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Domestic policy or global influences? The Swedish natural rate of interest and international spill-overs*

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Abstract

In Sweden, the Riksbank's policy of negative interest rates amid an economic upturn and a booming housing market has received a lot of criticism. The Riksbank, on the other hand, claims that it is not primarily policy but global factors that have had a negative influence on domestic interest rates. We use a version of the Laubach and Williams model to estimate the Swedish natural interest rate and find evidence that a large fraction of its decline can be traced to global spill-overs.

JEL: E43, E52, C32
Keywords: Natural interest rate, global spill-over

1 Introduction

In many advanced economies inflation has been low for an extended period of time while interest rates have reached new record lows. Although growth has picked up and output gaps are starting to close, policy rates are still negative in Japan, the euro area (EA), Sweden,

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Denmark and Switzerland.[1] In the United States (US), the Federal Reserve has started to gradually raise rates. However, both market pricing and Federal Open Market committee members' individual forecasts indicate that rates over the longer term will settle at a lower level than before the financial crisis. As argued by the Chair of the Board of Governors of the Federal Reserve System, Janet Yellen (2015, p. 11), this reflects a belief that the natural rate of interest in the US, "- defined as the value of the federal funds rate that would be neither expansionary nor contractionary if the economy were operating near its potential - is currently low by historical standards and is likely to rise only gradually over time". Others have noted that this shift downward in the natural rate of interest seems to be a global phenomenon (Rachel and Smith, 2015; Williams, 2016; Holston et al., 2017; Christensen and Rudebusch, 2017, to name a few).

In Sweden, the Riksbank seems to have been inspired by this line of reasoning. They have stated that global trends stemming from abroad have had a large influence on domestic rates, and that the decline in global rates can be traced to structural factors (Sveriges Riksbank, 2017b). Even if the Riksbank is right about global factors being an important part of the explanation for the general international interest rate levels, Holston et al. (2017) have shown that even over longer horizons about half of the variation in the natural rate is due to domestic factors in other small open economies like the United Kingdom (UK) and Canada. This implies that foreign proxies do not capture an important component of the domestic natural rate. In order to judge the expansiveness of domestic monetary policy, country-specific estimates are thus necessary.

Although the natural rate of interest is a crucial concept for gauging the expansiveness of monetary policy (see, for instance, Giammarioli and Valla, 2004, for an overview), and thus serves as an important input to macroeconomic forecasting and policy analysis, formal estimates of the natural rate are missing for many smaller countries, including Sweden. In this paper we estimate a model which makes it possible to disentangle the effect of global structural factors, i.e. the natural rate, and the expansiveness of monetary policy (the interest rate gap). We follow in spirit the methods outlined in Laubach and Williams (2003) and, more specifically, the Bayesian methods extended by Berger and Kempa (2014).

We apply our estimated natural interest rate and study the spill-overs from larger trading partners to Sweden, following the framework in Holston et al. (2017). Based on an error-correction model we find evidence of global spill-overs to the Swedish natural rate. Our findings have implications for both policy and forecasting in Sweden, and contribute to the wider discussion regarding the global dimension and spill-overs in interest rates.

The paper is organized as follows. Section 2 describes the model. Section 3 presents the estimation results. Section 4 studies the global spill-overs. Section 5 concludes. The outline of the estimation is placed in the appendix.

[1] For an overview regarding the implementation of negative policy rates, see, e.g., Bech and Malkhozov (2016).

2 The empirical framework

The concept of the natural interest rate in our model is similar to the Wicksellian (1936) concept of the natural rate as the real interest rate that is consistent with stable inflation and output. It is thus a more longer-run concept than for instance the natural interest rate in DSGE-models, which is the interest rate that would prevail in a world with flexible prices (see, e.g., Woodford, 2003).

Our semi-structural model assumes a relationship between potential growth (g) and the real natural rate of interest (r^*), consistent with basic economic theory, but also allows for lasting deviations between the two. To see this, we part from the household intertemporal utility maximization, where using a standard CES utility function, the solution yields the following (log-linear) relationship between the interest rate and steady state growth:

$$r^* = \frac{1}{\sigma} g + \rho, \qquad (1)$$

where σ is the intertemporal elasticity of substitution, and ρ is the rate of time preference.

As in the seminal paper by Laubach and Williams (2003), we link an unobserved time-varying version of Equation (1) to the observed economy, and then apply the Kalman filter to data on real gross domestic product (GDP), inflation, and the short-term interest rate to estimate jointly the natural rate of interest, the natural rate of output and trend growth. Additionally, to help identify the transitory and permanent unobserved components in a small open economy like Sweden, we follow the suggestions by Berger and Kempa (2014); that is, we incorporate the effective exchange rate - a weighted average of bilateral rates between the Swedish krona and a basket of foreign currencies - as well as apply Bayesian methods to penalize the likelihood in regions of the parameter space that are inconsistent with out-of sample information.

We use quarterly data on the log of real seasonally adjusted GDP and core inflation measured as the annualized log-difference of seasonally adjusted CPIF, the consumer price index with a fixed interest rate.[2] The effective exchange rate is the index KIX (an increase implies a depreciation of the krona, and a decrease implies an appreciation), and the short-term nominal interest rate is the quarterly average of the Riksbanks' policy rate, the repo rate.[3] We define the real interest rate as the nominal rate (i_t) minus expected inflation one year ahead (π^e_{t+4}), $r_t = i_t - \pi^e_{t+4}$. As suggested by Laubach and Williams (2003), inflation expectations are proxied by the prediction for the four-quarter-ahead percentage change in inflation, $\pi^e_{t+4} = \hat{\pi}_{t+4|t}$, where $\hat{\pi}_{t+4|t}$ is the ordinary least squares forecast from an univariate AR(3) process with a rolling estimation window of 40 quarters. Our sample covers 1996Q1 to 2016Q4; the data and modeled inflation expectations are shown in Figure 1. The starting point of our sample is suitable due to a series of important events in the beginning of the 1990's. Between 1990

[2]GDP and CPIF are produced by Statistics Sweden, and can be downloaded from www.scb.se. CPIF is the measure that the Riksbank is officially targeting since September 2017, but it also served as the implicit target prior to that; see Sveriges Riksbank (2017a).

[3]The repo rate and KIX can be downloaded from www.riksbank.se. Descriptions of KIX can be found in Erlandsson and Markowski (2006) and Alsterlind (2006).

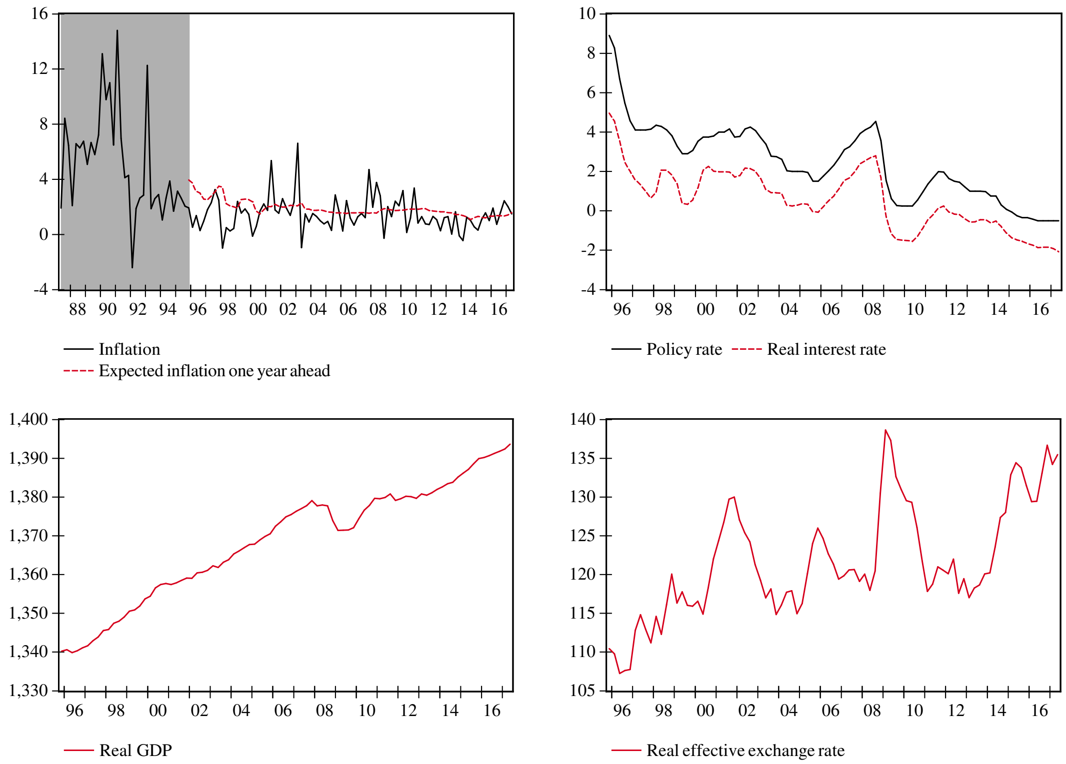

Figure 1. Data and modeled inflation expectations.

and 1992 Sweden reformed its tax system. Moreover, in 1993 Sweden introduced a flexible exchange rate system, and subsequently, in 1995 the Riksbank introduced an inflation target of 2 percent.

The log of real GDP (y_t), the real interest rate (r_t), and the real effective exchange rate (q_t) can each be expressed as the sum of two unobserved components: an equilibrium level, denoted by an asterisk, and a gap, denoted by a tilde,

$$y_t = y_t^* + \tilde{y}_t, \tag{2}$$
$$r_t = r_t^* + \tilde{r}_t, \tag{3}$$
$$q_t = q_t^* + \tilde{q}_t, \tag{4}$$

where y_t^* is the log potential GDP, r_t^* is the natural real rate of interest, and q_t^* is the natural real effective exchange rate. Potential GDP is specified as a random walk with a stochastic drift g_t (the log growth):

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*}, \tag{5}$$
$$g_t = (1 - \varphi_2)\varphi_1 + \varphi_2 g_{t-1} + \varepsilon_t^g. \tag{6}$$

A-priori, we would expect that the unobserved trend growth rate follows a stationary AR(1) process (that is, we expect φ_2 to be less than 1 in absolute value, such that the unconditional mean is $\mathrm{E}(g_t) = \varphi_1$). If the trend growth rate is $I(0)$, then potential log-GDP is $I(1)$, which, as advocated by Mésonnier and Renne (2007), is typically found for the EA.[4] In contrast, other studies, such as Laubach and Williams (2003) and Holston et al. (2017), explicitly model log-GDP as an $I(2)$-process.

