SECOND PRICE COMMON VALUE AUCTIONS AND BAYESIAN INFERENCE IN EBAY AUCTIONS

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Abstract

Second price common value auctions is the topic of this thesis. Estimation of such auctions are technically challenging and equilibrium bid functions in these settings are in general complex and not easy to analyze. In Paper 1 we derive closed form approximations of the bid function for two empirically important models. The approximate bid functions can be evaluated directly without time consuming numerical integration. This is crucial for speeding up likelihood/Bayesian estimations on auction data. In Paper 2 we explore the determinants of bidder and seller behaviour by modelling eBay auctions as independent second price common value auctions, and assume a similar (the same in Paper 1) hierarchical Gaussian valauation structure as in Bajari and Hortacsu (2003). We use an efficient Bayesian variable selection algorithm to assess the importance of the model's covariates. The good performance of the algorithm is documented on both real and simulated data. An important result of Paper 2 is the nearly identical inferences for the approximate bid function in Paper 1 with the exact bid function, which gives much faster and numerically more stable evaluations of the likelihood function. We apply the methodology to simulated data and to a carefully collected dataset of 1000 coin auctions at eBay. The structural estimates are reasonable, both in sign and magnitude, and the model fits the data well. Finally, we document good out-of-sample predictions from the estimated model.

Keywords: Closed form solution, Bid approximation, Normal valuations, Markov Chain Monte Carlo, Variable selection, Internet auctions.

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1 Introduction

Auctions have been used since antiquity for the sale of a variety of objects. An auction is a bidding mechanism, described by a set of auction rules that specify how the winner is determined and how much the bidder has to pay. There exists many different auction rules, but four basic types of auctions are particularly common and referred to as standard auctions, which are divided into oral (English, Dutch) and written (First price, Second price) auctions. In oral auctions bidders hear each other's bids and can make counteroffers. In a written or closed bid auction bidders submit only one bid simultaneously without revealing them to others. The open ascending auction or English auction is the oldest auction form and typically starts with low bids and increases in small predetermined portions until only one bidder is left. The dutch auction goes in the opposite direction. The auctioneer begins at a certain, usually high, price and gradually lowers it until someone makes a sign to claim the item. In first and second price auctions bids are sealed. The highest bidder wins in both auctions but pays an amount equal to the highest and the next highest bid in the first and the second price auctions, respectively. A key feature of auctions are the asymmetries in information. In the private values model each bidder knows the value to himself at the time of bidding and knowledge of other bidders' valuation would not affect his valuation. In the (pure) common value model, the value of the object is unknown but the same for all bidders *ex ante*, but bidders have different private information about the true value of the object.

Auction theory refers to the analysis of auctions as games of incomplete information. The first pioneering article in the field was the work of William Vickrey (1961). Since then the theory of auctions has developed extensively, especially over the last decades. Wilson (1969) introduced the common value model and developed the first closed-form equilibrium analysis of the winner's curse, which is a key feature in common value auctions. Each bidder must account for the fact that if she wins she has the highest signal among bidders and thus adjust her bids downwards accordingly. At the late seventies the major contributions came in the literature of auction theory. Roughly during the same time, independent of each other, Myerson (1981) and Riley and Samuelson (1981) generalize Vickrey's results about the equality in expected revenue for many auction settings with a privately known signal, including the four standard auctions. As Klemperer (1999) mention, in his broad survey of the literature in auction theory, the theorem is so fundamental that any reader who is unfamiliar with the result is strongly urged to learn it. In another article on the mechanism-design literature of auction theory, Maskin and Riley (1985), in the mechanism-design literature of auction theory, brings out many key ideas by focusing on only two bidders with private values. Apart from private values, in an influential article Milgrom and Weber (1982) derive the equilibrium bid function for a second price common value auction. In general common value models are much more technically challenging than these models counterpart of private values, why it is in practice difficult to specify distributional assumptions that yield closed-form solutions of the bid function or at least neat implicit forms. Quite a few closed-form solutions exists for specific distributional assumptions. In Paper 1 we look more into this as we come back to later on.

