Stockholms universitet

Research Report

Department of Statistics

No. 2013:7

Combination of sample surveys and projections of political opinions

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Abstract

In Sweden, political party preferences are surveyed almost every month by several institutes. The sample sizes are usually between 1000 and 2000 individuals, which means that the standard deviations are between 1 and 1.5 %. We study how these estimates can be improved by combining them and by modelling the behaviour over time. Our model is a combination of a dynamic model based on Wiener processes and sampling theory with design effects and measurement biases. The variances of our estimates are about 1/3 of those of the original polls when only previous polls are used and about 1/5 if the information in later polls is included. The proposed method leads to a smaller bias since the institute biases can be estimated. The party preferences are modelled as random processes, making it possible to study the probability for events like a party (or block) getting more than 50 % of the political preferences. Assuming that the same model will hold in the future, we can present intervals for future election results.

Keywords: Dynamic models, Meta analysis, Party preferences, Poll of polls, Sample surveys, Wiener process

1 Introduction and background

Political opinion polls in Sweden are made monthly by several institutes. Four of the important private actors are SIFO, Temo/Ipsos/Synovate, SKOP and Novus. In addition, Statistics Sweden makes a poll twice a year. In this paper, we study how these estimates can be combined to get improved estimates of the political opinion. Since the surveys have different sampling designs and sizes, interview periods and questionnaires, one has to find a method to pool them into a single figure. Our approach is a combination of dynamic modelling (Harrison & Stevens, 1976) based on Wiener processes and sampling theory with design effects. As a fringe benefit, we are able to predict an upcoming election outcome, given that the model will hold in the near future.

The idea of pooling polls, or "poll of polls", was suggested by Silver (2008) and by Salmond (2012). The aim was to get an optimal estimate of the prevailing opinion. Silver (2008) also discussed estimation of trends and used a simple Gibbs sampling technique based on regression analysis to forecast the outcome of the next election. We suggest the use of diffusion processes (see e.g. Øksendal, 2004) as the underlying model in order to avoid the rigidity of linear models and to be able to distinguish between imprecision due to measurement error and caused by obsolete data.

There are several papers on forecasting election results, e.g. Lewis-Beck (1988, 2005), Lewis-Beck & Jerome (2010), Lewis-Beck & Belanger (2012), Rattinger (1991), Sanders (1995) and Soroka & Wiezien (2005). They all use different economic, social or political factors to explain the election outcome, which in turn means that they have to forecast these factors in order to predict the upcoming elections. In contrast, we assume that the previous economic and political developments are reflected in the available polls and that future political actions, e.g. scandals, cannot be predicted but instead must be modelled in terms of unknown or random effects, so we rely on the fact that pollsters have tried to design polls to predict elections, see e.g. Lewis-Beck (2001) and Eklund & Järnbert (2011).

In the following section, we use a Wiener process as our model. In Sections 3 and 4, we introduce more features to make the model more realistic. Also, our approach is described in more detail. Most of the theoretical derivations are given in the appendix. In Subsection 4.1, we introduce design effects for the polling institutes to account for differences in questionnaire designs, sampling and estimation methods. In Subsection 4.3, the model is transformed to cope with political parties of different sizes. In Section 5, our data are described and estimation results are given in Section 6. In the final Section 7, we present some concluding comments and discuss further applications.

Table 1. Opinion polls in Sweden during a typical month (March 2012).
The proportion of uncertain/undecided is defined differently for different institutes.
Non-response is not always given and is excluded here. United minds, YouGov and Sentio Research are based on web-panels. (Source: Novus group, 2012)

Institute	M	FP	C	KD	S	V	MP	SD	Oth	Un-cert	Public. Date (2012)	Interview period	Size
Temo/ Ipsos/ Synovate	29.7	6.1	5	3.4	35.6	5.7	9.3	4.2	5.3	20.2	30 March	8 – 27/3	2502
Novus	30.5	5.7	4.8	3.6	32.4	7.3	8.8	5.7	6.9	8.6	21 March	21/2– 19/3	2000
Sifo	28.7	5.9	4.9	3.5	33.7	6.1	10.3	5.2	6.9	11.9	18 March	5 – 15/3	1918
Demoskop	32.9	5.5	4.9	3.6	29.7	6.3	10.9	5.2	6.2	11.2	14 March	29/2– 7/3	1000
YouGov	32.1	5.6	4.7	4.2	29.6	8.3	9.4	5.4	6.2		12 March	1/2 – 6/3	1754
United Minds	29.3	5.8	4.7	4.6	32.5	7.3	7.8	7.5	7.9	11	11 March	13/2– 11/3	1161
Skop	33.4	6.9	5	3.3	30.8	6.5	9.1	4.5	5.1	9	7 March	19/2– 6/3	1000
Sentio Research	29.7	4.3	4.2	4.6	31.8	7.4	8.1	8.6	9.9		5 March	23/2– 1/3	1035