Based on the relation (1), and in line with Laubach and Williams (2003), we assume that the natural rate depends on potential growth and a remaining unobserved component that follows a random walk,

$$r_t^* = cg_{t-1} + z_{t-1}, \tag{7}$$
$$z_t = z_{t-1} + \varepsilon_t^z. \tag{8}$$

The component z_t can be thought of as everything that affects the natural rate that is not related to growth, such as an increased desire to save, changes in fiscal policy, changes in the demand for safe assets, etcetera; see, e.g., Armelius et al. (2014), Rachel and Smith (2015) and Bean et al. (2015), for overviews. Because z_t is $I(1)$, the natural interest rate is $I(1)$. We consider alternative specifications for g_t and z_t later on.

In the absence of a strong view regarding the level of the long run exchange rate, the natural effective exchange rate is assumed to follow a random walk,

$$q_t^* = q_{t-1}^* + \varepsilon_t^q. \tag{9}$$

We assume that the GDP gap, the real interest rate gap, and the real effective exchange rate gap are stationary, implying that real GDP, the real interest rate and the real effective exchange rate each cointegrate with their equilibrium level. The gaps are expected to interact, and are therefore explained by a first-order VAR,

$$\tilde{x}_t = \Psi \tilde{x}_{t-1} + \tilde{\varepsilon}_t, \tag{10}$$

where $\tilde{x}_t = (\tilde{y}_t, \tilde{r}_t, \tilde{q}_t)'$ is a time series vector of gaps, Ψ is a 3×3 parameter matrix, and $\tilde{\varepsilon}_t = (\varepsilon_t^{\tilde{y}}, \varepsilon_t^{\tilde{r}}, \varepsilon_t^{\tilde{q}})'$ is a time series vector of specific errors. The VAR (10) is consistent with economic theory in that deviations from fundamental values in all the macroeconomic variables are allowed to influence the other variables. For instance, the first equation,

$$\tilde{y}_t = \psi_{11}\tilde{y}_{t-1} + \psi_{12}\tilde{r}_{t-1} + \psi_{13}\tilde{q}_{t-1} + \varepsilon_t^{\tilde{y}}, \tag{11}$$

is a reduced-form of an aggregate demand equation, an "IS-curve", where the output gap is determined by its own lag, and lags from, respectively, the real interest rate gap and the exchange rate gap. When the actual interest rate is above the natural rate, monetary policy is contractionary, which will have a negative impact on the output gap. We therefore expect the output gap to be negatively correlated with, not too distant, lags of the interest rate gap.

[4]Indeed, the augmented Dickey-Fuller (ADF) test cannot reject that log-GDP has a unit root, with a p-value of 0.666, but rejects that the difference of log-GDP has a unit root, with a p-value of less than 0.001.

Meanwhile, an appreciation of the effective exchange (i.e., a fall in KIX) is expected to decrease the economic activity through reduced exports. Hence, we expect the output gap to be positively correlated with some lags of the exchange rate gap.

Similarly, the second and third equations in the VAR determine, respectively, how the real interest rate gap reacts to cyclical variations in output and the exchange rate, and how the real exchange rate gap reacts to variations in the output gap and the interest rate gap. The natural interest rate is thus the rate that will prevail when the output gap is closed, and the exchange rate is not over- or undervalued, in the absence of other shocks.

Finally, we have an aggregate supply equation, a ”Phillips curve”, given by

$$\pi_t = \delta_1 + \delta_2 \pi_{t-1} + \delta_3 \Delta q^n_{t-1} + \delta_4 \tilde{y}_t + \varepsilon^\pi_t, \tag{12}$$

where π_t is inflation at time t, that is dependent on its own lag, changes in the *nominal* effective exchange rate q^n_t and the output gap $\tilde{y}_t$. Here, the nominal exchange rate should capture changes in international prices and the contribution of imports. A depreciation of the exchange rate ($\Delta q^n_t > 0$) means that foreign goods are more expensive, and should therefore increase inflation. Thus, the expected sign of δ_3 is positive. The effective exchange rate affects inflation directly in the Philips curve (12) in nominal terms, as well as through the output gap in the VAR (11) in real terms. Monetary policy, however, only affects inflation through the output gap. Because, fluctuations in economic activity should be positively correlated with inflationary pressure, we expect δ_4 to be positive, but not particularly large. Note, lastly, that if inflation is trending, then inflation targeting is not working. We therefore expect prices to be $I(1)$, so that inflation is $I(0)$. Thus, we expect δ_2 to be less than 1, and positive.

3 Estimation results

The model is estimated by the Kalman smoother using a state space representation; the mathematical details are outlined in the appendix. It is well-known that identification can be difficult when the unobserved natural rate in turn depends on other unobserved components (see, for instance, Laubach and Williams, 2003, and Mésonnier and Renne, 2007). In particular, the maximum likelihood estimator of the error variances for g_t and z_t (provided by the Kalman filter) are typically biased towards zero, due to nonstationarity in the underlying processes. Therefore, we follow Berger and Kempa (2014) and apply Bayesian methods, in which prior information can be used to penalize the likelihood in regions of the parameter space that concede with variances of the innovations that are close to zero. As such, a Bayesian approach could be suitable for dealing with the problems of model identification (see discussion in Pedersen, 2015).

As in Berger and Kempa (2014), we use independent Gaussian prior distributions for all parameters except the variance parameters, for which we use independent gamma prior distributions. Table 1 shows the means and associated 90 percent intervals for the prior and posterior parameter distributions. We set the prior mean of the potential growth coefficient c in (7) to 4, approximating a one-to-one relationship between the natural real rate of interest and annual potential growth. In the growth rate equation (6), we set the prior mean of φ_1

Table 1. Prior and posterior parameter distributions.

Parameter	Prior distribution		Posterior distribution	
	Mean	90% interval	Mean	90% interval
Potential output and growth				
$\sigma^2_{y^*}$	0.50	$[0.06, 1.28]$	0.146	$[0.061, 0.304]$
φ_1	0.57	$[0.41, 0.73]$	0.569	$[0.504, 0.634]$
φ_2	0.80	$[0.64, 0.96]$	0.687	$[0.627, 0.746]$
σ^2_g	0.25	$[0.11, 0.43]$	0.147	$[0.102, 0.214]$
Natural rate of interest				
c	4.00	$[2.34, 5.65]$	0.333	$[0.231, 0.441]$
σ^2_z	0.25	$[0.11, 0.43]$	0.063	$[0.048, 0.082]$
Natural exchange rate				
σ^2_q	0.25	$[0.11, 0.43]$	0.236	$[0.160, 0.346]$
Output gap				
ψ_{11}	0.50	$[0.09, 0.91]$	1.011	$[0.937, 1.086]$
ψ_{12}	0.00	$[-0.41, 0.41]$	-0.389	$[-0.492, -0.290]$
ψ_{13}	0.00	$[-0.41, 0.41]$	0.011	$[0.002, 0.020]$
$\sigma^2_{\tilde{y}}$	0.50	$[0.06, 1.28]$	0.245	$[0.167, 0.354]$
Real interest rate gap				
ψ_{21}	0.00	$[-0.41, 0.41]$	0.306	$[0.243, 0.370]$
ψ_{22}	0.50	$[0.09, 0.91]$	0.627	$[0.557, 0.697]$
ψ_{23}	0.00	$[-0.41, 0.41]$	-0.016	$[-0.024, -0.009]$
$\sigma^2_{\tilde{r}}$	0.50	$[0.06, 1.28]$	0.006	$[0.001, 0.029]$
Real exchange rate gap				
ψ_{31}	0.00	$[-0.41, 0.41]$	-0.114	$[-0.290, 0.073]$
ψ_{32}	0.00	$[-0.41, 0.41]$	0.257	$[0.095, 0.419]$
ψ_{33}	0.50	$[0.09, 0.91]$	0.916	$[0.882, 0.949]$
$\sigma^2_{\tilde{q}}$	1.00	$[0.13, 2.57]$	5.727	$[5.165, 6.350]$
Phillips curve				
δ_1	1.00	$[0.18, 1.82]$	1.298	$[1.154, 1.444]$
δ_2	0.50	$[0.09, 0.91]$	0.120	$[0.043, 0.193]$
δ_3	0.25	$[-0.16, 0.66]$	0.050	$[0.012, 0.086]$
δ_4	0.25	$[-0.16, 0.66]$	0.180	$[0.094, 0.272]$
σ^2_π	2.00	$[0.68, 3.88]$	1.419	$[1.277, 1.578]$

to 0.57 (the AR unconditional mean), corresponding to an annual steady state growth rate of around 2.3 percent, and the prior mean of φ_2 to 0.8, since we expect a somewhat persistent stationary potential growth. The prior means of the parameters in the gap VAR (10) are set to 0.5 for own lags, and 0 for the other lags, so that, a-priori, the gaps are independent stationary AR(1) processes. The prior mean for the slope of the Phillips curve (12), δ_2, is set to 0.5, whereas the prior means for the coefficients relating to the nominal exchange rate and the output gap are set so that the 90 percent intervals cover 0. We do not, for any parameter, restrict the sampling to draws which ensure stationarity of the underlying processes.