Recently, structural estimations of auction data have become increasingly popular over the last decades. Laffont and Vuong (1996) came with major contributions in this field and emphasize that auction models are particularly suited for structural estimation where many datasets are readily available and well-defined game forms exists. Bajari and Hortacsu (2005) mention three conditions that must apply for structural estimation. First, the bidders' goal is to maximize their expected utility. This is basicly an assumption of rational bidders. If bidders' maximize their expected utility by maximizing their profits they are risk-neutral bidders, which is commonly assumed in the literature. In equilibrium, the bidders' maximize their utility with an optimal bid strategy as a function of values. Second, bidders are able to compute the relationship between their bid and the probability of winning the auction. That is, they are able to compute the optimal combination of the probability of winning and the amount of the profit if they win. Third, given their beliefs, bidders are able to correctly maximize expected utility. These assumptions of rationally are quite strong, but there exists a number of papers that test for necessary conditions. In principle, Guerre, Perrigne, and Vuong (2000) point out that a necessary condition for rationality in private value auction models is to the test if the bid function is increasing.

More recently, over the last decade Internet auctions have gained wide popularity. Lucking-Reiley (2000) surveys 142 online auctions and estimate eBay as the world's largest auction site by far. At eBay, millions of items are sold every day in thousands of categories from which high-quality datasets become available to buyers and sellers through completed auction listings. To explore the determinants of bidder and seller behaviour, Bajari and Hortacsu (2003) examine a dataset of coin auctions from eBay. According to several empirical findings for auctions with a fixed end time, e.g. Wilcox (2000), Schindler (2003), and Ockenfels and Roth (2006), bids tend to arrive very late in these auctions. In the spirit of Wilson (1977), Bajari and Hortacsu (2003) show for their independent symmetric common value model of an eBay auction that late bidding is a Nash equilibrium. As a consequence, they estimate eBay auctions as independent second price common value auctions. In this environment each bidder is assumed to place only one bid in the very last minute of the auction, so that no other bidders have time to revise their bids.

In this thesis, we model eBay auctions as independent second price common value auctions. Equilibrium bid functions in common value auctions are in general complex and not easy to analyze. A handful closed form solutions have been derived, but only for highly specialized models, e.g. Kagel and Levin (1986), Matthews (1984), and Levin and Smith (1991). The lack of closed form solutions has two major drawbacks. First, it is hard to see how the bid function depends on various distributional components of the model, which makes it more difficult to bring out model characteristics. Second, to evaluate the bid function one has to make use of numerical integration which is very time demanding. This is a crucial step for econometric analysis of auction data (e.g. likelihood/Bayesian estimation) where the equilibrium bid function has to be evaluated over and over again (Bajari and Hortacsu, 2003). By exploiting a linearization property, Bajari and Hortacsu (2003) reduce the computational complexity significantly in their model, but the inverse bid function in the very complicated likelihood function still needs to be evaluated by time-consuming numerical integration.

In Paper 1, to simplify the computational complexity of likelihood/Bayesian estimation we obtain convenient closed form solutions, for both a known and a stochastic number of bidders, by approximating the equilibrium bid function for two realistic distributional assumptions. First, a linear bid approximation is derived for the hierarchical normal model, defined in Bajari and Hortacsu (2003), and then a non-linear approximation is obtained for the Gamma-Gamma model, as defined by Gordy (1997). The accuracy of both approximations is quite good, especially for the normal model, and yield straightforward and fast explicit solutions of the equilibrium bid functions that can be inverted analytically. We confirm this fact in Paper 2 by using several simulated datasets that orginates from a similar eBay auction model as in Bajari and Hortacsu (2003). We obtained nearly identical estimation results for the approximate and exact bid function.

Bajari and Hortacsu (2003) develop an interesting model for eBay auctions. In their model, the common values are modelled as a heteroscedastic Gaussian regression, and entry into the auctions is stochastically determined by a Poisson regression. The use of auction specific covariates makes it possible to analyze aspects such as the effect of the seller's minimum bid on participation. Following Bajari and Hortacsu (2003), we model eBay auctions as independent second price common value auctions and assume a similar hierarchical Gaussian valuation structure as in their model. Bajari and Hortacsu (2003) specify rather ad hoc which covariates to include in the model. One of the contributions of Paper 2 is the use of Bayesian inference methodology that lets the data make this decision. Therefore, we use an efficient Bayesian variable selection algorithm that simultaneously samples the posterior distribution of the model parameters and does inference on the choice of covariates. The performance of the variable selection algorithm performed well, the estimation results agree with intuition, model evaluations are excellent and even predictions performs well.

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