample s is partitioned into subsamples $s_1, s_2, \ldots, s_I$.
Stage 3: Observed values y_k are obtained for $k \in s_i$, $i = 1, 2, \ldots, I$.

The randomness in the first stage comes from the sampling design, denoted p, which in the actual case means a stratified simple random sampling without replacement of n_h elements from U_h ($h = 1,\ldots,H$). The randomness in the second stage comes from the random division of the sample into subsets assigned to the interviewers. The randomness in the third stage comes from the measurement process described by the measurement error model M. Sometimes it will be found convenient to consider the first two stages jointly as constituting one stage, which will then be denoted p^*.

In what follows, estimators will usually be judged by their bias and variance with respect to the joint distribution induced by the three stages above, which will be called the p^*M-distribution. It will sometimes be found convenient to express expected values and variances using conditional probabilities in the following way:

$$E_{p^*M}(\ .\ ) = E_{p^*}[E_M(\ .\ |\ s;\ s_1,\ \ldots,\ s_I)]$$

and

$$Var_{p^*M}(\ .\ ) = E_{p^*}[Var_M(\ .\ |\ s;\ s_1,\ \ldots,\ s_I)] + Var_{p^*}[E_M(\ .\ |\ s;\ s_1,\ \ldots,\ s_I)]$$

where $E_{p^*M}(\ .\ )$ denotes expectation with respect to the stochastic mechanisms in stage 1, 2, and 3 simultaneously, $E_{p^*}(\ .\ )$ denotes expectation with respect to stage 1 and 2 only, and $E_M(\ .\ |\ s;\ s_1,\ \ldots,\ s_I)$ denotes conditional expectation with respect to stage 3, given the outcome of stage 1 and 2. Analogous principles of notation hold for the variances. Note, this set-up implies that the order of E_{p^*} and E_M are not interchangeable. In the rest of this paper we will, for the sake of simplicity, write $E_M(\ .\ )$ instead of the longer and more exact expression $E_M(\ .\ |\ s;\ s_1,\ \ldots,\ s_I)$. Thus, in what follows,

$$E_M(\ .\ ) = E_M(\ .\ |\ s;\ s_1,\ \ldots,\ s_I)$$

The assumptions of the measurement error model can now be expressed formally. Notice that the model is not affected by the sample design, i.e. the interviewer and response errors are not affected by the stratification.

Model assumptions:

For a given sample s and given subsamples $s_1,\ s_2,\ \ldots,\ s_I$,

$y_k = \mu_k + b_i + \varepsilon_k$ for $k \in s_i,\ \ i = 1,\ \ldots,\ I$

$E_M(b_i) = B_b$ and $Var_M(b_i) = \sigma_b^2$ for $i = 1,\ \ldots,\ I$

$E_M(\varepsilon_k) = B_\varepsilon$ and $Var_M(\varepsilon_k) = \sigma_\varepsilon^2$ for $k \in s_i,\ \ i = 1,\ \ldots,\ I$

$b_1,\ b_2,\ \ldots,\ b_I,\ \varepsilon_k\ \ (k \in s)$ are independent random variables

The following result follows immediately from the model assumptions.

Result 2.1: Under Model M, it holds that

$$E_M(y_k) = \mu_k + B_b + B_\varepsilon \qquad \text{for } k \in s_i, \;\; i = 1, \ldots, I$$

$$Var_M(y_k) = \sigma_b^2 + \sigma_\varepsilon^2 \qquad \text{for } k \in s_i, \;\; i = 1, \ldots, I$$

$$Cov_M(y_k, y_l) = \begin{cases} \sigma_b^2 & \text{for } \; k \neq l, \; k \in s_i, \; l \in s_i, \; i = 1, \ldots, I \\ 0 & \text{for } \; k \neq l, \; k \in s_i, \; l \in s_j, \; i = 1, \ldots I, \; j = 1, \ldots, I, \; i \neq j \end{cases}$$

Thus, conditionally on the sample s and on the interviewer assignments s_1, s_2, …, s_I, observed values for different elements obtained by different interviewers are uncorrelated, while values for different elements obtained by the same interviewer are correlated.

3. Estimating the population mean

Based on the p^*M-distribution, specified in Section 2, it is possible to estimate the population mean, $\bar{\mu}_U$. We will in the following use abbreviated notation in our expressions: $\sum_h$ is the short form for the summation over the h strata $\sum_{h=1}^{H}$, $\sum_i$ is used for the summation over the interviewers $\sum_{i=1}^{I}$ and finally $\sum_A$ stands for $\sum_{k \in A}$ where $A \subseteq U$ is any subset of U.

If, hypothetically, we had sample data, μ_k for $k \in s$, *without* measurement errors, the true population mean would, under stratified simple random sampling and in the absence of auxiliary information, usually be estimated by

$$\bar{\mu}_{st} = \sum_h W_h \bar{\mu}_{s_h}$$

where $W_h = N_h / N$ denotes the relative size of stratum U_h and $\bar{\mu}_{s_h} = \frac{1}{n_h} \sum_{s_h} \mu_k$ for $h = 1, \ldots, H$.

The estimator that we are going to consider is the stratified sample mean based on data *with* measurement errors

$$\bar{y}_{st} = \sum_h W_h \bar{y}_{s_h} \tag{3.1}$$

where $\bar{y}_{s_h} = \frac{1}{n_h} \sum_{s_h} y_k$ for $h = 1, \ldots, H$.

We will now find expressions for the expected value and the variance of the estimator (3.1) with respect to the p^*M-distribution. Notice that the interviewer allocations give us the possibility to use the following relations

$$\bar{y}_{st} = \sum_h W_h \bar{y}_{s_h} = \sum_h W_h \frac{1}{I} \sum_i \bar{y}_{s_{hi}} = \frac{1}{I} \sum_i \sum_h W_h \bar{y}_{s_{hi}} = \frac{1}{I} \sum_i \bar{y}_{st,i} \tag{3.2}$$

The main result on expectation is the following:

Result 3.1: Under the p^*M-distribution, the expectation of the stratified sample mean estimator $\overline{y}_{st}$ is

$$E_{p^*M}\left(\overline{y}_{st}\right) = \overline{\mu}_U + B_b + B_\varepsilon \tag{3.3}$$

Thus, the bias of the estimator is

$$B_{p^*M}(\overline{y}_{st}) = E_{p^*M}(\overline{y}_{st}) - \overline{\mu}_U = B_b + B_\varepsilon \tag{3.4}$$

Result 3.1 can be obtained by using conditional probabilities

$$\begin{aligned} E_{p^*M}\left(\overline{y}_{st}\right) &= E_{p^*}\left[E_M\left(\sum_h W_h \frac{1}{n_h}\sum_i \sum\nolimits_{s_{hi}} y_k\right)\right] \\ &= E_{p^*}\left[\sum_h W_h \frac{1}{n_h}\sum\nolimits_{s_h}\left(\mu_k + B_b + B_\varepsilon\right)\right] \\ &= E_{p^*}\left(\overline{\mu}_{st} + B_b + B_\varepsilon\right) \\ &= \overline{\mu}_U + B \end{aligned}$$

because, $\overline{\mu}_{st}$ is unbiased for the true population mean $\overline{\mu}_U$. In the derivations above we utilize that $E_{p^*}(\overline{\mu}_{st}) = E_p(\overline{\mu}_{st})$ which can be easily justified.

In the following, we will sometimes use the shorter notation $B = B_b + B_\varepsilon$ for the bias.

The main result on the variance of $\overline{y}_{st}$ is:

Result 3.2: Under the p^*M-distribution, the variance of the stratified sample mean estimator $\overline{y}_{st}$ is

$$Var_{p^*M}\left(\overline{y}_{st}\right) = \frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h} + \sum_h W_h^2 \left(1 - \frac{n_h}{N_h}\right)\frac{S_{\mu U_h}^2}{n_h} \tag{3.5}$$

where

$$S_{\mu U_h}^2 = \frac{1}{N_h - 1}\sum\nolimits_{U_h}\left(\mu_k - \overline{\mu}_{U_h}\right)^2$$

Proof.

To obtain Result 3.2 we first note that the variance of the estimator $\bar{y}_{st}$ is seen as a combination of the variation from the measurement error model and the sampling design including the interviewer assignment. The variance can be written as follows

$$Var_{p^*M}\left(\bar{y}_{st}\right) = \underbrace{E_{p^*}\left[Var_M\left(\bar{y}_{st}\right)\right]}_{V_1} + \underbrace{Var_{p^*}\left[E_M\left(\bar{y}_{st}\right)\right]}_{V_2} = V_1 + V_2$$

and we consider the *measurement variance* V_1 and the *sampling variance* V_2 separately. For V_1 we have

$$\begin{aligned}
Var_M\left(\bar{y}_{st}\right) &= Var_M\left(\sum_h W_h \frac{1}{n_h}\sum_i \sum\nolimits_{s_{hi}} y_k\right) = \sum_i Var_M\left(\sum_h W_h \frac{1}{n_h}\sum\nolimits_{s_{hi}} y_k\right) \\
&= \sum_i \left\{\sum_h W_h^2 \frac{1}{n_h^2} Var_M\left[\sum\nolimits_{s_{hi}} y_k\right] + \sum_{h_1 \neq h_2}\sum W_{h_1} W_{h_2} \frac{1}{n_{h_1} n_{h_2}} Cov_M\left[\sum\nolimits_{s_{h_1 i}} y_k, \sum\nolimits_{s_{h_2 i}} y_k\right]\right\} \\
&= \sum_i \left\{\sum_h W_h^2 \frac{1}{n_h^2}\left(m_h^2 \sigma_b^2 + m_h \sigma_\varepsilon^2\right) + \sum_{h_1 \neq h_2}\sum W_{h_1} W_{h_2} \frac{1}{n_{h_1} n_{h_2}} m_{h_1} m_{h_2} \sigma_b^2\right\} \\
&= \sum_i \left\{\left(\sum_h W_h \frac{m_h}{n_h}\right)^2 \cdot \sigma_b^2 + \sum_h W_h^2 \frac{m_h}{n_h^2} \sigma_\varepsilon^2\right\} \\
&= \frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h}
\end{aligned}$$

where we have used the model property, independence between interviewers, and that $n_h = I \cdot m_h$ and $\sum\nolimits_h W_h = 1$. It now follows that, noticing that this expression is constant when taking the expectation over p^*, the measurement variance is

$$V_1 = \frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h}$$

The sampling variance component, V_2, is given by first finding the conditional *M*-expectation. From Result 3.1 we have that

$$E_M\left(\bar{y}_{st}\right) = \sum_h W_h \frac{1}{n_h}\sum\nolimits_{s_h} \mu_k + B_b + B_\varepsilon = \bar{\mu}_{st} + B$$

and because the sampling design is stratified simple random sampling ($\overline{\mu}_{st}$ is unaffected by the interviewer allocation) it follows that

$$V_2 = Var_{p*}\left(\overline{\mu}_{st}\right) = \sum_h W_h^2 \left(1 - \frac{n_h}{N_h}\right) \frac{S_{\mu U_h}^2}{n_h}$$

Combining the measurement variance and the sampling variance gives the result

$$Var_{p*M}\left(\overline{y}_{st}\right) = V_1 + V_2 \qquad \square$$

Under the actual measurement error model, the sampling variance, V_2, of $\overline{y}_{st}$ is equal to the variance of an estimator based on $\overline{\mu}_{st}$. The sampling variance is zero under complete enumeration. When there is no measurement variability, then $V_1 = 0$ ($\sigma_b^2 = 0$ and $\sigma_\varepsilon^2 = 0$) and V_2 is the only contribution to the variance.