Figure 2 shows the estimated gaps and trends together with the respective 90 percent posterior intervals. As can be seen, the intervals are quite wide for most estimates. Similar uncertainties are typical findings in these types of models, and were one of the major conclusions in Laubach and Williams (2003). At the same time, however, the posterior distributions for the error variances are considerably more condensed than the their prior distributions (see Table 1), suggesting that data provide meaningful information for our model. As in many previous studies (see, e.g., Rachel and Smith, 2015; Laubach and Williams, 2016, and ref-

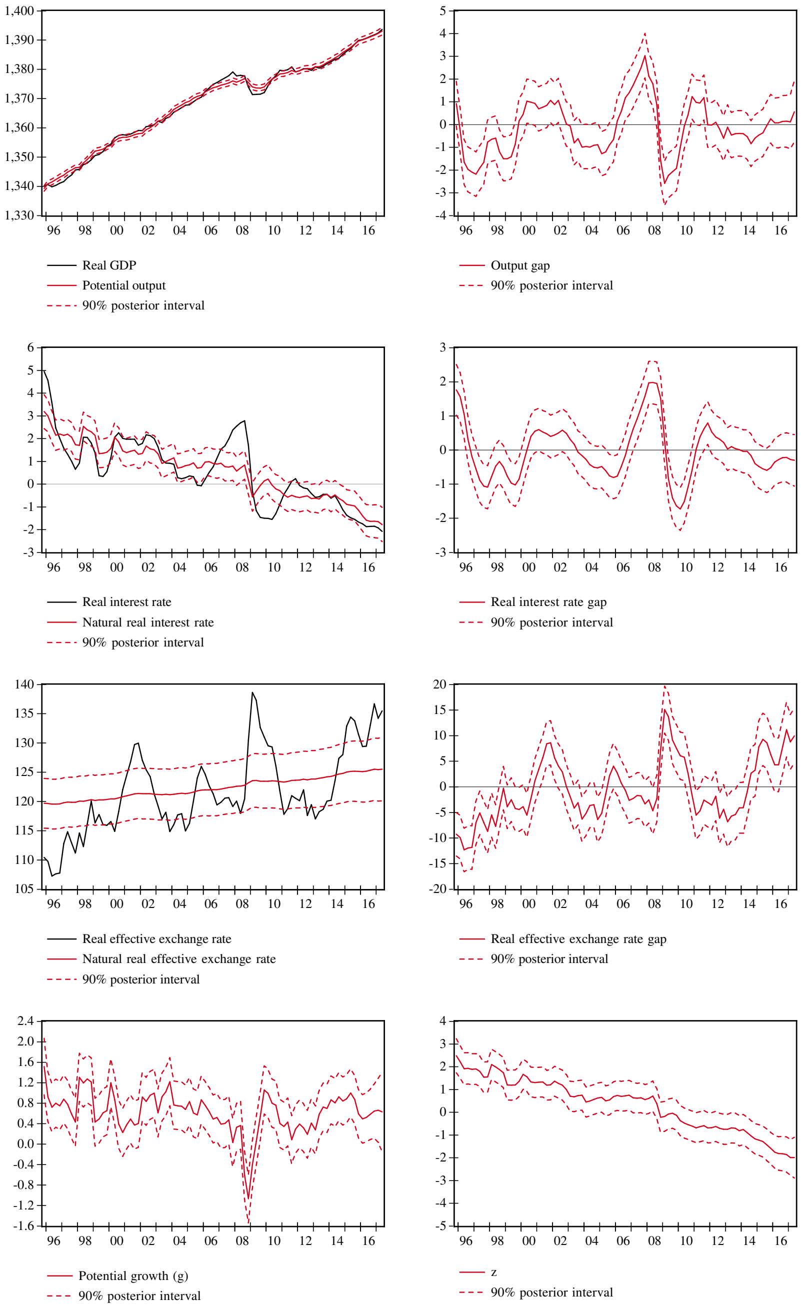

Figure 2. Estimated gaps and trends with posterior intervals.

erences therein), there is a clear downward sloping trend in the natural rate of interest since the beginning of the sample period. Moreover, in line with Holston et al. (2017), there is no sign of a recent pick-up in the natural interest rate in our estimate, which has been below

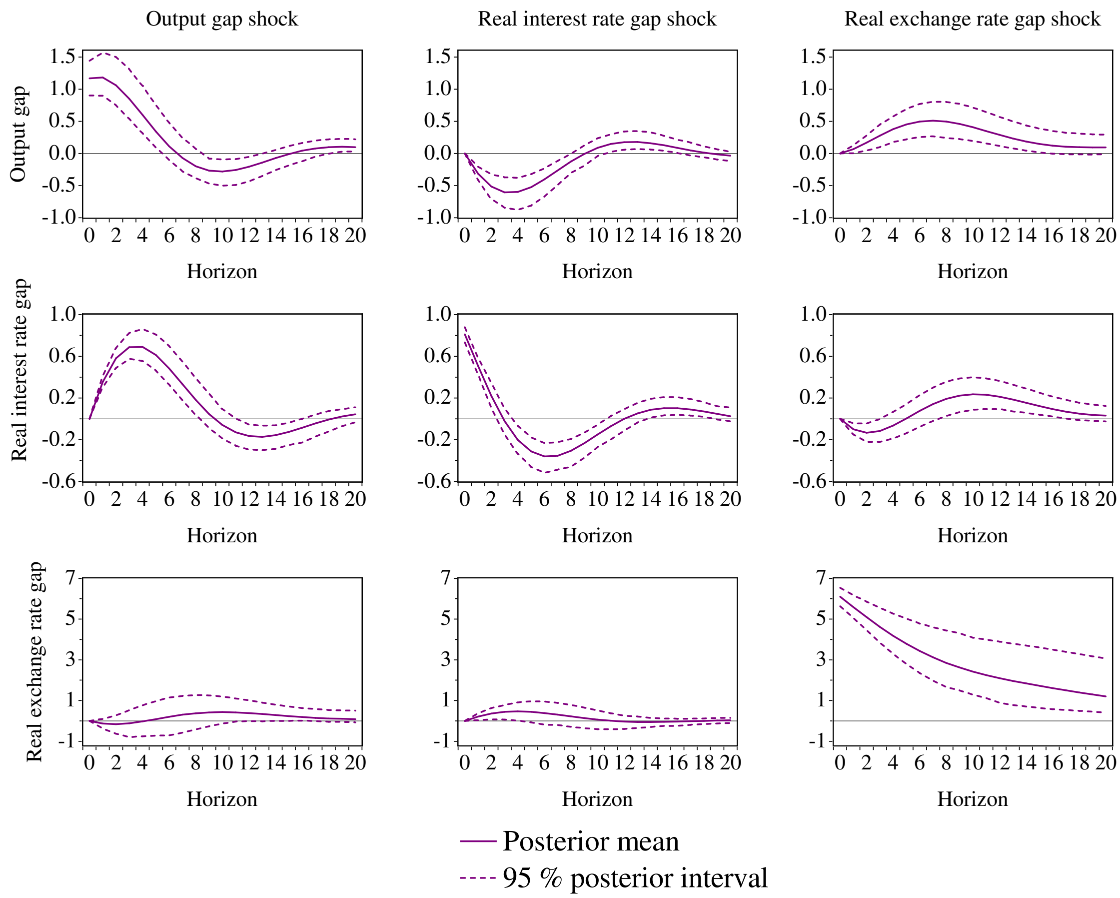

Figure 3. Gap impulse responses.

zero since the third quarter of 2010. It is also clear that a large fraction of the decline in the natural interest interest rate originates from the unobserved component z_t. Potential growth, on the other hand, has been more stable, where the posterior distribution for φ_2 is in line with our stationary presumption. Meanwhile, as in, e.g., Hamilton et al. (2015), the connection between potential growth and the natural interest rate appears weak, with a posterior mean for the growth parameter c estimated to 0.33, quite far from the prior mean.

Looking at the real interest rate gap in Figure 2, the rate hikes in 2010 and 2011 seem to have made monetary policy quite tight, leading to a positive interest rate gap. In the years following that episode inflation undershot the target level for almost five years, until the end of 2016 (see Figure 1). It is possible that central banks underestimated the downturn in the natural rate that according to our model (and others) had already occurred at this time. Furthermore, the interest rate gap indicates that Swedish monetary policy has been expansionary since 2014.

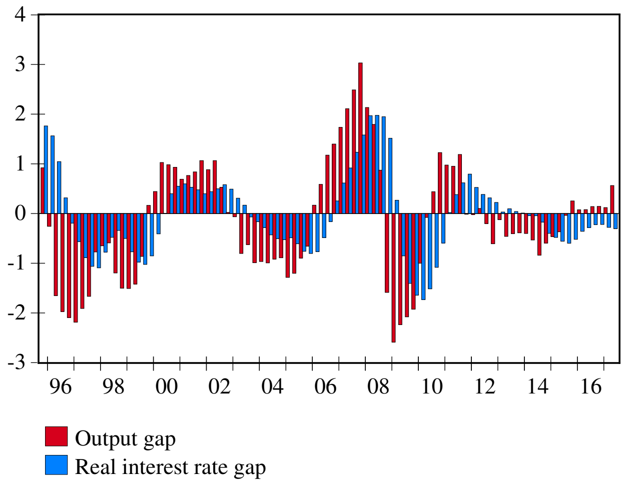

Figure 4. Estimated output and interest rate gaps.

It is worth noting that in the literature there are those who believe that the focus on inflation as the main signal of whether output is at potential or not is misguided. Juselius et al. (2016) and Borio (2017) emphasize the importance of possible financial distortions that could make these macroeconomic relationships more complicated. According to them, output cannot be at a sustainable level if distortions are building on the financial side, and central banks cannot simply say that rates are low due to forces out of their control - represented by z_t in our analysis. While it is beyond the scope of the small model we are using to analyze the contents of these ”other factors”contained in z_t in more detail, it would be natural to expect that the unobserved component consists mainly of influences from abroad for a small open economy like Sweden. Economic developments in Sweden have been much more favorable than in for instance the EA, the major trading partner of Sweden. We explore this idea briefly in Section 4.