2 A basic model for political preferences

2.1 A Wiener process model

In our basic model, the proportion of voters supporting a party/party block behaves like a Wiener process over time (also referred to as Brownian motion or random walk). According to the Wiener process, it is equally likely that the proportion of voters for a party decreases as it is that it increases from the present level. This is a mathematical model which does not explicitly take all available extra information into account. For example, the time when a party changes its leader, should it be predictable or not, will be modelled as taking place at an unknown random time in the future. Similarly, other influential events such as financial crises, sex scandals, political debates or special campaigns are considered as random events that cannot be foretold neither in time, nor in political magnitude. Such factors will, however, implicitly be taken into account as soon as they have an impact on the opinion polls. The model thus uses only observed data and does not incorporate any correction for subjective (but generally held) beliefs that the opinion usually swings away from the incumbents in the middle of a parliamentary term and then swings back in favour of the sitting government as the election gets closer. A specialist in political science can very likely improve on our model by using his/her expert knowledge. Our aim, however, is to make estimates and predictions by only using observed polls.

The proportion of a party/party block at time t is denoted P_t, which is assumed to be a normally distributed variable with expected value $E(P_t)$ and variance $C(P_{t,}P_t)$, and its *development* is modelled by a random process given by the stochastic differential equation

$$dP_t = \gamma \, dW(t),$$

where $W(t)$ is a Wiener process that accumulates all factors that affect voter opinions until time t.

The solution to this equation says that if a party at a certain day t has the share P_t of the votes, then the proportion s days ahead will be normally distributed around P_t with the variance increment γ s, i.e. the distribution at t+s has mean and variance

$$E(P_t), C(P_t, P_t) + (\gamma \, \mathrm{s}) \tag{1}$$

if there is no new information available before t+s.

2.2 Updating when new polls arrive

The proportion P_t is unknown in advance and measured with a random error by opinion polls and exactly by general elections. Our knowledge thus has to be updated when new opinion poll results are published. Since our model assumptions lead to normally distributed increments, we only have to update the expected values and variances. Suppose that at time t the result X_t of an opinion poll becomes available, measuring P_t with error V_t. Combining the prior belief with the observed poll, the true proportion P_t is normally distributed with a mean that is weighted between the prior proportion and the observed poll. The weights should be inversely proportional to the variances, which gives the new proportion

$$\frac{V_t E(P_t) + C(P_{t,}P_t) X_t}{V_t + C(P_t, P_t)}, \tag{2}$$

and the new variance is given by the now smaller value

$$\frac{V_t C(P_t, P_t)}{V_t + C(P_t, P_t)}. \tag{3}$$

With this updating step, the model may also be viewed as a dynamic linear model based on the Kalman filter (Harrison & Stevens, 1976). This model is akin to the models used by financial experts for modelling prices at the stock exchange. In that case, although there is a lot of information around, most of the effects are already capitalised in the stock prices, and it is difficult to predict the future development.

This section can be summarized as follows. We start with a vague prior distribution. If no opinion polls are available, the updating is done using (1) and when a new poll becomes known, the updating is done by (2) and (3). Estimation is done successively, starting with the first poll and continuing to the present date.

3 A model with a trend

3.1 Background

In the following, we will introduce a trend into the simple model above. The best predictor of the future was in the preceding section merely the estimate of the present level. One might argue that there may also be a trend involved. If a party has been steadily increasing its share of voters for the last period, one may expect that it will continue to do so. However, such a trend cannot go on forever so the trend is expected to decrease gradually and to be replaced by another other trend. The following model contains three positive parameters: γ, measuring the size of the short term fluctuations, i.e. the instability (or volatility) of voters; α, a duration parameter that measures how fast the trend disappears (e.g. α = 0.05 means that half the trend will remain after roughly 1/0.05 = 20 days); and β, a measure of how the randomness can explained by the (evolving) trend.

3.2 The process

As before, let P_t be the true level of the sympathy for a party (or party block) and let T_t be the trend in P_t. We assume that the trend behaves like an Ornstein-Uhlenbeck process, i.e. it follows the stochastic differential equation

$$dT_t = -\alpha T_t dt + \beta dW_1(t), \tag{4}$$

where $W_1(t)$ is a Wiener process so that $dW_1(t)$ is white noise. This formula says that there is a trend which will be fading out (depending on α), and the second term describes how large the variation is that will be described by the trend. The changes in the party's share of the voters has a constant noise term as before but also a drift proportional to T_t

described by (4), i.e. it follows the stochastic differential equation

$$dP_t = T_t dt + \gamma dW_2(t)\,, \tag{5}$$

where $W_2(t)$ is another independent white noise process. From (4) and (5), the behaviour of the process and expected values, variances and covariances can be obtained, as well as predictions, deferred to the appendix.