4. Estimating the variance of $\overline{y}_{st}$

Two estimators of $Var_{p^*M}(\overline{y}_{st})$, given in Result 3.2, will be considered. The first is the traditional design based estimator (see for example Särndal, Swensson and Wretman 1992, p. 103)

$$\hat{V}_{st} = \sum_h W_h^2 \left(1 - \frac{n_h}{N_h}\right) \frac{S_{ys_h}^2}{n_h} \tag{4.1}$$

where

$$S_{ys_h}^2 = \frac{1}{n_h - 1} \sum\nolimits_{s_h} (y_k - \overline{y}_{s_h})^2$$

Recall that $\hat{V}_{st}$ would be the appropriate choice if there were no measurement errors. The second estimator is based on the means of the subsamples assigned to the different interviewers

$$\hat{V}_B = \frac{1}{I(I-1)} \sum_i \left(\overline{y}_{st,i} - \overline{y}_{st}\right)^2 \tag{4.2}$$

where $\overline{y}_{st,i}$ could be obtained from equation (3.2). This estimator is called the random group estimator in Wolter (1985) and will only be approximately unbiased when there are no measurement errors. It will be shown that both these two estimators are biased with respect to the p^*M-distribution. For simple random sampling, Lundquist and Wretman (2002) found that the traditional design based estimator has a negative bias and the estimator based on the means of the subsamples has a positive bias. In the main results given below it is shown that this will be the case also under stratified simple random sampling.

Result 4.1: Under the p^*M-distribution, the expectation of the variance estimator $\hat{V}_{st}$ is

$$E_{p^*M}\left(\hat{V}_{st}\right) = \sum_h W_h^2 \left(1 - \frac{n_h}{N_h}\right) \left[\frac{I-1}{I\left(n_h - 1\right)} \sigma_b^2 + \frac{\sigma_\varepsilon^2}{n_h} + \frac{S_{\mu U_h}^2}{n_h}\right]$$

$$= Var_{p^*M}\left(\overline{y}_{st}\right) - \sum_h W_h \left[\frac{N\left(n_h - 1\right) - \left(N_h - n_h\right)(I-1)}{NI\left(n_h - 1\right)}\right] \sigma_b^2 - \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{N_h}$$

The bias is zero when we do not have any measurement errors, but when measurement errors are present we will have a negative bias. The negative bias results in underestimation of the true variance.

Result 4.2: Under the p^*M-distribution, the expectation of the variance estimator $\hat{V}_B$ is

$$E_{p^*M}\left(\hat{V}_B\right) = \frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h} + \sum_h W_h^2 \frac{S^2_{\mu U_h}}{n_h} = Var_{p^*M}\left(\overline{y}_{st}\right) + \sum_h W_h^2 \frac{S^2_{\mu U_h}}{N_h}$$

The estimator based on the means of the subsamples, $\hat{V}_B$, has a positive bias that would remain even in a situation without measurement errors. The bias is introduced by the interviewer assignments, because the subsamples s_i ($i = 1, ..., I$) are dependent. However, the bias is small in most situations when N_h is large.

Proof of Result 4.1.

First note that the p^*M-expectation of $\hat{V}_{st}$ could be written

$$E_{p^*M}\left(\hat{V}_{st}\right) = \sum_h W_h^2 (1 - \frac{n_h}{N_h}) \frac{1}{n_h\left(n_h - 1\right)} E_{p^*M}\left[\sum\nolimits_{s_h} (y_k - \overline{y}_{s_h})^2\right]$$

Using the Auxiliary Result in the paper by Lundquist and Wretman (2002) for the subsamples s_h, we get

$$\frac{1}{(n_h - 1)} E_M\left[\sum\nolimits_{s_h} (y_k - \overline{y}_{s_h})^2\right] = \frac{m_h(I-1)}{n_h - 1}\sigma_b^2 + \sigma_\varepsilon^2 + \frac{1}{n_h - 1}\sum\nolimits_{s_h} (\mu_k - \overline{\mu}_{s_h})^2$$

The p^*M-expectation of $\hat{V}_{st}$ is then given by

$$E_{p^*}\left\{\sum_h W_h^2 (1 - \frac{n_h}{N_h}) \frac{1}{n_h}\left[\frac{m_h(I-1)}{n_h - 1}\sigma_b^2 + \sigma_\varepsilon^2 + \frac{1}{n_h - 1}\sum\nolimits_{s_h} (\mu_k - \overline{\mu}_{s_h})^2\right]\right\} =$$

$$= \sum_h W_h^2 (1 - \frac{n_h}{N_h}) \frac{1}{n_h}\left\{\frac{m_h(I-1)}{n_h - 1}\sigma_b^2 + \sigma_\varepsilon^2 + E_{p^*}\left[\frac{1}{n_h - 1}\sum\nolimits_{s_h} (\mu_k - \overline{\mu}_{s_h})^2\right]\right\}$$

$$= \sum_h W_h^2 (1 - \frac{n_h}{N_h})\left[\frac{I-1}{I(n_h - 1)}\sigma_b^2 + \frac{\sigma_\varepsilon^2}{n_h} + \frac{S^2_{\mu U_h}}{n_h}\right] \qquad \square$$

Proof of Result 4.2.

Starting with the model expectation we have from Result 3.1 and 3.2 that

$$\begin{aligned} E_M\left(\overline{y}_{st}^2\right) &= Var_M\left(\overline{y}_{st}\right) + \left[E_M\left(\overline{y}_{st}\right)\right]^2 \\ &= \frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h} + \left(\overline{\mu}_{st} + B\right)^2 \end{aligned} \tag{4.3}$$

and by simple algebra

$$\begin{aligned} E_M(\overline{y}_{st,i}^2) &= Var_M(\overline{y}_{st,i}) + \left[E_M(\overline{y}_{st,i})\right]^2 \\ &= \sigma_b^2 + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{m_h} + \left(\overline{\mu}_{st,i} + B\right)^2 \end{aligned} \tag{4.4}$$

Using (4.3) and (4.4) in the *M*-expectation of the sums of squares between subsamples we have

$$\begin{aligned} E_M\left[\sum_i \left(\overline{y}_{st,i} - \overline{y}_{st}\right)^2\right] &= E_M\left[\sum_i \overline{y}_{st,i}^2 - I\overline{y}_{st}^2\right] = \sum_i E_M(\overline{y}_{st,i}^2) - I \cdot E_M(\overline{y}_{st}^2) \\ &= \sum_i \left[\sigma_b^2 + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{m_h} + \left(\overline{\mu}_{st,i} + B\right)^2\right] - I\left[\frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h} + \left(\overline{\mu}_{st} + B\right)^2\right] \\ &= (I-1)\sigma_b^2 + (I-1)\sum_h W_h^2 \frac{\sigma_\varepsilon^2}{m_h} + \left[\sum_i \left(\overline{\mu}_{st,i} + B\right)^2 - I\left(\overline{\mu}_{st} + B\right)^2\right] \\ &= (I-1)\sigma_b^2 + (I-1)\sum_h W_h^2 \frac{\sigma_\varepsilon^2}{m_h} + \left[\sum_i \overline{\mu}_{st,i}^2 - I\overline{\mu}_{st}^2\right] \end{aligned} \tag{4.5}$$

To find the p*-expectations of equation (4.5) (i.e. of $\overline{\mu}_{st,i}^2$ and $\overline{\mu}_{st}^2$) we use the findings in Result 3.1 and 3.2 again (with simple algebra for $\overline{\mu}_{st,i}^2$)

$$E_{p^*}(\overline{\mu}_{st}^2) = Var_{p^*}(\overline{\mu}_{st}) + \left[E_{p^*}(\overline{\mu}_{st})\right]^2 = \sum_h W_h (1 - \frac{n_h}{N_h}) \frac{S_{\mu U_h}^2}{n_h} + \overline{\mu}_U^2 \tag{4.6}$$

and

$$E_{p^*}(\overline{\mu}_{st,i}^2) = Var_{p^*}(\overline{\mu}_{st,i}) + \left[E_{p^*}(\overline{\mu}_{st,i})\right]^2 = \sum_h W_h (1 - \frac{m_h}{N_h}) \frac{S_{\mu U_h}^2}{m_h} + \overline{\mu}_U^2 \tag{4.7}$$

Using (4.6) and (4.7) the p^*M-expectation of $\hat{V}_B$ is given by

$$
\begin{aligned}
E_{p^*}\left[E_M\left(\hat{V}_B\right)\right] &= \frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h} + \frac{I}{I(I-1)}\left\{\sum_h W_h^2\left(1-\frac{m_h}{N_h}\right)\frac{S_{\mu U_h}^2}{m_h} - W_h^2\left(1-\frac{n_h}{N_h}\right)\frac{S_{\mu U_h}^2}{n_h}\right\} \\
&= \frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h} + \sum_h W_h^2 \frac{S_{\mu U_h}^2}{n_h}
\end{aligned}
$$

□

In survey sampling a variance estimator with a small positive bias is usually preferred to one with a negative bias. The conclusion is that we prefer the variance estimator (4.2) to (4.1). Notice that the variance estimator $\hat{V}_B$ only is an alternative when the initial sample is divided into nonoverlapping subsamples.

However, the primary reason to use the interviewer allocation is that we believe that there is an interviewer effect present in the measured variable y_k. This setup, the three-stage process, will make it possible to estimate a potential interviewer influence. We will in the following sections focus on the estimation of the interviewer effect.