All in all, the posterior parameter distributions are largely in line with our a-priori assumptions. In the Phillips curve (12), the posterior means of the coefficients have the expected signs. In the gap VAR (10), the gaps interact. It is readily verified that the VAR based on the posterior parameter means is stationary (i.e., the three eigenvalues of the posterior mean of Ψ lie within the unit circle, in modulus for complex pairs), despite that the posterior mean of φ_{11} is larger than 1. To illustrate the properties of the estimated VAR, we produce impulse responses in terms of effects from one standard deviation positive shock in the respective gaps (see Appendix A3). As can be seen in Figure 3, a positive shock to the output gap leads to a contractive monetary policy (positive real interest rate gap), which contributes to a faster return to balanced resource utilization. A positive shock in the real interest rate gap leads to a negative development of the output gap, which within four quarters leads to a negative real interest rate gap to get the economy back to equilibrium. The effects of these shocks on the real exchange rate gap are, however, relatively small. Meanwhile shocks to the exchange rate gap appear quite persistent; a positive shock to the real exchange rate gap (i.e., a short-run depreciation) is associated with an expansionary monetary policy and a positive output gap. The dynamic interaction between the estimated output and real interest rate gaps are also illustrated in Figure 4. When an output gap opens up, monetary policy responds with a lag, creating an interest rate gap. Periods with a positive interest rate gap are followed by a declining output gap, whereas periods with a negative interest rate gap gap are followed by a rising output gap.

3.1 Sensitivity analysis: comparison to alternative models

Based on Economic theory, our model allows the natural rate of interest to vary over time in response to shifts in exogenous factors and the growth rate of output. Naturally, it is of interest to study the robustness of our results to changes in the underlying assumptions. In this section, we compare the results from our baseline model and four models with alternative specifications for g_t and z_t, affecting the specifications for potential output y_t^* and the real natural interest rate r_t^*. The alternative specifications have been considered by other authors, e.g., Laubach and Williams (2003); Mésonnier and Renne (2007); Berger and Kempa (2014); Holston et al. (2017).

First alternative model (g_t is constant): We consider constant potential growth, estimated as a parameter. The model is formed by removing (6) from the baseline model, and then replacing g_t in (5) and (7) with a parameter φ_0, such that

$$r_t^* = c\varphi_0 + z_{t-1},$$
$$y_t^* = y_{t-1}^* + \varphi_0 + \varepsilon_t^{y^*}.$$

The prior mean of φ_0 is set identical to the prior mean of the parameter φ_1 in (6) of the baseline model. Note that, even though potential growth is constant, potential output is still allowed to vary due to the error $\varepsilon_t^{y^*}$.

Second alternative model ($g_t \sim I(1)$): We let potential growth follow a random walk,

$$g_t = g_{t-1} + \varepsilon_t^g.$$

All other series are defined according to the baseline model. This way, the natural rate of interest is the sum of two independent random walks, and, because g_t is $I(1)$, potential output is $I(2)$.

Third alternative model ($g_t \sim I(1)$, z_t possibly $I(0)$): We let potential growth follow a random walk, but allow z_t to be stationary by letting

$$g_t = g_{t-1} + \varepsilon_t^g,$$
$$z_t = \phi z_{t-1} + \varepsilon_t^z.$$

Thus, the real natural interest rate is still $I(1)$ due to the nonstationary trend growth, and potential output is $I(2)$. As we expect z_t to be persistent, we set the prior mean of ϕ to 0.8.

Table 2. Prior and posterior parameter distributions for alternative models.

	Prior distribution		Posterior distribution							
			Alternative model 1		Alternative model 2		Alternative model 3		Alternative model 4	
Parameter	Mean	90% interval	Mean	90% interval	Mean	90% interval	Mean	90% interval	Mean	90% interval
Potential output and growth										
$\sigma^2_{y^*}$	0.50	$[0.06, 1.28]$	0.417	$[0.321, 0.534]$	0.198	$[0.089, 0.382]$	0.205	$[0.109, 0.361]$	0.079	$[0.070, 0.091]$
φ_0	0.57	$[0.41, 0.73]$	0.630	$[0.572, 0.687]$	-	-	-	-	-	-
φ_1	0.57	$[0.41, 0.73]$	-	-	-	-	-	-	0.551	$[0.530, 0.572]$
φ_2	0.80	$[0.64, 0.96]$	-	-	-	-	-	-	0.658	$[0.622, 0.693]$
σ^2_g	0.25	$[0.11, 0.43]$	-	-	0.083	$[0.042, 0.156]$	0.104	$[0.059, 0.175]$	0.176	$[0.159, 0.195]$
Natural rate of interest										
c	4.00	$[2.34, 5.65]$	3.948	$[2.243, 5.665]$	0.523	$[0.238, 0.838]$	0.483	$[0.283, 0.690]$	0.366	$[0.317, 0.415]$
ϕ	0.80	$[0.64, 0.96]$	-	-	-	-	0.929	$[0.893, 0.961]$	0.950	$[0.948, 0.953]$
σ^2_z	0.25	$[0.11, 0.43]$	0.068	$[0.047, 0.097]$	0.056	$[0.036, 0.084]$	0.055	$[0.037, 0.081]$	0.064	$[0.058, 0.072]$
Natural exchange rate										
σ^2_q	0.25	$[0.11, 0.43]$	0.230	$[0.133, 0.384]$	0.223	$[0.130, 0.368]$	0.237	$[0.146, 0.375]$	0.225	$[0.202, 0.250]$
Output gap										
ψ_{11}	0.50	$[0.09, 0.91]$	1.116	$[1.042, 1.191]$	1.012	$[0.923, 1.100]$	1.068	$[0.990, 1.145]$	1.021	$[0.990, 1.053]$
ψ_{12}	0.00	$[-0.41, 0.41]$	-0.551	$[-0.694, -0.414]$	-0.399	$[-0.533, -0.275]$	-0.357	$[-0.487, -0.231]$	-0.368	$[-0.390, -0.345]$
ψ_{13}	0.00	$[-0.41, 0.41]$	0.012	$[-0.002, 0.026]$	0.008	$[-0.004, 0.021]$	0.013	$[0.002, 0.025]$	0.001	$[-0.003, 0.006]$
$\sigma^2_{\tilde{y}}$	0.50	$[0.06, 1.28]$	0.175	$[0.102, 0.287]$	0.238	$[0.142, 0.385]$	0.241	$[0.152, 0.376]$	0.291	$[0.283, 0.300]$
Real interest rate gap										
ψ_{21}	0.00	$[-0.41, 0.41]$	0.262	$[0.196, 0.333]$	0.269	$[0.194, 0.349]$	0.315	$[0.238, 0.393]$	0.288	$[0.263, 0.314]$
ψ_{22}	0.50	$[0.09, 0.91]$	0.604	$[0.507, 0.698]$	0.671	$[0.585, 0.756]$	0.637	$[0.548, 0.724]$	0.604	$[0.572, 0.636]$
ψ_{23}	0.00	$[-0.41, 0.41]$	-0.007	$[-0.017, 0.003]$	-0.013	$[-0.024, -0.002]$	-0.015	$[-0.024, -0.006]$	-0.020	$[-0.024, -0.017]$
$\sigma^2_{\tilde{r}}$	0.50	$[0.06, 1.28]$	0.035	$[0.020, 0.058]$	0.024	$[0.014, 0.044]$	0.007	$[0.000, 0.037]$	6×10^{-13}	$[3 \times 10^{-13}, 10^{-12}]$
Real exchange rate gap										
ψ_{31}	0.00	$[-0.41, 0.41]$	-0.096	$[-0.274, 0.084]$	-0.087	$[-0.283, 0.106]$	-0.079	$[-0.243, 0.084]$	-0.126	$[-0.175 - 0.077]$
ψ_{32}	0.00	$[-0.41, 0.41]$	0.313	$[0.078, 0.545]$	0.244	$[0.019, 0.472]$	0.266	$[0.075, 0.464]$	0.113	$[0.094, 0.132]$
ψ_{33}	0.50	$[0.09, 0.91]$	0.920	$[0.872, 0.967]$	0.930	$[0.885, 0.973]$	0.917	$[0.877, 0.956]$	0.951	$[0.933, 0.968]$
$\sigma^2_{\tilde{q}}$	1.00	$[0.13, 2.57]$	5.645	$[4.877, 6.561]$	5.679	$[4.944, 6.545]$	5.675	$[5.035, 6.414]$	5.459	$[5.213, 5.711]$
Phillips curve										
δ_1	1.00	$[0.18, 1.82]$	1.274	$[1.054, 1.494]$	1.296	$[1.083, 1.513]$	1.379	$[1.175, 1.581]$	1.449	$[1.423, 1.474]$
δ_2	0.50	$[0.09, 0.91]$	0.141	$[0.032, 0.250]$	0.140	$[0.032, 0.248]$	0.118	$[0.023, 0.212]$	0.065	$[0.050, 0.080]$
δ_3	0.25	$[-0.16, 0.66]$	0.050	$[-0.000, 0.101]$	0.055	$[0.006, 0.109]$	0.046	$[0.003, 0.089]$	0.067	$[0.051, 0.082]$
δ_4	0.25	$[-0.16, 0.66]$	0.123	$[0.029, 0.217]$	0.154	$[0.047, 0.263]$	0.175	$[0.088, 0.267]$	0.169	[0.203, 0.236]
$\sigma^2_{\tilde{\pi}}$	2.00	$[0.68, 3.88]$	1.481	$[1.266, 1.734]$	1.437	$[1.223, 1.697]$	1.452	$[1.266, 1.671]$	1.344	$[1.226, 1.476]$

Fourth alternative model (g_t and z_t possibly $I(0)$): We allow both g_t and z_t to be stationary, according to

$$
\begin{aligned}
g_t &= (1 - \varphi_2)\varphi_1 + \varphi_2 g_{t-1} + \varepsilon^g_t, \\
z_t &= \phi z_{t-1} + \varepsilon^z_t.
\end{aligned}
$$

That is, the natural interest rate is allowed to be stationary, and potential output is either $I(1)$ or $I(2)$.

For each of the alternative models, all remaining series are defined according to the baseline model, with unchanged prior parameter distributions. The prior and posterior distributions for the four alternative models are collected in Table 2. The alternative specifications for the potential growth affect the posterior parameter distributions of all error variances. This is expected, as they are central for identification. For the first alternative model, the posterior distribution of the potential growth coefficient c is very close to the prior distribution. This result - which suggests that data do not provide much information to the prior assumption - was not found for the baseline model, and it is also not found for any of the other alternative models. By imposing constant growth, an identification problem arises. It seems, however, that it can be largely neutralized by a level shift in z_t; see Figure 5.

For all four alternative models, the posterior mean of the own autoregressive parameter in the VAR output gap equation (11) is larger than 1. Yet, it is readily verified that each VAR

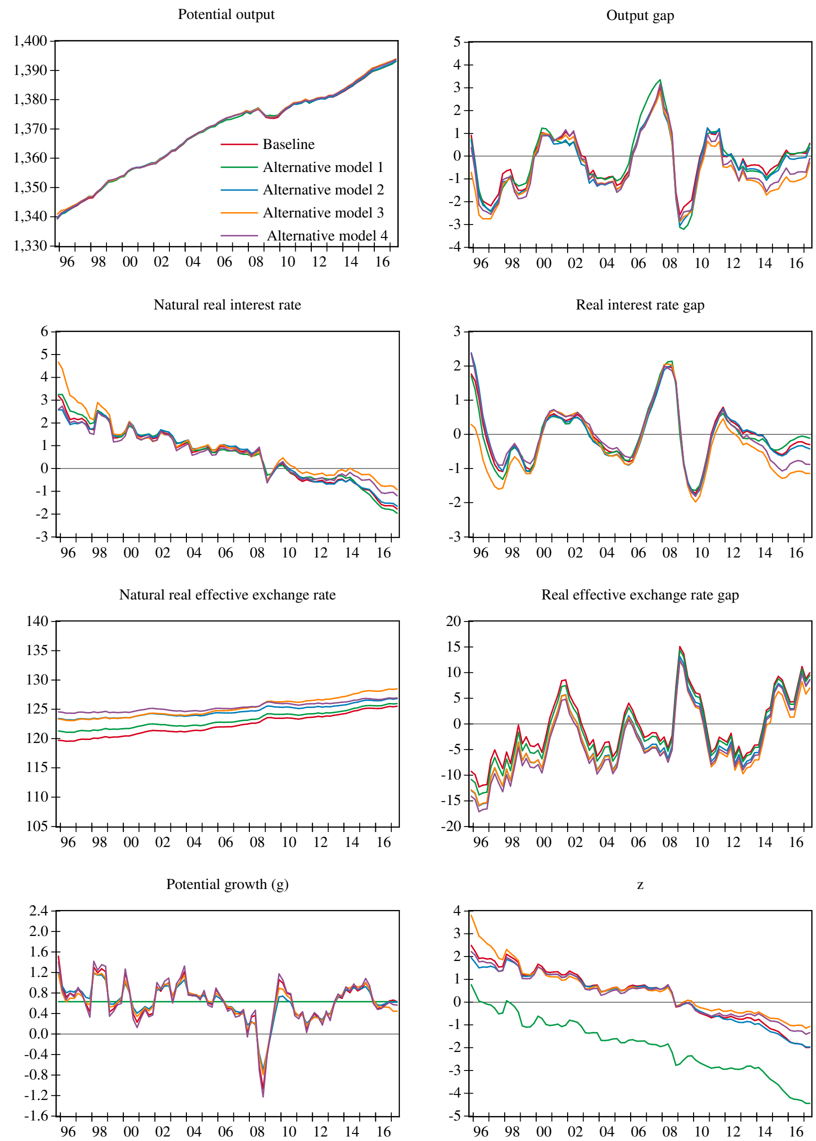

Figure 5. Estimated gaps and trends under various specifications for g_t and z_t.

is stationary. The posterior parameter distributions associated with the Phillips curve are not affected much by the different model specifications.

Although the different specifications have some overall effects on the estimated series, they do not affect the estimate of the natural interest rate much, with the exception of the periods at the start and end of the sample. There is still a clear downward sloping trend in the natural rate, and the current estimate is still below zero. Thus, the dynamic specifications of potential growth and z_t do not seem to be of major importance for our results.

4 International spill-overs

In this section, we study international spill-overs from some important trading partners to Sweden. More precisely, we study if our estimate of the Swedish natural real rate of interest cointegrates with the natural interest rates for the US, the EA and the UK estimated by Holston et al. (2017).[5] The natural interest rates are displayed in Figure 6.

As noted by Holston et al. (2017), because the natural interest rates are estimated under the presumption that they are nonstationary, we cannot use traditional methods to measure their comovement, such as correlations or principal components. The natural choice is to consider cointegration and the error-correction framework. Because the interest rates are themselves estimated, some care should be taken when interpreting the results. However, we expect both estimation and inference to be asymptotically valid, at least approximately. For all natural interest rates, ADF tests cannot reject the null hypotheses of a unit root (see Table 3). Hence, the question of cointegration becomes interesting.

We apply a vector error-correction model (VECM),

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \cdots + \Gamma_{p-1} \Delta x_{t-p+1} + v_t, \tag{13}$$

where x_t is an $n \times 1$ time series vector, $\Pi, \Gamma_1, ..., \Gamma_{p-1}$ are $n \times n$ parameter matrices, and v_t is an error term. It is well-established (see Johansen, 1995) that $I(1)$-cointegration is a restriction on the matrix Π, which under reduced rank r $(0 < r < n)$ can be decomposed into $\Pi = \alpha\beta'$, where α $(n \times r)$ is the adjustment matrix, and β $(n \times r)$ is the cointegrating matrix.

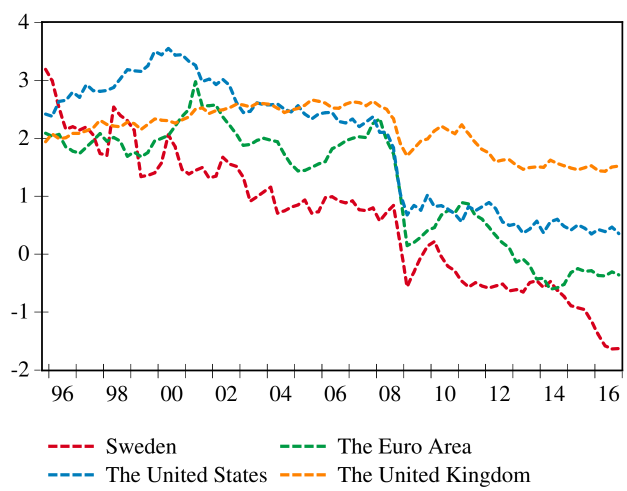

Figure 6. Estimated natural interest rates.

[5]The estimated natural interest rates by Holston et al. (2017) were downloaded from John Williams' San Francisco Fed website: http://www.frbsf.org/economic-research/economists/john-williams.

Table 3. VECMs for natural interest rates.

	constant	r_t^*	$r_{t,US}^*$	$r_{t,EA}^*$	$r_{t,UK}^*$
ADF, p-values		0.98	0.96	0.80	0.76
VECM I: one cointegrating vector	constant				
Long-run parameters, cointegrating vector β	0.886	1	**-0.470**	-0.553	0.032
			(0.228)	(0.324)	(0.472)
Adjustment parameters to $\beta' x_{t-1}$		**-0.184**	0.066	-0.007	0.008
		(0.053)	(0.036)	(0.041)	(0.021)
VECM II: two cointegrating vectors					
Long-run parameters, first cointegrating vector β_1	-0.300	1	0	**-1.264**	0.583
				(0.219)	(0.551)
Long-run parameters, second cointegrating vector β_2	-2.525	0	1	**-1.512**	1.173
				(0.243)	(0.613)
Adjustment parameters to $\beta_1' x_{t-1}$		**-0.184**	0.065	-0.012	0.006
		(0.053)	(0.037)	(0.021)	(0.021)
Adjustment parameters to $\beta_2' x_{t-1}$		0.097	-0.005	**0.156**	0.041
		(0.066)	(0.045)	(0.047)	(0.026)
VECM III: three cointegrating vectors					
Long-run parameters, first cointegrating vector β_1	5.422	1	0	0	**-2.832**
					(0.708)
Long-run parameters, second cointegrating vector β_2	4.322	0	1	0	**-2.914**
					(0.762)
Long-run parameters, third cointegrating vector β_3	4.527	0	0	1	**-2.703**
					(0.516)
Adjustment parameters to $\beta_1' x_{t-1}$		**-0.206**	0.060	-0.008	-0.012
		(0.059)	(0.041)	(0.043)	(0.023)
Adjustment parameters to $\beta_2' x_{t-1}$		0.130	0.002	**0.150**	**0.068**
		(0.076)	(0.052)	(0.055)	(0.029)
Adjustment parameters to $\beta_3' x_{t-1}$		0.114	-0.069	**-0.225**	-0.046
		(0.100)	(0.069)	(0.072)	(0.039)

Note: Conventional standard errors in parentheses; bold numbers are significant at the 5 percent level.

The r cointegrating (long-run) equations are given by $\beta' x_t$, and disequilibria occur when $\beta' x_t \neq 0$. We denote the columns of β - the cointegrating vectors - by β_j ($j = 1, 2, ..., r$).

Let $x_t = (r_t^*, r_{t,US}^*, r_{t,EA}^*, r_{t,UK}^*)'$, where r_t^* is our estimate of the Swedish real natural interest rate, and $r_{t,US}^*$, $r_{t,EA}^*$ and $r_{t,UK}^*$ are the estimated natural interest rates by Holston et al. (2017) for, respectively, the US, the EA and the UK. We use the trace test by Johansen (1991) to estimate the number of cointegrating vectors, allowing for a constant in the cointegrating relationship and a linear trend in the data. Because the trace test tends to be more sensitive, in terms of size distortions, to under-parametrization than to over-parametrization (see Cheung and Lai, 1993), we set the lag number in (13) to a rather large number, $p = 8$. All models are, however, estimated using the lags suggested by the Akaike information criterion ($p = 1$ or $p = 2$ for the reported models in Table 3 and Table 4). The trace test suggests two cointegrating relationships (i.e., $r = 2$). However, because Holston et al. (2017) reported their results for one and three cointegrating vectors, we report our results for $r = 1, 2, 3$. The most central output for each VECM - the cointegrating vectors and adjustment vectors