5. Estimating the interviewer variance and intra-interviewer correlation

To estimate the interviewer variance, σ_b^2, the estimator of the sum of squares within subsamples, $\hat{V}_W$, suggested for simple random sampling in Lundquist and Wretman (2002) has to be changed to match the stratified simple random sampling design and the accompanied interviewer allocation. We will in this section give two methods to estimate the interviewer variance. Both methods use findings from the earlier paper. The first method imitates the procedure in the earlier paper, but with necessary changes for the p*-distribution in the sums of squares. The second estimator of the interviewer variance is given as the unweighted mean of H interviewer variances, where for each subpopulation U_h ($h = 1, ..., H$) an interviewer variance estimator is produced similar to the estimator given in the earlier paper.

Another measure of interviewer influence is the intra-interviewer correlation, which is defined as

$$\rho_w = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_\varepsilon^2 + S_{\mu U}^2} = \frac{\sigma_b^2}{\sigma_{tot}^2} \tag{5.1}$$

This correlation can be interpreted as the correlation of the measurement made on two elements that are drawn at random from the population and then interviewed by the same interviewer. For each method of estimating σ_b^2, an approximate estimator of the intra-interviewer correlation, ρ_w, is given. However, it will be seen that the estimator of ρ_w associated with the first method is to be preferred.

Method 1

An estimator of the interviewer variance σ_b^2 is

$$\hat{\sigma}_{b,ST1}^2 = I \cdot \hat{V}_B - \hat{V}_W^a \tag{5.2}$$

where

$$\hat{V}_W^a = \frac{1}{I}\sum_i\sum_h W_h^2 \frac{1}{m_h\left(m_h - 1\right)}\sum_{s_{hi}}\left(y_k - \overline{y}_{s_{hi}}\right)^2 \tag{5.3}$$

and $\hat{V}_B$ is given in equation (4.2).

The expectation of the variance estimator $\hat{\sigma}_{b,ST1}^2$ is given in the following result.

Result 5.1: Under the p^*M-distribution, the expectation of the interviewer variance estimator $\hat{\sigma}_{b,ST1}^2$ is

$$E_{p^*M}\left(\hat{\sigma}_{b,ST1}^2\right) = \sigma_b^2$$

Proof of Result 5.1.

To prove that $\hat{\sigma}_{b,ST1}^2$ is unbiased we have to find the p^*M-expectation of the two components of the right hand side in equation (5.2). From Result 4.2 we have that

$$E_{p^*M}\left(I \cdot \hat{V}_B\right) = I\left[\frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{n_h} + \sum_h W_h^2 \frac{S_{\mu U_h}^2}{n_h}\right] = \sigma_b^2 + \sum_h W_h^2 \frac{\sigma_\varepsilon^2}{m_h} + \sum_h W_h^2 \frac{S_{\mu U_h}^2}{m_h}$$

and to prove that $\hat{\sigma}_{b,ST1}^2$ is unbiased it is sufficient to show that

$$E_{p^*M}\left(\hat{V}_W^a\right) = E_{p^*M}\left(I \cdot \hat{V}_B\right) - E_{p^*M}\left(\hat{\sigma}_{b,ST1}^2\right) = \sum_h W_h^2 \frac{1}{m_h}\left(\sigma_\varepsilon^2 + S_{\mu U_h}^2\right) \tag{5.4}$$

Starting with the M-expectation of $\sum_{s_{hi}}\left(y_k - \overline{y}_{s_{hi}}\right)^2$ and using $B = B_b + B_\varepsilon$ we have

$$\begin{aligned}
E_M\left[\sum_{s_{hi}}\left(y_k - \overline{y}_{s_{hi}}\right)^2\right] &= E_M\left[\sum_{s_{hi}} y_k^2 - m_h\overline{y}_{s_{hi}}^2\right] \\
&= m_h\left(\sigma_b^2 + \sigma_\varepsilon^2\right) + \sum_{s_{hi}}\left(\mu_k + B\right)^2 - m_h\left[\sigma_b^2 + \frac{\sigma_\varepsilon^2}{m_h} + \left(\overline{\mu}_{s_{hi}} + B\right)^2\right] \\
&= \left(m_h - 1\right)\sigma_\varepsilon^2 + \sum_{s_{hi}}\mu_k^2 - m_h\overline{\mu}_{s_{hi}}^2 \\
&= \left(m_h - 1\right)\sigma_\varepsilon^2 + \sum_{s_{hi}}\left(\mu_k - \overline{\mu}_{s_{hi}}\right)^2
\end{aligned}$$

Thus,

$$E_{p^*M}\left[\sum_{s_{hi}}\left(y_k-\overline{y}_{s_{hi}}\right)^2\right]=E_{p^*}\left[E_M\left(\sum_{s_{hi}}(y_k-\overline{y}_{s_{hi}})^2\right)\right]$$

$$=(m_h-1)\sigma_\varepsilon^2+(m_h-1)E_{p^*}\left[\frac{1}{m_h-1}\sum_{s_{hi}}(\mu_k-\overline{\mu}_{s_{hi}})^2\right]$$

Since each stratified subsample s_{hi} is a simple random sample from the population of N_h elements, the p*-expectation for the sum of squares within subsamples is given by

$$E_{p^*}\left[\frac{1}{m_h-1}\sum_{s_{hi}}(\mu_k-\overline{\mu}_{s_{hi}})^2\right]=S^2_{\mu U_h}$$

Using the derivations above, we have that

$$\begin{aligned}E_{p^*M}\left(\hat{V}_W^a\right)&=\frac{1}{I}\sum_i\sum_h W_h^2\frac{1}{m_h(m_h-1)}E_{p^*M}\left[\sum_{s_{hi}}\left(y_k-\overline{y}_{s_{hi}}\right)^2\right]\\&=\frac{1}{I}\sum_i\sum_h W_h^2\frac{1}{m_h(m_h-1)}\left[(m_h-1)\left(\sigma_\varepsilon^2+S^2_{\mu U_h}\right)\right]\\&=\sum_h W_h^2\frac{1}{m_h}\left(\sigma_\varepsilon^2+S^2_{\mu U_h}\right)\end{aligned}$$

which proves Result 5.1

$$E_{p^*M}\left(\hat{\sigma}^2_{b,ST1}\right)=E_{p^*M}\left(I\cdot\hat{V}_B\right)-E_{p^*M}\left(\hat{V}_W^a\right)=\sigma_b^2$$

□

We now turn to the intra-interviewer correlation in equation (5.1). The correlation ρ_w is the ratio between two variance expressions. The nominator is the interviewer variance that is estimated by $\hat{\sigma}^2_{b,ST1}$ in equation (5.2) and we have to find an estimator of the denominator σ^2_{tot}. We will in the following show that it is possible by a linear combination of equation (4.2) and (5.3) to find an approximately p^*M-unbiased estimator

$$\hat{\sigma}^2_{tot,st}=\hat{\sigma}^2_{b,ST1}+\hat{V}_B^++\hat{V}_W^+ \tag{5.5}$$

where

$$\hat{V}_B^+=\sum_h W_h\left(\overline{y}_{s_h}-\overline{y}_{st}\right)^2$$

and

$$\hat{V}_W^+ = \frac{1}{I}\sum_i\sum_h\left(W_h - \frac{W_h - W_h^2}{n_h}\right)\frac{1}{m_h-1}\sum_{s_{hi}}\left(y_k - \overline{y}_{s_{hi}}\right)^2$$

The intra-interviewer correlation estimator to be considered (for large populations, N, subpopulations, N_h, and when the sampling fraction $n_h / N_h \doteq 0$ in all strata) is the ratio estimator

$$\hat{\rho}_{w,ST1} = \frac{\hat{\sigma}^2_{b,ST1}}{\hat{\sigma}^2_{tot,st}} \tag{5.6}$$

The approximate p^*M-unbiasedness of the estimator $\hat{\sigma}^2_{tot,st}$ is stated in the following result.

Result 5.2: With respect to the p^*M-distribution

$$E_{p^*M}\left(\hat{\sigma}^2_{tot,st}\right) \doteq \sigma^2_b + \sigma^2_\varepsilon + S^2_{\mu U} = \sigma^2_{tot}$$

for large populations, N, subpopulations, N_h, and when the sampling fraction $n_h / N_h \doteq 0$ in all strata.

Proof of Result 5.2.

We first notice that

$$E_{p^*M}(\hat{\sigma}^2_{tot,st}) = E_{p^*M}(\hat{\sigma}^2_{b,ST1} + \hat{V}_B^+ + \hat{V}_W^+) = \sigma^2_b + E_{p^*M}(\hat{V}_B^+ + \hat{V}_W^+)$$

and what we actually have to prove is that

$$E_{p^*M}(\hat{V}_B^+ + \hat{V}_W^+) \doteq \sigma^2_\varepsilon + S^2_{\mu U} \tag{5.7}$$

To prove equation (5.7) (i.e. Result 5.2) we use the population sum of squares, which can be expressed in different ways

$$\sum_U\left(\mu_k - \overline{\mu}_U\right)^2 = \sum_h\sum_{U_h}\left(\mu_k - \overline{\mu}_{U_h}\right)^2 + \sum_h N_h\left(\overline{\mu}_{U_h} - \overline{\mu}_U\right)^2$$
$$= N\left[\sum_h W_h\frac{1}{N_h}\sum_{U_h}\left(\mu_k - \overline{\mu}_{U_h}\right)^2 + \sum_h W_h\left(\overline{\mu}_{U_h} - \overline{\mu}_U\right)^2\right]$$

For large populations U and subpopulations U_h, using that $1/N \doteq 1/(N-1)$ and $1/N_h \doteq 1/(N_h-1)$ we approximate the second term by

$$\sum_h W_h \left(\bar{\mu}_{U_h} - \bar{\mu}_U\right)^2 \doteq S^2_{\mu U} - \sum_h W_h S^2_{\mu U_h} \tag{5.8}$$