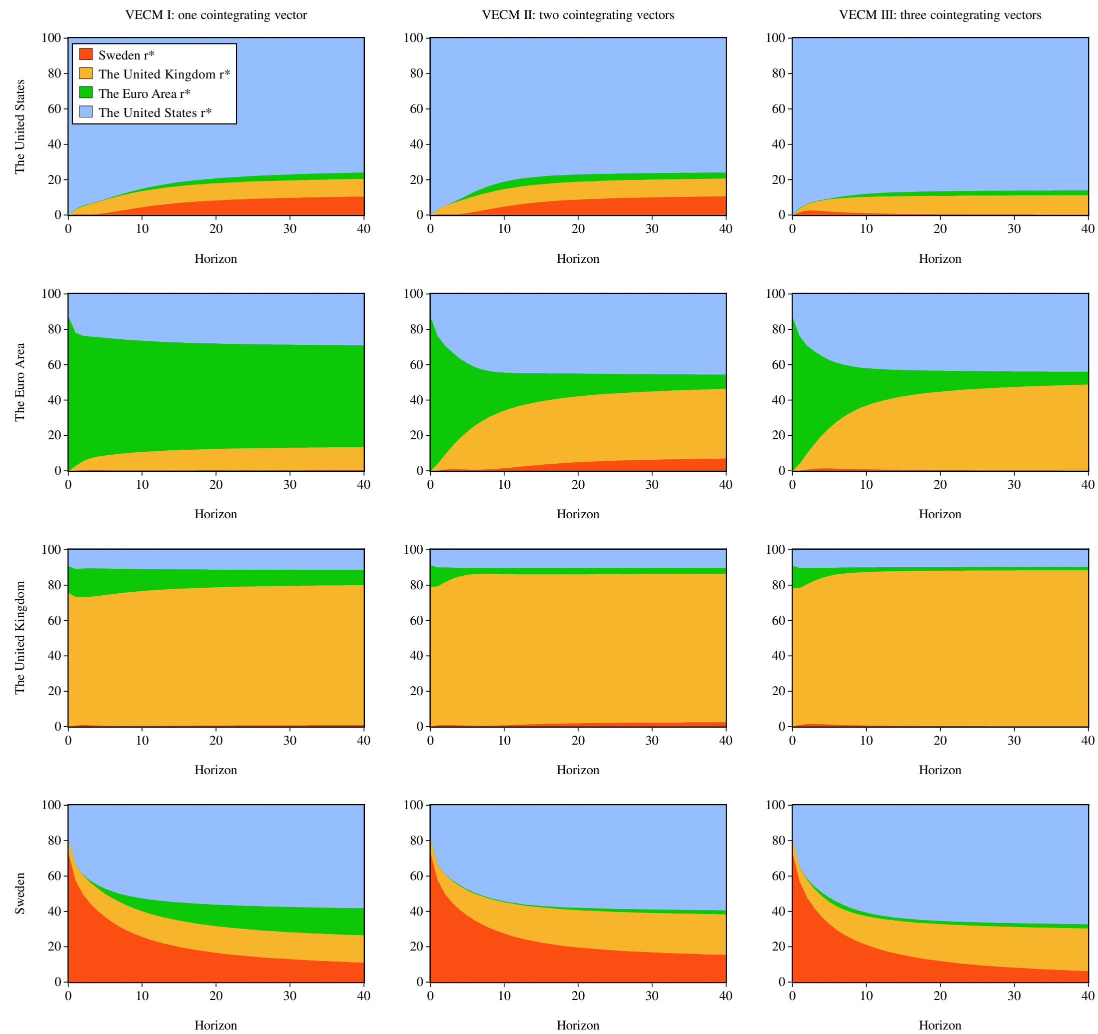

Figure 7. Variance decompositions for estimated natural interest rates.

- are shown in Table 3. Note that, without imposing identifying restrictions, the estimated coefficients are not necessarily interpretable in an economic sense. Moreover, since all the series in the long-run relationship of the model are endogenously affecting each other, the coefficients would tend to change somewhat with respect to the lag length (the short-run dynamics).

Table 3 indicates two main conclusions, even when keeping in mind that inference is weakened by the fact that the interest rates are estimated.[6] First, the estimated Swedish

[6]We expect the uncertainty surrounding the estimation of the natural interest rates to carry over to increases in variances of the finite-sample distributions of the statistics that we use. Thus, in general, we expect increases in size (as well as power) for these statistics.

Table 4. VECMs for z and international natural interest rates.

VECM IV: two cointegrating vectors	constant	g_t	z_t	$r^*_{t,US}$	$r^*_{t,EA}$	$r^*_{t,UK}$
Long-run parameters, first cointegrating vector β_1	-0.612	1	0	0	0	0
Long-run parameters, second cointegrating vector β_2	2.554	0	1	**-0.984**	0.036	-0.526
				(0.267)	(0.391)	(0.564)
Adjustment parameters to $\beta_1' x_{t-1}$		**-0.410**	**-0.196**	0.009	0.005	-0.018
		(0.084)	(0.044)	(0.050)	(0.055)	(0.029)
Adjustment parameters to $\beta_2' x_{t-1}$		-0.001	**-0.085**	0.050	-0.067	0.003
		(0.062)	(0.032)	(0.037)	(0.040)	(0.021)
VECM V: three cointegrating vectors						
Long-run parameters, first cointegrating vector β_1	-0.612	1	0	0	0	0
Long-run parameters, second cointegrating vector β_2	-0.326	0	1	0	**-1.494**	0.832
					(0.254)	(0.761)
Long-run parameters, third cointegrating vector β_3	-2.927	0	0	1	**-1.554**	**1.380**
					(0.216)	(0.535)
Adjustment parameters to $\beta_1' x_{t-1}$		**-0.456**	**-0.201**	-0.022	-0.059	-0.037
		(0.089)	(0.047)	(0.053)	(0.055)	0.030
Adjustment parameters to $\beta_2' x_{t-1}$		0.035	**-0.081**	0.075	-0.017	0.018
		(0.066)	(0.035)	(0.040)	(0.041)	(0.022)
Adjustment parameters to $\beta_3' x_{t-1}$		0.082	**0.093**	0.006	**0.179**	0.032
		(0.083)	(0.044)	(0.050)	(0.051)	(0.028)
VECM VI: four cointegrating vectors						
Long-run parameters, first cointegrating vector β_1	-0.612	1	0	0	0	0
Long-run parameters, second cointegrating vector β_2	5.107	0	1	0	0	**-2.576**
						(0.644)
Long-run parameters, third cointegrating vector β_3	2.725	0	0	1	0	**-2.165**
						(0.604)
Long-run parameters, fourth cointegrating vector β_4	3.636	0	0	0	1	**-2.282**
						(0.406)
Adjustment parameters to $\beta_1' x_{t-1}$		**-0.462**	**-0.203**	-0.023	-0.061	-0.048
		(0.091)	(0.048)	(0.054)	(0.056)	(0.030)
Adjustment parameters to $\beta_2' x_{t-1}$		0.032	**-0.083**	0.074	-0.017	0.012
		(0.067)	(0.035)	(0.040)	(0.042)	(0.022)
Adjustment parameters to $\beta_3' x_{t-1}$		0.098	**0.101**	0.010	**0.183**	**0.062**
		(0.092)	(0.048)	(0.055)	(0.057)	(0.030)
Adjustment parameters to $\beta_4' x_{t-1}$		-0.172	-0.018	-0.119	**-0.251**	-0.061
		(0.128)	(0.068)	(0.077)	(0.080)	(0.042)

Note: Conventional standard errors in parentheses; bold numbers are significant at the 5 percent level.

natural rate of interest error-corrects significantly to disequilibria from the first cointegrating vector for all of the three VECMs that are reported. About one fifth of the disequilibrium is expected to be restored in each period. Second, the US natural interest rate does not error-correct significantly to any of the VECMs, speaking in favor of weak exogeneity, that is, that the US natural interest rate is influencing the long-run development of the other natural interest rates, but is not influenced by them.

Variance decompositions for the three VECMs are shown in Figure 7, each VECM corresponding to a column. Clearly, the US natural interest rate is a major source of variance in the natural interest rates. Meanwhile, shocks to the Swedish natural interest rate do not

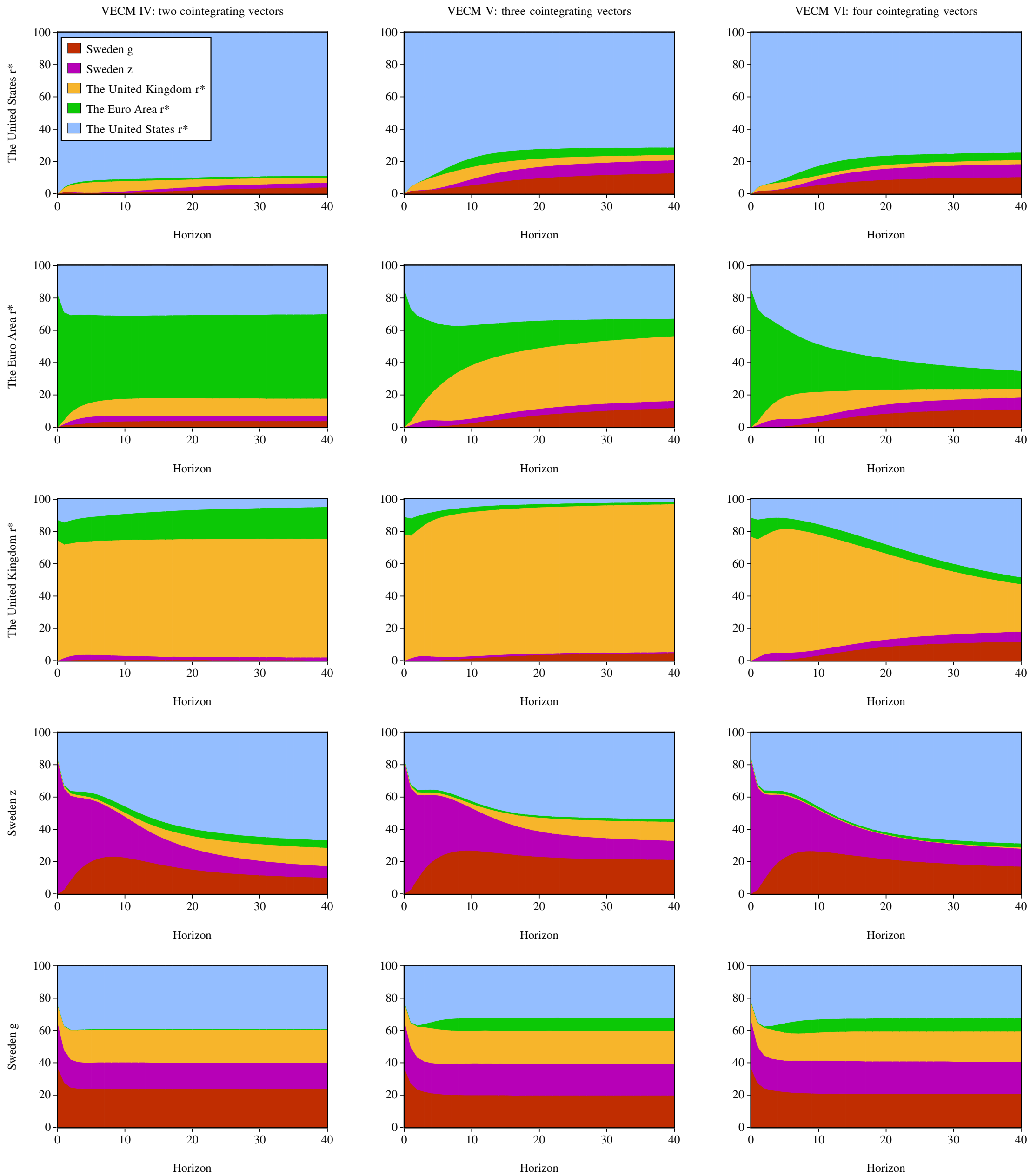

Figure 8. Variance decompositions for z_t and estimated international natural interest rates.

contribute much to the variation in the other natural rates. Moreover, except for at very short horizons, shocks to the Swedish natural rate of interest do not contribute particularly much even to its own variation. This suggest that it is largely dependent on the international natural interest rates, confirming the relatively large adjustment terms reported in Table 3.

Next, we let $x_t = (g_{t-1}, z_{t-1}, r^*_{t,US}, r^*_{t,EA}, r^*_{t,UK})'$, where g_t and z_t are our baseline estimates of the natural interest rate components in (7). By construction, g_t is stationary and z_t is nonstationary.[7] While the stationary trend growth cannot cointegrate with the nonstationary components in x_t in the typical meaning of the term (i.e., they cannot share stochastic trends), the cointegrating rank will by definition be increased by one from the cointegrating rank that exists between z_t and the international interest rates. Therefore, we let g_t be a cointegrating vector by itself. This time the trace test suggests four cointegrating vectors, that is, three cointegrating vectors excluding g_t. In Table 4, we have reported the cases with $r = 2, 3, 4$.

Replacing our estimated natural interest rate with the estimated components g_t and z_t leads to some interesting results. Both g_t and z_t error-corrects significantly to disequilibria in every VECM reported in Table 4. The component g_t only error-corrects significantly to disequilibria in itself, relating to the behavior of a mean reverting process. The component z_t error-corrects significantly to disequilibria in g_t as well as in other cointegrating equations, suggesting that (i) departure from steady-state domestic potential growth affects the "other factors" contained in z_t, and (ii) these "other factors" are affected by international natural interest rates. As before, the US natural interest rate does not error-correct significantly to any disequilibria, in any of the VECMs, suggesting that it is weakly exogenous. The decomposition of r^*_t into g_t and z_t also has implications for the variance decompositions, as shown in Figure 8. Although shocks to g_t and z_t do not contribute much to the variance in the international natural interest rates, they both contribute non-trivially to the variances in both g_t and z_t.

To further analyze the relationship between the interest rates, we perform Granger-causality tests using VAR models with, respectively, vectors $x_t = (z_{t-1}, r^*_{t,US}, r^*_{t,EA}, r^*_{t,UK})'$ and $x_t = (r^*_t, r^*_{t,US}, r^*_{t,EA}, r^*_{t,UK})'$. As Granger-non-causality implies zero-restrictions on the corresponding off-diagonal elements in the parameter matrices of the VAR, the tests may be executed by Wald tests (see, e.g., Lütkepohl, 2007, Section 3.6). If cointegration exists, then Granger-causality must exist in at least one direction between the elements in x_t (see Granger, 1988). Table 5 shows the results from tests with the null hypothesis of non-Granger-causality (that is, we reject in favor of Granger-causality) based on regular Wald statistics from a VAR estimated in levels. Because x_t is $I(1)$-nonstationary, we follow Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996) and add an extra lag to estimate the VAR, but apply the tests to the parameters up to the original choice of lag number. This procedure ensures us that the Wald test is asymptotically valid.[8] We report results for 1 and 2 lags in the VARs (the Schwarz criterion suggests 1 lag and the Akaike criterion suggests 2 lags, for both cases of x_t). The left pane of Table 5 suggests that the Swedish natural interest rate does not Granger-cause any of the international natural interest rate. Meanwhile, the results supports that US

[7] The ADF test cannot reject the null hypothesis of a unit root in z_t, with a p-value of 0.979, but rejects null hypothesis of a unit root in g_t, with a p-value of 0.002.

[8] There are three main approaches for testing Granger-non-causality in the present framework when the series are potentially integrated. First, we can use the VECM (13) directly. Second, we can transform the series to stationarity and use a VAR model. Third, we can estimate a VAR in levels following Toda and Yamamoto (1995). The latter approach is generally seen as the best in terms of stability (e.g. size), due to that it involves known asymptotic distributions under the null hypothesis without nuisance parameters.

Table 5. Granger non-causality tests.

Lag order	VARs that include r_t^*		VARs that include z_t	
	Null hypothesis	Chi-square statistic	Null hypothesis	Chi-square statistic
1	$r_{t,US}^*$ does not Granger-cause r_t^*	**7.946**	$r_{t,US}^*$ does not Granger-cause z_t	**6.684**
	$r_{t,EA}^*$ does not Granger-cause r_t^*	0.293	$r_{t,EA}^*$ does not Granger-cause z_t	0.053
	$r_{t,UK}^*$ does not Granger-cause r_t^*	0.743	$r_{t,UK}^*$ does not Granger-cause z_t	0.408
	r_t^* does not Granger-cause $r_{t,US}^*$	1.142	z_t does not Granger-cause $r_{t,US}^*$	0.787
	r_t^* does not Granger-cause $r_{t,EA}^*$	0.535	z_t does not Granger-cause $r_{t,EA}^*$	1.079
	r_t^* does not Granger-cause $r_{t,UK}^*$	2.124	z_t does not Granger-cause $r_{t,UK}^*$	2.508
2	$r_{t,US}^*$ does not Granger-cause r_t^*	4.283	$r_{t,US}^*$ does not Granger-cause z_t	**6.210**
	$r_{t,EA}^*$ does not Granger-cause r_t^*	1.091	$r_{t,EA}^*$ does not Granger-cause z_t	2.697
	$r_{t,UK}^*$ does not Granger-cause r_t^*	0.396	$r_{t,UK}^*$ does not Granger-cause z_t	0.416
	r_t^* does not Granger-cause $r_{t,US}^*$	0.303	z_t does not Granger-cause $r_{t,US}^*$	0.234
	r_t^* does not Granger-cause $r_{t,EA}^*$	2.845	z_t does not Granger-cause $r_{t,EA}^*$	3.420
	r_t^* does not Granger-cause $r_{t,UK}^*$	0.839	z_t does not Granger-cause $r_{t,UK}^*$	1.425

Note: Bold statistics are significant at the 5 percent level using conventional critical values. The degrees of freedom of the chi-square distribution is equal to the lag order.

natural interest rate Granger-causes the Swedish natural interest rate, even though the Wald statistic is not significant for the case with two lags. Likewise, the right pane suggests that z_t does not Granger-cause the international natural interest rates, whereas the US natural interest rate (significantly) Granger-causes z_t.

Put together, our results suggest that the longer-term decline in the natural interest rate for the last decades seems to be largely driven by international factors, in line with what the Riksbank has been arguing (see, e.g., Sveriges Riksbank, 2017b). Our analysis thus gives some formal support for that type of statement.

5 Conclusions

We have produced an estimate of the natural rate of interest in Sweden, using a small-scale macroeconomic model. We found that the natural rate is currently negative in Sweden, and that it has been on a declining trend for the past two decades. Most of the decline in the Swedish natural rate can in our model be traced to unobserved components that are unrelated to growth of potential GDP. The Riksbank has in its' communication stated that the decline in the Swedish natural interest rate has its origin in global factors. We assess this hypothesis by testing for cointegrating relationships between our estimate of the natural rate in Sweden and estimated natural rates of the United States, the euro area and the United Kingdom. Although the cointegration tests should be interpreted with some caution, since the time series are themselves estimated, we do find evidence of cointegrating relationships. We also find a significant error correction term for the Swedish natural rate. About one fifth of a disequilibrium from a linear combination of the other rates is expected to be recovered each quarter. Additionally, we find significant influence coming from the natural rate in the United States.

This finding could have important implications for monetary policy in Sweden, and in other small open economies.

Appendix: Technical details

Our estimation method follows, in large, Berger and Kempa (2014). The method is explained in detail in the following subsections.

A1. A state space representation

Let x_t be a $p \times 1$ vector of time series observed over time periods $t = 1, 2, ..., T$. Linear time series can be cast in state space form:

$$x_t = \mu + H\alpha_t + Aw_t + u_t, \tag{A1}$$

$$\alpha_{t+1} = \kappa + B\alpha_t + R\eta_t, \tag{A2}$$

where w_t is a $l \times 1$ vector of exogenous observable time series, α_t is a $s \times 1$ latent state vector, H, A and B are coefficient matrices of appropriate dimensions, R is a selection matrix (that usually consists of a subset of the columns of the identity matrix), μ ($p \times 1$) and κ ($s \times 1$) are vectors of constants, and u_t ($p \times 1$) and η_t ($d \times 1$) are error time series. Equations (A1) and (A2) are referred to as the signal equation and state equation, respectively. By imposing assumptions on the signal and state errors, the latent state vector α_t can be estimated for $t = 1, 2, ..., T$ by the Kalman filter and smoother. The Kalman filter is a (one-sided) forward recursion that uses estimation only up to time t, whereas the Kalman smoother is a (two-sided) backward recursion that, based on the output from the Kalman filter, uses information from the whole sample; see, e.g., Harvey (1989) and Durbin and Koopman (2012). We use the Kalman smoother to estimate the states under the assumption that u_t and η_t are mutually independent Gaussian white noise processes with contemporary distributions $u_t \sim \mathcal{N}(0, \Sigma_u)$ and $\eta_t \sim \mathcal{N}(0, \Sigma_\eta)$. The Kalman filter is initialized using the diffuse Kalman filter developed by de Jong (1991). All modelling was done in the EViews 9.5 programming language; the codes can be sent upon request.