It is now possible to find an approximation of the expectation in (5.7). We inspect $\hat{V}_B^+$ under the $p*M$-distribution, and we have by Result 3.1 and 3.2 that

$$\begin{aligned}
E_{p^*M}\left(\hat{V}_B^+\right) &= E_{p^*M}\left[\sum_h W_h\left(\bar{y}_{s_h} - \bar{y}_{st}\right)^2\right] = E_{p^*M}\left[\sum_h W_h \bar{y}^2_{s_h} - \bar{y}^2_{st}\right] \\
&= \sum_h W_h\left[Var_{p^*M}(\bar{y}_{s_h}) + E_{p^*M}(\bar{y}_{s_h})^2\right] - \left[Var_{p^*M}(\bar{y}_{st}) + E_{p^*M}(\bar{y}_{st})^2\right] \\
&= \sum_h W_h\left\{\left[\frac{\sigma_b^2}{I} + \frac{\sigma_\varepsilon^2}{n_h} + \left(1 - \frac{n_h}{N_h}\right)\frac{S^2_{\mu U_h}}{n_h}\right] + \left[\bar{\mu}_{U_h} + B\right]^2\right\} \\
&\quad - \left\{\left[\frac{\sigma_b^2}{I} + \sum_h W_h^2 \frac{1}{n_h}\left(\sigma_\varepsilon^2 + \left(1 - \frac{n_h}{N_h}\right) S^2_{\mu U_h}\right)\right] + \left[\bar{\mu}_U + B\right]^2\right\} \\
&= \sum_h \left(W_h - W_h^2\right)\frac{1}{n_h}\left[\sigma_\varepsilon^2 + \left(1 - \frac{n_h}{N_h}\right) S^2_{\mu U_h}\right] + \sum_h W_h\left(\bar{\mu}_{U_h} - \bar{\mu}_U\right)^2
\end{aligned}$$

Using equation (5.8) and assuming that the sample in every stratum, n_h, is small compared to the subpopulation, N_h, we could approximate $E_{p^*M}\left(\hat{V}_B^+\right)$ with

$$E_{p^*M}\left(\hat{V}_B^+\right) \doteq \sum_h \left(W_h - W_h^2\right)\frac{1}{n_h}\left(\sigma_\varepsilon^2 + S^2_{\mu U_h}\right) + S^2_{\mu U} - \sum_h W_h S^2_{\mu U_h} \tag{5.9}$$

The first sum in equation (5.9) bears a resemblance to the $p*M$-expectation of $\hat{V}_W^a$. Using equation (5.4) the $p*M$-expectation of $\hat{V}_W^+$ is

$$E_{p^*M}\left(\hat{V}_W^+\right) = \sum_h \left(W_h - \frac{W_h - W_h^2}{n_h}\right)\left(\sigma_\varepsilon^2 + S^2_{\mu U_h}\right) \tag{5.10}$$

Result 5.2 now follows from combining (5.9) and (5.10),

$$\begin{aligned}
E_{p^*M}(\hat{V}_B^+ + \hat{V}_W^+) &\doteq \sum_h \left(W_h - W_h^2\right)\frac{1}{n_h}\left(\sigma_\varepsilon^2 + S^2_{\mu U_h}\right) + S^2_{\mu U} - \sum_h W_h S^2_{\mu U_h} + \sum_h \left(W_h - \frac{W_h - W_h^2}{n_h}\right)\left(\sigma_\varepsilon^2 + S^2_{\mu U_h}\right) \\
&= \sigma_\varepsilon^2 + S^2_{\mu U}
\end{aligned}$$

□

Method 2

To find an estimator of the interviewer variance, σ_b^2, we notice that the stratified simple random sampling design makes it possible for us to use the result in Lundquist and Wretman (2002) on each subpopulation U_h. We then have

$$\hat{\sigma}_{b,h}^2 = I\left(\hat{V}_{B,h} - \frac{\hat{V}_{W,h}}{n_h}\right) \qquad \text{for } h = 1,\ldots,H. \tag{5.11}$$

where $\hat{V}_{B,h}$ and $\hat{V}_{W,h}$ are estimators defined for the subpopulations U_h. As in the earlier paper we have for each stratum h ($h = 1,\ldots,H$.)

$$\hat{V}_{B,h} = \frac{1}{I(I-1)}\sum_i \left(\overline{y}_{s_{hi}} - \overline{y}_{s_h}\right)^2 \qquad \text{and} \qquad \hat{V}_{W,h} = \frac{1}{I\left(m_h - 1\right)}\sum_i \sum_{s_{hi}} \left(y_k - \overline{y}_{s_{hi}}\right)^2$$

It could be shown using the results in Lundquist and Wretman (2002) that the estimators expectation with respect to the p^*M-distribution for s_h are

$$E_{p^*M}\left(\hat{V}_{B,h}\right) = \frac{\sigma_b^2}{I} + \frac{\sigma_\varepsilon^2 + S_{\mu U_h}^2}{n_h} \qquad \text{and} \qquad E_{p^*M}\left(\hat{V}_{W,h}\right) = \sigma_\varepsilon^2 + S_{\mu U_h}^2$$

and we then have that

$$E_{p^*M}\left(\hat{\sigma}_{b,h}^2\right) = \sigma_b^2 \qquad \text{for } h = 1,\ldots,H.$$

The model assumptions imply that each stratum produces an estimator of the interviewer variance. An estimator that uses the whole sample would be more stable and one choice is the mean of the H estimators in formula (5.11). The estimator of σ_b^2 for *Method 2* (which also is p^*M-unbiased) that uses the whole stratified simple random sample is:

$$\hat{\sigma}_{b,ST2}^2 = \frac{1}{H}\sum_h \hat{\sigma}_{b,h}^2 \tag{5.12}$$

Note that this simple way of estimating the interviewer variance is not practicable when estimating the intra-interviewer correlation. The H strata would produce different estimators of σ^2_{tot} given by

$$\hat{\sigma}^2_{tot,h} = I \cdot \hat{V}_{B,h} + \frac{m_h - 1}{m_h} \hat{V}_{W,h} \qquad \text{for } h = 1,\ldots,H. \tag{5.13}$$

If we now take the $p*M$-expectation on (5.13) we would get

$$E_{p^*M}\left(\hat{\sigma}^2_{tot,h}\right) = \sigma^2_b + \sigma^2_\varepsilon + S^2_{\mu U_h}$$

and the intra-interviewer correlation is estimated differently for the different subpopulations

$$\hat{\rho}_{w,h} = \frac{\hat{\sigma}^2_{b,h}}{\hat{\sigma}^2_{tot,h}} \qquad \text{for } h = 1,\ldots,H. \tag{5.14}$$

Equation (5.14) will produce the intra-interviewer correlation in the subpopulation U_h. The estimator of ρ_w for *Method 2* is also based on averaging over the H different strata (as for the interviewer variance (5.12)):

$$\hat{\rho}_{w,ST2} = \frac{1}{H} \sum_h \hat{\rho}_{w,h} \tag{5.15}$$

This estimator is an average of the intra-interviewer correlation estimators in the different subpopulations, which is not necessarily equal to the intra-interviewer correlation in the whole population unless the subpopulation variances $S^2_{\mu U_h}$ are all equal.

6. Simulation study

This section describes a simulation study with the purpose to compare the interviewer estimators for Method 1 and 2 by simulations. Two artificial populations are considered; one population is symmetric and the other is skew. The skew population is meant to be more realistic in some situations, such as when dealing with economic data. Each population consists of eight strata and only one sample size is used, with equal sample size in all strata.

In Lundquist and Wretman (2002) it was shown, for simple random sampling, that the accuracy of the estimated interviewer variance and estimated intra-interviewer correlation varied for a fix sample size when the subsample sizes and number of interviewers varied. Eight different combinations of m_h and I for n_h are created to investigate if this also holds for the stratified simple random sampling.

The simulation study is done in the following way.

- Two artificial finite populations of the size $N = 100{,}000$ are created by generating 100,000 independent random numbers, μ_k:
    - **Population 1**, drawn from a standard normal distribution, N(0,1)

  and

    - **Population 2**, drawn from a gamma distribution, Gamma(0.5, 1) with expectation 0.5 and variance 0.5.

  For each population there is an auxiliary variable, x_k ($k = 1, \ldots, 100{,}000$), which is used in the stratification. The auxiliary variables are correlated with the "true values" and the correlation, r, is fixed to 0.4 in both populations. The correlated auxiliary variable x_k is achieved by using the following relation: Let μ_k and z_k be i.i.d. and calculate $x_k = r\mu_k + (1 - r^2)^{0.5} z_k$. This means that we in Population 1 first generate two independent variables μ_k and z_k for $k = 1, \ldots, 100{,}000$ from the standard normal distribution and then use the relation to create the auxiliary values x_k. In Population 2 the procedure is similar but with two independent variables μ_k and z_k from

Gamma(0.5, 1). For the artificial population where μ_k are realized from N(0,1) we found the following covariance matrix:

$$\begin{pmatrix} S^2_{\mu U} & S_{\mu x U} \\ S_{\mu x U} & S^2_{xU} \end{pmatrix} = \begin{pmatrix} 0.99284498 & 0.39680338 \\ 0.39680338 & 0.99976496 \end{pmatrix}$$

And for the skew population where μ_k are realized from Gamma(0.5, 1) we found:

$$\begin{pmatrix} S^2_{\mu U} & S_{\mu x U} \\ S_{\mu x U} & S^2_{xU} \end{pmatrix} = \begin{pmatrix} 0.49138668 & 0.19853051 \\ 0.19853051 & 0.50150039 \end{pmatrix}$$

To create the stratification the populations are sorted by the auxiliary variables and divided into eight subpopulations. The following sizes on the subpopulations are chosen: N_1 = 5,000, N_2 = 10,000, N_3 = 15,000, N_4 = 20,000, N_5 = 20,000, N_6 = 15,000, N_7 = 10,000 and N_8 = 5,000. (The 5,000 lowest values in the first subpopulation and the successive 10,000 values in the second subpopulation etc.) Means and variances for the eight subpolulations are presented in Table A.1 and the distributions for the different subpopulations are shown in Figure A.1 and Figure A.2 in Appendix A.

- From the two finite populations 5,000 samples of the same size, n = 2,400, are drawn by stratified simple random sampling without replacement. Each sample is replaced before the next sample is drawn, so that all samples are independent. The sample size in every stratum is fixed to n_h = 300 (h = 1,…,8).

- For each sample, the sampled elements are assigned at random to I fictitious interviewers, in conformity with assumptions made in Section 2, so that every interviewer gets the same number, $m_h = n_h/I$, of respondents (h = 1,…,8). Eight different values of I are used (I =150, 75, 60, 50, 30, 20, 15 and 10).

- For each sample, with given I, and given interviewer assignments, interviewer effects, b_i (i = 1,…, I) are obtained by generating I independent random numbers from an $N(0,\sigma_b^2)$ distribution. Different values of σ_b^2 were chosen as described below.

- For each sample, response errors, ε_k ($k \in s$), are obtained by generating n independent random numbers from an N(0, σ_ε^2) distribution. Different values of σ_ε^2 were chosen as described below.

- The following combinations of values were used for σ_b^2 and σ_ε^2:

		$\mu_k \sim$ N(0,1)	$\mu_k \sim$ Gamma(0.5,1)
	$\sigma_b^2 / \sigma_\varepsilon^2$	σ_b^2	σ_b^2
$\rho_w = 0.01$	0.1	0.01115556	0.00552120
	1.0	0.01013107	0.00501415
	10.0	0.01003888	0.00496852
$\rho_w = 0.02$	0.1	0.02545756	0.01259966
	1.0	0.02068427	0.01023722
	10.0	0.02030358	0.01004881
$\rho_w = 0.04$	0.1	0.07091750	0.03509905
	1.0	0.04316717	0.02136464
	10.0	0.04154163	0.02056011

  These combinations of values were chosen in order to illustrate various relations between the two variances involved, while at the same time, together with the actual values of S_μ^2, giving a constant intra-interviewer correlation of ρ_w.

- Finally, for each sample s we obtain values y_k ($= \mu_k + b_i + \varepsilon_k$) for all $k \in s$. Using these values, we then calculate, for each sample, $\overline{y}_{st}$ (3.1), $\hat{\sigma}_{b,ST1}^2$ (5.2), $\hat{\rho}_{w,ST1}$ (5.6), $\hat{\sigma}_{b,ST2}^2$ (5.12) and $\hat{\rho}_{w,ST2}$ (5.15).

The measures considered are the expected values and the standard errors. To obtain them we use the following approximate results from the simulations. Let t_s denote some sample quantity calculated from observed data in a sample s, and let t_{sj} be the realized value of t_s for the jth simulation sample ($j = 1, \ldots, 5{,}000$). (For example, if t_s is the stratified sample mean $\overline{y}_{st}$, then $t_{sj} = \overline{y}_{st,j}$ is the observed mean in the jth simulation sample.) The simulation mean

$$\overline{t}_s = \frac{1}{5,000} \sum_{j=1}^{5,000} t_{sj}$$

is the simulation estimate of the expected value $E_{p*M}(t_s)$. We denote this estimated expectation with E_{sim}. The simulation variance

$$S_{t_s}^2 = \frac{1}{5,000-1}\sum_{j=1}^{5,000}(t_{sj} - \overline{t}_s)^2$$

is the simulation estimate of the variance $Var_{p*M}(t_s)$. We use the square root of this estimated variance $V_{sim}^{1/2}$, i.e. the estimate of the standard error of the estimator t_s, to compare the accuracy of the methods for the eight different combinations of m_h and I.

Let us first look at the simulation results for the sample mean of the true values,

$$\overline{\mu}_{st} = \sum_{h=1}^{H} W_h \overline{\mu}_{s_h}$$

We already know from elementary sampling theory that (because the sampling design is stratified simple random sampling without replacement):

$$E_p(\overline{\mu}_{st}) = \overline{\mu}_U = \begin{cases} 0.00430063 & \text{Population 1} \\ 0.49647738 & \text{Population 2} \end{cases}$$

$$\left[Var_p(\overline{\mu}_{st})\right]^{1/2} = \left[\sum_{h=1}^{H} W_h^2 (1 - \frac{n_h}{N_h}) \frac{S_{\mu U_h}^2}{n_h}\right]^{1/2} = \begin{cases} 0.02028429 & \text{Population 1} \\ 0.01216316 & \text{Population 2} \end{cases}$$

Since the true values are fixed constants, the measurement error model is not considered here.

The results of the simulation study are given in Table 6.1 below. The simulation estimates are seen to be rather close to the exact values of the quantities that they are supposed to approximate.

Table 6.1. Results from the simulation study on sample mean of the true values. 5,000 repeated samples.

		$\mu_k \sim$ N(0,1)	$\mu_k \sim$ Gamma(0.5,1)
$E_p(\overline{\mu}_{st})$	Exact value	0.00430063	0.49647738
	Simulation estimate	0.00410500	0.49636391
$\left[Var_p(\overline{\mu}_{st})\right]^{1/2}$	Exact value	0.02028429	0.01216316
	Simulation estimate	0.01993082	0.01206599

Simulation mean and simulation standard error for each of $\overline{y}_{st}$, $\hat{\sigma}^2_{b,ST1}$, $\hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ are given in Tables B.1 – B.18 in Appendix B, for the two populations; for various values of I (and $m_h = n_h/I$); for ρ_w = 0.01, 0.02 and 0.04 and for $\sigma^2_b / \sigma^2_\varepsilon$ = 0.1, 1, and 10. Some comments on these simulation results follow here.

- We note that the simulated standard error of $\overline{y}_{st}$ increases as the number of interviewers, I, decreases (that is when the subsample, m_h, increases). This is what is to be expected from equation (3.5).

- It seems that $\hat{\sigma}^2_{b,ST1}$ is more efficient than $\hat{\sigma}^2_{b,ST2}$ in this study. It is always possible to find a combination (of I and m_h) for $\hat{\sigma}^2_{b,ST1}$ with a lower standard error. The lowest standard error is not given for the same combination for the two interviewer variance estimators, $\hat{\sigma}^2_{b,ST2}$ always need a larger subsample, m_h, than $\hat{\sigma}^2_{b,ST1}$. This is true for the chosen populations and the studied values of the intra-interviewer combinations.

- The relation between the interviewer variance σ^2_b and the response error variance σ^2_ε is not of great importance in this simulation example. If the interviewer variance is a tenth of, equal to, or ten times the elementary error variance does not affect the results very much.

- If the population is skew or symmetric does not affect the results of $\hat{\sigma}^2_{b,ST1}$ as much as $\hat{\sigma}^2_{b,ST2}$. There are small differences for $\hat{\sigma}^2_{b,ST1}$ and the combination (of I and m_h) with the lowest standard error in the skew population is also among the ones with the lowest standard error for the symmetric population. There is a minor indication that we need a larger subsample size, m_h. For $\hat{\sigma}^2_{b,ST2}$ we always need a larger subsample size for the skew population for ρ_w = 0.02 and 0.04.

- Different values of ρ_w produce different minima of the standard error of the interviewer variance estimators. We note that a smaller subsample size, m_h, is needed when the intra-interviewer correlation is large.

- The findings for $\hat{\rho}_{w,ST1}$ in the simulation study are similar to what we found for $\hat{\sigma}^2_{b,ST1}$.

- $\hat{\rho}_{w,ST2}$ is not recommended as an estimator of the intra-interviewer correlation in these situations.

7. Discussion

This is the second paper where we have used the theoretical framework for interviewer variance studies for continuous data. We have extended the theory to be practicable for stratified simple random sampling. The basic concepts under the stratified sample design have made it possible to create interviewer effect estimators. Two different interviewer variance estimators have been investigated under the simple measurement error model. The first $\hat{\sigma}^2_{b,ST1}$ is to be preferred to $\hat{\sigma}^2_{b,ST2}$ according to the simulation study. Our judgment is based on that $\hat{\sigma}^2_{b,ST1}$ always has a lower standard error than $\hat{\sigma}^2_{b,ST2}$ for all combinations of I and m. The estimator is also rather stable in that sense that the minimum values for the combinations of number of interviewers and sizes of the subsample is not affected when the underlying population departs from a symmetric distribution. There may however exist situations where $\sigma^2_{b,ST2}$ works well. The results are based on one stratified sample design from two artificial populations. In a real experiment the number of strata will be different and maybe also the sample design within strata.

The simulations also indicated that it is possible to use $\hat{\rho}_{w,ST1}$ as an estimator of the intra-interviewer correlation ρ_w. Some care is needed in a real situation when estimating the intra-interviewer correlation. However, the approximations (for large populations, N, and subpopulations, N_h, and when the sampling fraction $n_h / N_h \doteq 0$ in all strata) used for the denominator $\hat{\sigma}^2_{tot,st}$ work well in this situation where for example $n_1/N_1 = 300/5{,}000$.

We use the simple measurement error model $y_k = \mu_k + b_i + \varepsilon_k$ for all $k \in s$. If on the other hand we would have additional information, about the interviewer effects or the response errors, another model could be preferred. The basic idea is that a survey is going to take place, and we have some doubts about some of the questions in the survey, but no extra information about the error structure. To investigate if there are interviewer effects we then use a simple model.

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Appendix A

Table A.1 The populations and eight subpopulations sorted according to the auxiliary variables.

	μ_k from N(0,1)	μ_k from Gamma(0.5,1)
$N = 100{,}000$	$\overline{\mu}_U = 0.00430063$	$\overline{\mu}_U = 0.49647738$
	$S^2_{\mu U} = 0.99284498$	$S^2_{\mu U} = 0.49138668$
$N_1 = 5{,}000$	$\overline{\mu}_{U_1} = -0.82670502$	$\overline{\mu}_{U_1} = 0.01903444$
	$S^2_{\mu U_1} = 0.84202525$	$S^2_{\mu U_1} = 0.00037749$
$N_2 = 10{,}000$	$\overline{\mu}_{U_2} = -0.49948376$	$\overline{\mu}_{U_2} = 0.07834953$
	$S^2_{\mu U_2} = 0.83327089$	$S^2_{\mu U_2} = 0.00417544$
$N_3 = 15{,}000$	$\overline{\mu}_{U_3} = -0.30194349$	$\overline{\mu}_{U_3} = 0.18513546$
	$S^2_{\mu U_3} = 0.83144030$	$S^2_{\mu U_3} = 0.02168764$
$N_4 = 20{,}000$	$\overline{\mu}_{U_4} = -0.09757170$	$\overline{\mu}_{U_4} = 0.35293309$
	$S^2_{\mu U_4} = 0.83593094$	$S^2_{\mu U_4} = 0.08363140$
$N_5 = 20{,}000$	$\overline{\mu}_{U_5} = 0.10855032$	$\overline{\mu}_{U_5} = 0.57214530$
	$S^2_{\mu U_5} = 0.84132404$	$S^2_{\mu U_5} = 0.25642728$
$N_6 = 15{,}000$	$\overline{\mu}_{U_6} = 0.30432987$	$\overline{\mu}_{U_6} = 0.81992360$
	$S^2_{\mu U_6} = 0.84728370$	$S^2_{\mu U_6} = 0.62592751$
$N_7 = 10{,}000$	$\overline{\mu}_{U_7} = 0.52208466$	$\overline{\mu}_{U_7} = 0.99418949$
	$S^2_{\mu U_7} = 0.84543291$	$S^2_{\mu U_7} = 1.2517950$
$N_8 = 5{,}000$	$\overline{\mu}_{U_8} = 0.81683091$	$\overline{\mu}_{U_8} = 1.0499443$
	$S^2_{\mu U_8} = 0.88246143$	$S^2_{\mu U_8} = 1.9237204$