The state space representation of equations (2)-(10) and (12) is outlined as follows. Equations (2)-(4) and (12) have left-hand side time series that are observed, and are therefore modeled as signal equations. Equations (5)-(10) have left-hand side time series that are unobserved, and are therefore modeled as state equations.

The time series components are

$$\begin{aligned} x_t &= (y_t, r_t, q_t, \pi_t)', \\ w_t &= (\pi_{t-1}, \Delta q^n_{t-1})', \\ \alpha_t &= (y^*_t, r^*_t, q^*_t, g_t, z_t, \tilde{y}_t, \tilde{r}_t, \tilde{q}_t)'. \end{aligned}$$

The coefficient vectors and matrices are

$$\mu = (0, 0, 0, \delta_1)',$$
$$\kappa = (0, 0, 0, (1 - \varphi_2)\varphi_1, 0, 0, 0, 0)',$$

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \delta_4 & 0 & 0 \end{pmatrix},$$

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \delta_2 & \delta_3 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varphi_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_{11} & \psi_{12} & \psi_{13} \\ 0 & 0 & 0 & 0 & 0 & \psi_{21} & \psi_{22} & \psi_{23} \\ 0 & 0 & 0 & 0 & 0 & \psi_{31} & \psi_{32} & \psi_{33} \end{pmatrix},$$

and the selection matrix is

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The error u_t is four-dimensional, where only the last element is non-zero, $u_t = (0, 0, 0, \varepsilon_t^{\pi})'$. Its contemporary covariance matrix is therefore diagonal, $\Sigma_u = \text{diag}(0, 0, 0, \sigma_\pi^2)$. The error η_t is seven-dimensional, $\eta_t = (\varepsilon_t^{y^*}, \varepsilon_t^{q}, \varepsilon_t^{g}, \varepsilon_t^{z}, \varepsilon_t^{\tilde{y}}, \varepsilon_t^{\tilde{r}}, \varepsilon_t^{\tilde{q}})'$. By assumption, its elements are independent, so that its contemporary covariance matrix is diagonal with non-zero elements, $\Sigma_\eta = \text{diag}(\sigma_{y^*}^2, \sigma_{q^*}^2, \sigma_g^2, \sigma_z^2, \sigma_{\tilde{y}}^2, \sigma_{\tilde{r}}^2, \sigma_{\tilde{q}}^2)$.

A2. Parameter estimation

Let the parameters of the model be collected in the vector $\theta = (\theta_{nv}, \theta_v)'$, where θ_{nv} is a vector of non-variance parameters and θ_v is a vector of the log of the variance parameters (for reasons to be explained below), and let X denote the stacked vector of observable time series, $X = (x'_1, x'_2, ..., x'_T)'$. Given X, the state vector α_t is estimated numerically for $t = 1, 2, ..., T$ by the Kalman smoother. In this paper, we treat θ and α_t ($t = 1, 2, ..., T$) as random parameter vectors, following, in large, the procedure in Berger and Kempa (2014).

For the sake of brevity, we first outline the method of estimation for the parameters in θ. The estimation of the states α_t follow the same principle, and are described later. By Bayes' theorem we have that

$$p(\theta)p(X|\theta) \propto p(\theta|X),$$

where $p(\theta)$ denotes the prior density of θ, $p(X|\theta)$ denotes the likelihood function and $p(\theta|X)$ denotes the posterior density of θ. Let $m(\theta)$ be a function of θ such that a moment of the posterior density is obtained by

$$m = \mathrm{E}[m(\theta)|X] = \int m(\theta)p(\theta|X)d\theta. \tag{A3}$$

We use importance sampling (see Särkkä, 2013, for a thorough treatment) with an importance density $i(\theta|X)$ as a proxy for $p(\theta|X)$. Let

$$f(\theta, X) = \frac{p(\theta)p(X|\theta)}{i(\theta|X)} \propto \frac{p(\theta|X)}{i(\theta|X)} \tag{A4}$$

be a weighting function such that expectations under $p(\theta|X)$ are the same as expectations under $f(\theta, X)i(\theta|X)$. After some manipulations, Equations (A3) and (A6) follow from

$$m = \frac{\int m(\theta)f(\theta, X)i(\theta, X)d\theta}{\int f(\theta, X)i(\theta|X)d\theta}.$$

We can sample θ from the known importance density a large number of times (n), and produce an estimate of m by

$$\tilde{m} = \sum_{i=1}^{n} w_i m(\theta^{(i)}), \tag{A5}$$

where $\theta^{(i)}$ is the ith draw from $i(\theta|X)$, and w_i is the weighting function

$$w_i = \frac{f(\theta^{(i)}, X)}{\sum_{i=1}^{n} f(\theta^{(i)}, X)}. \tag{A6}$$

If $i(\theta|X)$ is proportional to $p(\theta|X)$, then, under some weak regularity conditions (see Geweke, 1989), $\tilde{m}$ is an, almost surely, consistent estimator of m ($\tilde{m} \overset{a.s.}{\to} m$), as $n \to \infty$. As in Berger and Kempa (2014), we choose a normal distribution as the importance density,

$$i(\theta|X) = \mathcal{N}(\theta^{(g)}, \hat{\Omega}^{(g)}),$$

where g denotes the gth step in the sequential updating algorithm. The algorithm starts from $\theta^{(0)} = \mathcal{M}$ and $\hat{\Omega}^{(0)} = 2\mathcal{J}^{-1}$, where $\mathcal{M}$ is the estimated posterior mode and $\mathcal{J}$ is the approximate hessian obtained when maximizing

$$\log p(\theta|X) = \log p(X|\theta) + \log p(\theta) - \log p(X).$$

For this purpose, we use the BFGS-algorithm (see, e.g., Chapter 8 of Nocedal and Wright, 1999, for details). Since $p(X)$ is not a function of θ, it can be disregarded in the optimization. The inverted hessian is inflated with a factor of 2 to take into account the risk of thicker tails in the posterior distribution, as proposed by Bauwens et al. (1999). Defining the variance parameters in terms of logarithms ensures that the optimization searches over positive real numbers. Moreover, the logarithm function is itself defined over the whole real line, which makes it a suitable transformation under the importance density.

We update the importance density using the estimated posterior mean and posterior variance by

$$\hat{\theta}^{(g)} = \tilde{m}^{g-1}; \;\; m(\theta) = \theta,$$

and

$$\hat{\Omega}^{(g)} = \tilde{m}^{g-1}; \;\; m(\theta) = (\theta - \mathrm{E}[\theta|X])(\theta - \mathrm{E}[\theta|X])',$$

until a satisfying precision of $\hat{\theta}^{(g)}$ is obtained. The precision of an element in $\hat{\theta}^{(g)}$ (denoted $\hat{\theta}_j^{(0)}$) is measured by the 95 percent relative error bound given by Bauwens et al. (1999, Eq. 3.34). We update the importance density until this error bound does not exceed 10 percent for any $\hat{\theta}_j^{(0)}$.

The procedure defined above is also applied to estimate the posterior mean for each smoothed state by setting

$$\hat{\alpha}_t = \tilde{m}; \;\; m(\theta) = \alpha_t, \; (t = 1, 2, ..., T),$$

where $\alpha_t^{(i)}$ in equation (A5) is the state estimated by the Kalman smoother using $\theta^{(i)}$ from the importance density in the last step of the updating algorithm.

Percentiles for the posterior marginal densities can now be obtained using the following procedure. Let $F(\theta_j^{(a)|X}) = Pr(\theta_j^{(a)} \leq \theta_j|X)$, where $\theta_j^{(a)}$ is an arbitrary value. An estimate of $F(\theta_j^{(a)|X})$ is obtained by

$$\hat{F}(\theta_j^{(a)|X}) = \tilde{m}; \;\; m(\theta) = I(\theta_j^{(a)}),$$

where $I(\theta_j^{(a)})$ is an indicator function that equals one if $\theta_j^{(a)} \leq \theta_j$, and zero otherwise. An estimate of the bth percentile of the marginal posterior density is thereby given by the $\theta_j^{(a)}$ such that $\hat{F}(\theta_j^{(a)|X}) = b$.

The percentiles forming the 90 percent posterior intervals for the smoothed states are obtained by

$$\alpha_{j,t}^{95\%} = \tilde{m}; \;\; m(\theta^{(i)}) = \alpha_{j,t}^{(i)} + 1.645\sqrt{\hat{U}_{j,t}^{(i)}}, \;\; (t = 1, 2, ..., T),$$
$$\alpha_{j,t}^{5\%} = \tilde{m}; \;\; m(\theta^{(i)}) = \alpha_{j,t}^{(i)} - 1.645\sqrt{\hat{U}_{j,t}^{(i)}}, \;\; (t = 1, 2, ..., T),$$

where $\alpha_{j,t}^{(i)}$ is the jth element of the estimated smoothed state vector and $\hat{U}_{j,t}^{(i)}$ is the jth diagonal element of the estimated smoothed state covariance matrix using $\theta^{(i)}$ from the importance density. These posterior intervals capture both parameter and filter uncertainty.

A3. Impulse responses for the gap VAR

The impulse responses for the first-order VAR (10) in Figure (3) are created by averaging over the n importance samples (see aforementioned), based on the moving average representations. The mean impulse responses at horizon h to a shock in the jth gap ($j = 1, 2, 3$) are given by the 3×1 vector

$$IR_j(h) = \sum_{i=1}^{n} w_i (\Psi^{(i)})^h s_{i,j},$$

where w_i is the weighting function (A6), $\Psi^{(i)}$ is the ith draw of the VAR parameter matrix and $s_{i,j}$ is a 3×1 vector from the ith draw where the jth element is an innovation and the other elements are zero. For every draw, a shock is set equal to the standard deviation of the respective gap innovation.

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