Figure A.1 Subpopulations, $h = 1,\ldots,8$, for the first finite population N(0,1)

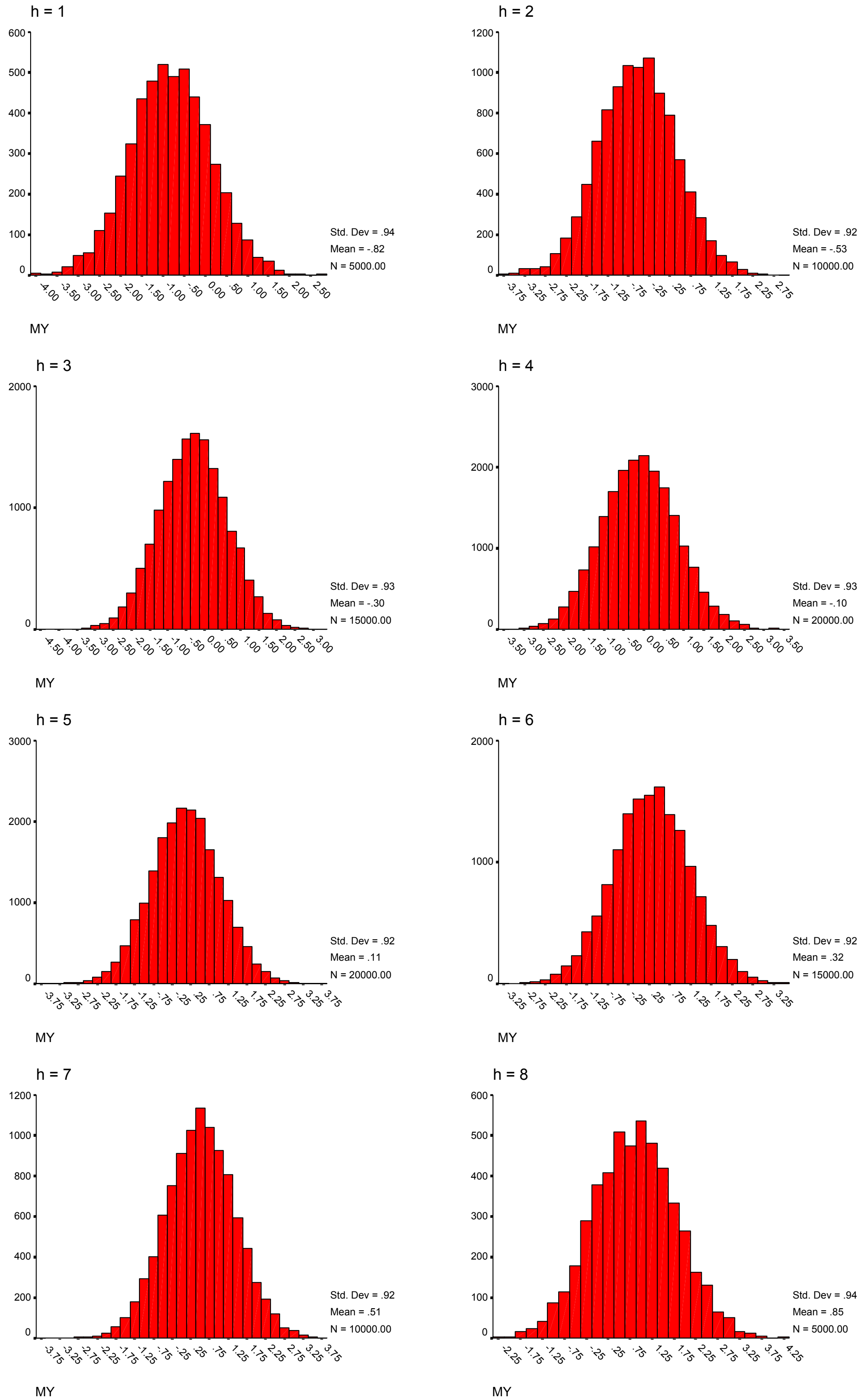
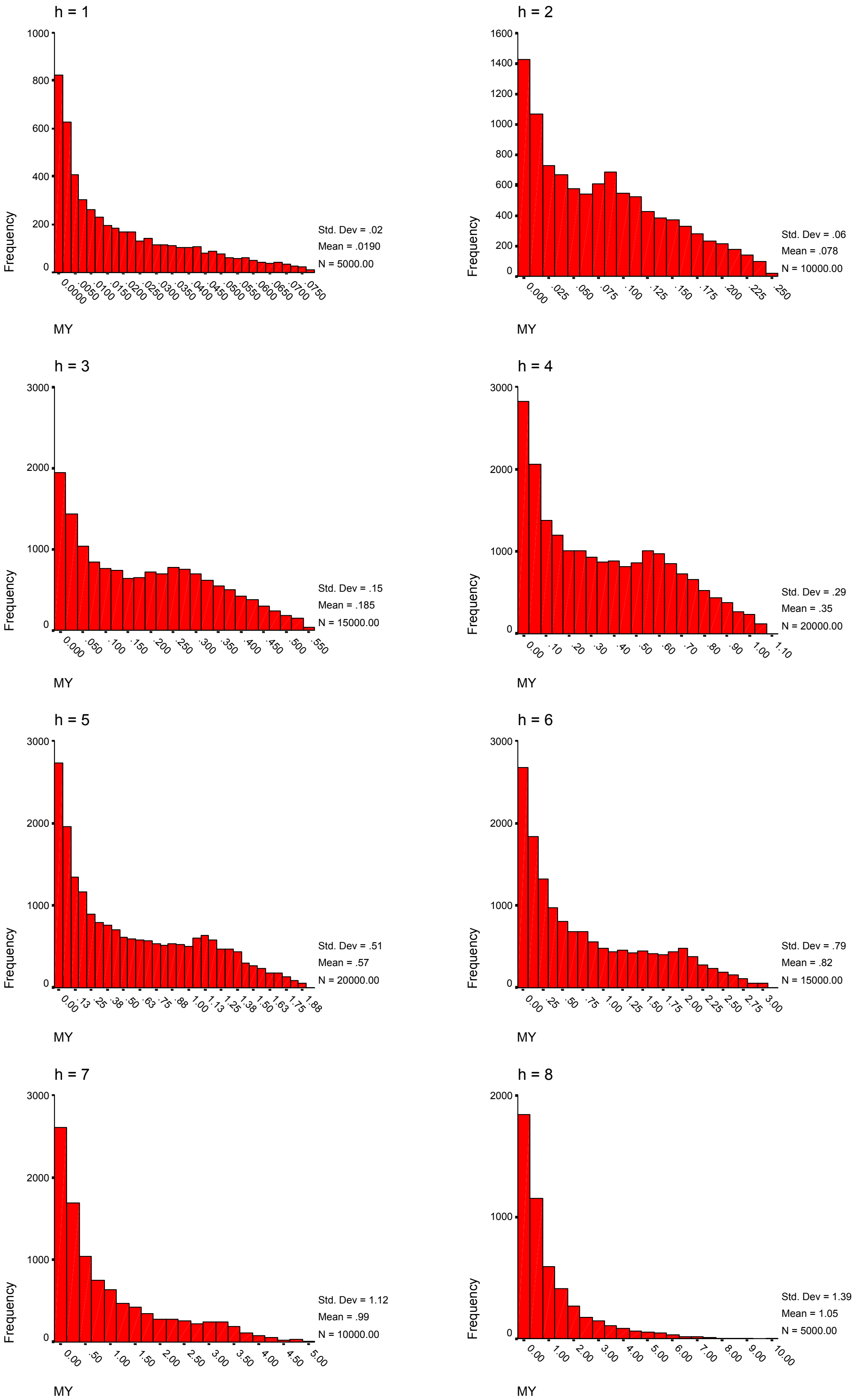


Figure A.2 Subpopulations, $h = 1,\ldots,8$, for the second finite population Gamma(0.5,1)



Appendix B

Table B.1. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ N(0,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 0.1, and ρ_w = 0.01. (σ_b^2 =0.011155562)

I =	150	75	60	50	30	20	15	10
m_h =	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	0.40	0.44	0.41	0.44	0.42	0.42	0.44	0.40
$V_{sim}^{1/2} \times 10^2$	2.29	2.42	2.51	2.62	2.84	3.20	3.43	3.98
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	1.10	1.12	1.12	1.12	1.12	1.11	1.12	1.12
$V_{sim}^{1/2} \times 10^2$	1.03	0.79	0.74	0.71	0.69	0.68	0.69	0.74
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	0.99	1.01	1.00	1.01	1.00	0.99	1.00	1.00
$V_{sim}^{1/2} \times 10^2$	0.93	0.71	0.66	0.64	0.62	0.60	0.61	0.65
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	1.12	1.10	1.10	1.10	1.11	1.08	1.11	1.11
$V_{sim}^{1/2} \times 10^2$	2.81	1.67	1.45	1.34	1.07	0.94	0.89	0.86
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	1.15	1.13	1.13	1.12	1.13	1.11	1.13	1.14
$V_{sim}^{1/2} \times 10^2$	2.90	1.72	1.50	1.38	1.10	0.97	0.91	0.88

Table B.2. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ N(0,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 1, and ρ_w = 0.01. (σ_b^2 =0.010131071)

I =	150	75	60	50	30	20	15	10
m_h =	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	0.40	0.44	0.41	0.44	0.42	0.42	0.44	0.41
$V_{sim}^{1/2} \times 10^2$	2.17	2.29	2.38	2.47	2.70	3.03	3.26	3.78
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	1.00	1.02	1.01	1.02	1.02	1.00	1.02	1.01
$V_{sim}^{1/2} \times 10^2$	0.92	0.70	0.67	0.64	0.62	0.61	0.62	0.67
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	0.99	1.01	1.00	1.00	1.00	0.99	1.00	1.00
$V_{sim}^{1/2} \times 10^2$	0.91	0.69	0.66	0.63	0.61	0.60	0.61	0.65
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	1.02	1.01	1.00	1.00	1.00	0.98	1.00	1.01
$V_{sim}^{1/2} \times 10^2$	2.53	1.50	1.31	1.21	0.97	0.85	0.80	0.78
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	1.17	1.16	1.15	1.14	1.15	1.12	1.15	1.15
$V_{sim}^{1/2} \times 10^2$	2.91	1.73	1.51	1.39	1.11	0.98	0.91	0.88

Table B.3. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ N(0,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 10, and ρ_w = 0.01. (σ_b^2 =0.010038877)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	0.40	0.44	0.41	0.44	0.42	0.42	0.44	0.41
$V_{sim}^{1/2} \times 10^2$	2.16	2.28	2.37	2.46	2.68	3.02	3.24	3.76
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	0.99	1.01	1.00	1.01	1.01	0.99	1.01	1.00
$V_{sim}^{1/2} \times 10^2$	0.91	0.70	0.66	0.64	0.61	0.60	0.62	0.66
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	0.99	1.01	1.00	1.00	1.00	0.99	1.00	1.00
$V_{sim}^{1/2} \times 10^2$	0.91	0.69	0.66	0.63	0.61	0.60	0.61	0.65
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	1.01	1.00	0.99	0.99	0.99	0.97	1.00	1.00
$V_{sim}^{1/2} \times 10^2$	2.51	1.49	1.30	1.20	0.96	0.84	0.79	0.77
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	1.17	1.16	1.15	1.15	1.15	1.13	1.15	1.15
$V_{sim}^{1/2} \times 10^2$	2.92	1.73	1.51	1.39	1.12	0.98	0.92	0.88

Table B.4. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ N(0,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 0.1, and ρ_w = 0.02. (σ_b^2 =0.025457564)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	0.39	0.45	0.40	0.45	0.42	0.42	0.45	0.40
$V_{sim}^{1/2} \times 10^2$	2.63	2.90	3.07	3.24	3.66	4.26	4.68	5.56
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	2.53	2.56	2.55	2.56	2.56	2.53	2.56	2.55
$V_{sim}^{1/2} \times 10^2$	1.33	1.11	1.08	1.08	1.14	1.18	1.28	1.43
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	1.99	2.01	2.00	2.01	2.00	1.98	2.00	1.99
$V_{sim}^{1/2} \times 10^2$	1.05	0.87	0.84	0.84	0.88	0.91	0.98	1.10
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	2.54	2.53	2.54	2.53	2.54	2.51	2.55	2.54
$V_{sim}^{1/2} \times 10^2$	3.27	2.01	1.77	1.67	1.46	1.39	1.42	1.51
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	2.24	2.23	2.24	2.23	2.24	2.20	2.24	2.23
$V_{sim}^{1/2} \times 10^2$	2.89	1.78	1.57	1.47	1.27	1.21	1.23	1.30

Table B.5. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\bar{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ N(0,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 1, and ρ_w = 0.02. (σ_b^2 =0.020684270)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\bar{y}_{st}$								
$E_{sim} \times 10^2$	0.40	0.45	0.41	0.45	0.43	0.42	0.45	0.40
$V_{sim}^{1/2} \times 10^2$	2.34	2.57	2.73	2.88	3.27	3.81	4.19	5.00
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	2.05	2.08	2.07	2.08	2.08	2.05	2.08	2.07
$V_{sim}^{1/2} \times 10^2$	1.05	0.88	0.86	0.86	0.91	0.95	1.03	1.16
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	1.99	2.01	2.00	2.00	2.00	1.98	2.00	1.99
$V_{sim}^{1/2} \times 10^2$	1.01	0.84	0.83	0.83	0.87	0.91	0.97	1.09
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	2.07	2.07	2.06	2.05	2.06	2.03	2.07	2.06
$V_{sim}^{1/2} \times 10^2$	2.60	1.59	1.42	1.34	1.17	1.11	1.14	1.21
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	2.32	2.31	2.30	2.30	2.31	2.27	2.30	2.30
$V_{sim}^{1/2} \times 10^2$	2.92	1.79	1.59	1.50	1.30	1.23	1.25	1.32

Table B.6. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\bar{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ N(0,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 10, and ρ_w = 0.02. (σ_b^2 =0.020303578)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\bar{y}_{st}$								
$E_{sim} \times 10^2$	0.40	0.46	0.41	0.45	0.43	0.43	0.45	0.41
$V_{sim}^{1/2} \times 10^2$	2.32	2.55	2.71	2.85	3.24	3.77	4.14	4.95
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	2.01	2.04	2.03	2.04	2.04	2.01	2.04	2.03
$V_{sim}^{1/2} \times 10^2$	1.02	0.86	0.85	0.85	0.89	0.94	1.01	1.13
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	1.98	2.01	2.00	2.00	2.00	1.98	2.00	1.99
$V_{sim}^{1/2} \times 10^2$	1.01	0.84	0.83	0.83	0.86	0.90	0.97	1.09
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	2.03	2.03	2.02	2.02	2.02	1.99	2.03	2.02
$V_{sim}^{1/2} \times 10^2$	2.55	1.56	1.39	1.31	1.15	1.09	1.12	1.19
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	2.33	2.32	2.31	2.31	2.31	2.27	2.31	2.30
$V_{sim}^{1/2} \times 10^2$	2.92	1.79	1.60	1.50	1.30	1.23	1.25	1.32

Table B.7. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ N(0,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 0.1, and ρ_w = 0.04. (σ_b^2 =0.070917499)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	0.38	0.48	0.40	0.47	0.43	0.43	0.47	0.39
$V_{sim}^{1/2} \times 10^2$	3.51	4.08	4.40	4.69	5.49	6.58	7.35	8.89
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	7.05	7.11	7.10	7.13	7.11	7.05	7.12	7.08
$V_{sim}^{1/2} \times 10^2$	2.26	2.13	2.16	2.23	2.54	2.80	3.16	3.65
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	3.98	4.00	4.00	4.01	4.00	3.96	3.99	3.96
$V_{sim}^{1/2} \times 10^2$	1.27	1.17	1.18	1.21	1.37	1.51	1.70	1.95
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	7.07	7.08	7.10	7.08	7.09	7.03	7.12	7.07
$V_{sim}^{1/2} \times 10^2$	4.73	3.18	2.89	2.81	2.80	2.94	3.24	3.70
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	4.32	4.32	4.34	4.32	4.32	4.28	4.32	4.28
$V_{sim}^{1/2} \times 10^2$	2.89	1.93	1.74	1.69	1.66	1.73	1.89	2.13

Table B.8. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ N(0,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 1, and ρ_w = 0.04. (σ_b^2 =0.043167173)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	0.40	0.47	0.41	0.47	0.43	0.43	0.47	0.40
$V_{sim}^{1/2} \times 10^2$	2.67	3.11	3.37	3.60	4.25	5.09	5.69	6.91
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	4.29	4.34	4.32	4.33	4.33	4.29	4.34	4.31
$V_{sim}^{1/2} \times 10^2$	1.32	1.26	1.29	1.33	1.53	1.69	1.90	2.20
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	3.97	4.01	3.99	4.00	3.99	3.95	3.99	3.96
$V_{sim}^{1/2} \times 10^2$	1.21	1.14	1.16	1.19	1.35	1.50	1.67	1.93
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	4.31	4.32	4.31	4.30	4.31	4.26	4.33	4.30
$V_{sim}^{1/2} \times 10^2$	2.76	1.84	1.71	1.67	1.68	1.77	1.95	2.22
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	4.59	4.60	4.59	4.58	4.59	4.53	4.58	4.54
$V_{sim}^{1/2} \times 10^2$	2.94	1.94	1.80	1.75	1.73	1.81	1.98	2.23

Table B.9. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim \mathrm{N}(0,1)$. Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 10, and ρ_w = 0.04. (σ_b^2 =0.041541631)

I =	150	75	60	50	30	20	15	10
m_h =	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	0.40	0.48	0.41	0.47	0.43	0.43	0.47	0.40
$V_{sim}^{1/2} \times 10^2$	2.62	3.04	3.30	3.53	4.16	4.98	5.57	6.78
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	4.12	4.18	4.15	4.17	4.16	4.12	4.17	4.15
$V_{sim}^{1/2} \times 10^2$	1.26	1.21	1.24	1.28	1.46	1.62	1.82	2.12
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	3.97	4.01	3.99	4.00	3.99	3.95	3.99	3.95
$V_{sim}^{1/2} \times 10^2$	1.21	1.13	1.16	1.19	1.35	1.50	1.67	1.93
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	4.14	4.16	4.14	4.14	4.15	4.10	4.16	4.14
$V_{sim}^{1/2} \times 10^2$	2.65	1.76	1.64	1.61	1.62	1.70	1.88	2.13
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	4.62	4.63	4.62	4.61	4.62	4.55	4.61	4.57
$V_{sim}^{1/2} \times 10^2$	2.95	1.94	1.81	1.76	1.74	1.82	1.98	2.23

Table B.10. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ Gamma(0.5,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2 = 0.1$, and $\rho_w = 0.01$. (σ_b^2 =0.0055211987)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.63	49.66	49.63	49.65	49.64	49.64	49.65	49.63
$V_{sim}^{1/2} \times 10^2$	1.46	1.56	1.62	1.67	1.87	2.14	2.32	2.71
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	0.54	0.55	0.55	0.55	0.55	0.54	0.56	0.54
$V_{sim}^{1/2} \times 10^2$	0.40	0.32	0.30	0.29	0.29	0.29	0.32	0.34
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	0.99	1.00	1.00	1.01	1.00	0.98	1.01	0.98
$V_{sim}^{1/2} \times 10^2$	0.74	0.58	0.54	0.53	0.52	0.52	0.57	0.62
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	0.51	0.56	0.54	0.55	0.57	0.57	0.58	0.55
$V_{sim}^{1/2} \times 10^2$	2.59	1.53	1.32	1.18	0.92	0.77	0.70	0.62
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	3.85	3.90	3.89	3.90	3.90	3.85	3.90	3.81
$V_{sim}^{1/2} \times 10^2$	2.87	1.89	1.69	1.62	1.54	1.56	1.71	1.90

Table B.11. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ Gamma(0.5,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2 = 1$, and $\rho_w = 0.01$. (σ_b^2 =0.0050141498)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.63	49.66	49.64	49.66	49.64	49.64	49.66	49.63
$V_{sim}^{1/2} \times 10^2$	1.36	1.45	1.51	1.55	1.75	2.01	2.18	2.56
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	0.49	0.50	0.50	0.50	0.50	0.49	0.51	0.49
$V_{sim}^{1/2} \times 10^2$	0.35	0.28	0.26	0.26	0.25	0.26	0.28	0.30
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	0.99	1.00	1.00	1.00	1.00	0.98	1.01	0.98
$V_{sim}^{1/2} \times 10^2$	0.72	0.56	0.53	0.52	0.50	0.51	0.56	0.61
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	0.46	0.50	0.48	0.49	0.52	0.51	0.52	0.50
$V_{sim}^{1/2} \times 10^2$	2.50	1.47	1.26	1.14	0.88	0.74	0.67	0.58
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	13.37	13.40	13.36	13.36	13.28	13.11	13.13	12.83
$V_{sim}^{1/2} \times 10^2$	2.87	2.26	2.22	2.27	2.58	2.97	3.44	4.05

Table B.12. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ Gamma(0.5,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 10, and ρ_w = 0.01. (σ_b^2 =0.0049685206)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.63	49.66	49.64	49.66	49.64	49.64	49.66	49.63
$V_{sim}^{1/2} \times 10^2$	1.35	1.44	1.50	1.54	1.74	2.00	2.17	2.54
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	0.49	0.50	0.50	0.50	0.50	0.49	0.50	0.49
$V_{sim}^{1/2} \times 10^2$	0.35	0.27	0.26	0.25	0.25	0.25	0.28	0.30
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	0.99	1.00	1.00	1.00	1.00	0.98	1.01	0.98
$V_{sim}^{1/2} \times 10^2$	0.72	0.55	0.52	0.51	0.50	0.51	0.56	0.60
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	0.45	0.50	0.47	0.49	0.52	0.51	0.52	0.50
$V_{sim}^{1/2} \times 10^2$	2.49	1.46	1.26	1.13	0.87	0.74	0.67	0.58
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	20.34	20.36	20.33	20.31	20.23	20.03	20.00	19.66
$V_{sim}^{1/2} \times 10^2$	2.72	2.04	2.00	2.05	2.31	2.69	3.13	3.77

Table B.13. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ Gamma(0.5,1). Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 0.1, and ρ_w = 0.02. (σ_b^2 =0.012599659)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.62	49.67	49.63	49.66	49.64	49.64	49.66	49.63
$V_{sim}^{1/2} \times 10^2$	1.73	1.93	2.03	2.13	2.47	2.92	3.22	3.85
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	1.25	1.26	1.26	1.27	1.26	1.24	1.27	1.24
$V_{sim}^{1/2} \times 10^2$	0.55	0.48	0.47	0.47	0.50	0.53	0.61	0.69
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	1.99	2.00	2.00	2.01	2.00	1.97	2.01	1.97
$V_{sim}^{1/2} \times 10^2$	0.88	0.76	0.74	0.75	0.79	0.83	0.95	1.07
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	1.21	1.27	1.25	1.26	1.28	1.27	1.30	1.26
$V_{sim}^{1/2} \times 10^2$	2.74	1.65	1.43	1.30	1.06	0.93	0.90	0.88
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	4.65	4.71	4.70	4.71	4.70	4.64	4.70	4.60
$V_{sim}^{1/2} \times 10^2$	2.88	1.95	1.77	1.71	1.70	1.76	1.97	2.22

Table B.14. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ Gamma(0.5,1). Sample size $n =$ 2,400, $n_h = 300$, $h = 1,\ldots 8$, $\sigma_b^2 / \sigma_\varepsilon^2 = 1$, and $\rho_w = 0.02$. (σ_b^2 =0.010237223)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.63	49.67	49.64	49.66	49.65	49.65	49.66	49.63
$V_{sim}^{1/2} \times 10^2$	1.50	1.68	1.78	1.86	2.18	2.59	2.86	3.44
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	1.01	1.02	1.02	1.03	1.02	1.01	1.03	1.01
$V_{sim}^{1/2} \times 10^2$	0.42	0.36	0.36	0.36	0.39	0.42	0.49	0.55
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	1.99	2.00	2.00	2.01	2.00	1.97	2.01	1.97
$V_{sim}^{1/2} \times 10^2$	0.83	0.71	0.70	0.71	0.76	0.82	0.94	1.05
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	0.98	1.03	1.00	1.02	1.04	1.03	1.05	1.02
$V_{sim}^{1/2} \times 10^2$	2.52	1.50	1.30	1.17	0.94	0.82	0.79	0.75
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	16.27	16.30	16.25	16.26	16.15	15.95	15.96	15.60
$V_{sim}^{1/2} \times 10^2$	2.90	2.45	2.46	2.55	2.99	3.49	4.07	4.80

Table B.15. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim$ Gamma(0.5,1). Sample size $n =$ 2,400, $n_h = 300$, $h = 1,\ldots 8$, $\sigma_b^2 / \sigma_\varepsilon^2 = 10$, and $\rho_w = 0.02$. (σ_b^2 =0.010048807)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.63	49.67	49.64	49.67	49.65	49.65	49.67	49.63
$V_{sim}^{1/2} \times 10^2$	1.48	1.66	1.76	1.83	2.16	2.56	2.83	3.41
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	0.99	1.01	1.01	1.01	1.00	0.99	1.01	0.99
$V_{sim}^{1/2} \times 10^2$	0.41	0.36	0.35	0.36	0.38	0.41	0.48	0.54
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	1.99	2.01	2.00	2.01	2.00	1.97	2.01	1.97
$V_{sim}^{1/2} \times 10^2$	0.83	0.71	0.70	0.71	0.75	0.81	0.93	1.05
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	0.96	1.01	0.98	1.00	1.02	1.01	1.03	1.00
$V_{sim}^{1/2} \times 10^2$	2.50	1.49	1.28	1.16	0.93	0.81	0.78	0.74
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	25.08	25.10	25.06	25.04	24.94	24.71	24.67	24.25
$V_{sim}^{1/2} \times 10^2$	2.69	2.14	2.14	2.22	2.58	3.05	3.58	4.33

Table B.16. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim \text{Gamma}(0.5,1)$. Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 0.1, and ρ_w = 0.04. (σ_b^2 =0.035099049)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.62	49.69	49.63	49.68	49.65	49.65	49.68	49.62
$V_{sim}^{1/2} \times 10^2$	2.39	2.80	3.01	3.18	3.80	4.58	5.13	6.22
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	3.49	3.50	3.52	3.53	3.51	3.47	3.53	3.48
$V_{sim}^{1/2} \times 10^2$	1.01	0.99	1.00	1.04	1.19	1.32	1.55	1.79
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	3.98	3.99	4.00	4.01	4.00	3.94	4.00	3.94
$V_{sim}^{1/2} \times 10^2$	1.15	1.10	1.11	1.15	1.31	1.45	1.68	1.94
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	3.45	3.52	3.51	3.52	3.54	3.51	3.57	3.50
$V_{sim}^{1/2} \times 10^2$	3.29	2.09	1.86	1.75	1.62	1.60	1.73	1.91
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	5.77	5.82	5.82	5.83	5.81	5.74	5.81	5.70
$V_{sim}^{1/2} \times 10^2$	2.90	2.05	1.89	1.85	1.93	2.06	2.33	2.67

Table B.17. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim \text{Gamma}(0.5,1)$. Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2 / \sigma_\varepsilon^2$ = 1, and ρ_w = 0.04. (σ_b^2 =0.021364638)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.63	49.68	49.64	49.68	49.65	49.65	49.68	49.63
$V_{sim}^{1/2} \times 10^2$	1.76	2.08	2.25	2.38	2.90	3.51	3.94	4.81
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	2.12	2.14	2.14	2.15	2.14	2.11	2.15	2.11
$V_{sim}^{1/2} \times 10^2$	0.55	0.55	0.57	0.59	0.69	0.78	0.92	1.07
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	3.98	4.01	4.01	4.02	4.00	3.94	4.01	3.93
$V_{sim}^{1/2} \times 10^2$	1.05	1.02	1.05	1.09	1.25	1.41	1.65	1.91
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	2.08	2.14	2.12	2.14	2.16	2.13	2.17	2.13
$V_{sim}^{1/2} \times 10^2$	2.57	1.57	1.38	1.28	1.12	1.07	1.13	1.20
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	19.61	19.65	19.59	19.59	19.46	19.21	19.22	18.78
$V_{sim}^{1/2} \times 10^2$	2.95	2.68	2.75	2.88	3.47	4.08	4.79	5.66

Table B.18. Simulation means, E_{sim}, and standard errors, $V_{sim}^{1/2}$, of $\overline{y}_{st}, \hat{\sigma}^2_{b,ST1}, \hat{\rho}_{w,ST1}$, $\hat{\sigma}^2_{b,ST2}$ and $\hat{\rho}_{w,ST2}$ from 5,000 repeated stratified simple random samples assuming $\mu_k \sim \text{Gamma}(0.5,1)$. Sample size n = 2,400, n_h = 300, h = 1,…8, $\sigma_b^2/\sigma_\varepsilon^2$ = 10, and ρ_w = 0.04. (σ_b^2 =0.020560112)

$I =$	150	75	60	50	30	20	15	10
$m_h =$	2	4	5	6	10	15	20	30
$\overline{y}_{st}$								
$E_{sim} \times 10^2$	49.63	49.68	49.64	49.68	49.65	49.65	49.68	49.63
$V_{sim}^{1/2} \times 10^2$	1.72	2.03	2.20	2.33	2.83	3.44	3.86	4.71
$\hat{\sigma}^2_{b,ST1}$								
$E_{sim} \times 10^2$	2.04	2.06	2.06	2.06	2.06	2.03	2.07	2.03
$V_{sim}^{1/2} \times 10^2$	0.52	0.53	0.54	0.57	0.66	0.75	0.89	1.02
$\hat{\rho}_{w,ST1}$								
$E_{sim} \times 10^2$	3.98	4.01	4.01	4.02	4.00	3.94	4.01	3.93
$V_{sim}^{1/2} \times 10^2$	1.05	1.02	1.04	1.08	1.25	1.41	1.65	1.90
$\hat{\sigma}^2_{b,ST2}$								
$E_{sim} \times 10^2$	2.00	2.06	2.03	2.05	2.07	2.05	2.09	2.04
$V_{sim}^{1/2} \times 10^2$	2.54	1.54	1.36	1.25	1.09	1.04	1.10	1.16
$\hat{\rho}_{w,ST2}$								
$E_{sim} \times 10^2$	30.48	30.49	30.45	30.43	30.30	30.03	29.98	29.49
$V_{sim}^{1/2} \times 10^2$	2.66	2.26	2.31	2.41	2.89	3.45	4.08	4.96