3.3 Updating the knowledge through polls

Before the observation at time *t*, the expected values are denoted $E(P_t)$ and $E(T_t)$, the variances are denoted $C(P_t, P_t)$ and $C(T_t, T_t)$, and the covariance is $C(P_t, T_t)$ After observing X_t from a poll with the variance $Var(X_t - P_t) = V_t$, the prior is updated by combining the observation and the prior to obtain the posterior. The expected levels are updated exactly as in formulas (2) and (3), and updating the distribution of the trend is also straightforward:

$$E(T_t \mid X_t) \;=\; E(T_t) + \frac{C(P_t, T_t)}{C(P_t, P_t) + V_t}(X_t - E(P_t)) \tag{6}$$

with the variance

$$Var(T_t \mid X_t) \;=\; C(T_t, T_t) - \frac{C(P_t, T_t)^2}{C(P_t, P_t) + V_t} \tag{7}$$

and the covariance

$$Cov(P_t, T_t) \;=\; C(P_t, T_t) - \frac{C(P_t, T_t)C(P_t, P_t)}{C(P_t, P_t) + V_t}\,. \tag{8}$$

In short, this extended model works similar to the basic model: we start with some vague distribution and if no opinion polls are available, the update is done using the formulas in the appendix. When a new opinion poll arrives, the update is done by (2), (3) and (6)-(8).

This procedure works when the goal is to estimate the present (or future) situation using all available polls. Sometimes, one may be interested in studying the development during an earlier period, for example the previous year or the period between two elections. In that case, one should use all available polls. The proportion P_t at time *t* is thus estimated using polls performed after the time point.

When additionally using subsequent polls to estimate the level at a certain earlier date, the procedure updates from both ends of the observed period and the estimates from both directions are combined by similar formulas. This procedure will not be presented here since it is straightforward and not central to this presentation.

4 Political parties, data and model estimation features

4.1 Parties in Sweden

The parties in the Swedish Parliament (Riksdagen) can be divided into two blocks. The Bourgeois block, called the Alliance, forms the government since 2006 and comprises the conservative Moderates (Moderaterna, M), the Christian Democrats (Kristdemokraterna, KD), the Liberals (Folkpartiet, FP) and the former agrarian Centre Party (Centern, C). The opposition consists of three parties: the Social Democrats (Socialdemokraterna, S), the Left Party (Vänsterpartiet, V) and the Green Party (Miljöpartiet, MP). During the later half of the last parliamentary term, these three opposition parties formed a coalition with the objective of winning the elections in 2010 and forming the government together. However, as they did not succeed, they are now acting as three separate parties. Yet, it is still common among political commentators to compare their total size to that of the Alliance. These seven parties were represented in the parliament in 2006. In 2010, a new populist party, the Sweden Democrats (Sverigedemokraterna, SD), took seats in the parliament but do not belong to any of the two blocks. There are also some other smaller parties that are not represented in the parliament, supported by less than 4 % of the voters. More information about the Swedish parliamentary system and the political parties can be found on the homepage of the Swedish Parliament (Riksdagen, 2012).

We estimate the support both of the parties and the party blocks with our models. When studying the party blocks we have removed the Sweden Democrats and smaller parties not in the Parliament, hence we study the support of the Alliance to all seven parties: (M+KD+FP+C)/(M+KD+FP+C+MP+S+V). This is the usual approach in Sweden and there seems to be a consensus that the larger of these two blocks shall form the government, disregarding the Sweden Democrats.

A special note should be made on the political development during this period. The Social Democrats elected a new leader on March 25, 2011. Soon afterwards, he made some unfortunate statements and the party's popularity decreased rapidly. In late January 2012, he was replaced by Stefan Löfvén and the party regained much of its support. As early as in April 2012, the situation was back to normal. Our models do not take such information into account explicitly, but the models should adapt fast to such shocks.

4.2 The data

We use all Swedish party preference studies from June 2006 until October 2012, altogether 313 observed polls from the five institutes. During this period, there were two general elections: in (September) 2006 and 2010. The data have been retrieved from the public home page of the Novus Group (2012). An excerpt from our database is given in Table 1. It contains not only polls from the five institutes used in our study but also other polls during a period. The complete database can be downloaded from Novus Group (2012). Although the Swedish polling institutes use slightly varying techniques, their methods are fairly similar and are based on probability sampling. Polls based on web panels do not use probability sampling and are not comparable with those based on random telephone interviews. Not only was there differences in levels, which may be adjusted by institute-specific bias corrections, but the differences also seemed to change over time. This may be explained e.g. by changes in the web panels, or by different reaction times to political moves and scandals between the two data collection modes. As should be noted, some of the smaller opinion research institutes could have been included in the study but due to their small sample sizes, they would not contribute much to the results.

Data collection for a poll is usually done during one or two weeks. We have used the date in the middle of the collection period as the date of the study. Statistics Sweden has a longer data collection period than the private institutes. One of the institutes (SKOP) changed their weighing method at a specific moment during the study period. Before the change, their estimates differed clearly from the others, but after the change there were only minor differences in the levels of the five institutes' estimates. We corrected SKOP estimates with an additive constant (η) before the change: $X_t = (X_t - \eta \mid k = SKOP)$.

4.3 Design effects

The party preference study of Statistics Sweden is based on a simple random sample from the Swedish population register and is performed by telephone interviewing with about one-third non-response, of which one half are refusals and the other half are persons who can not be found. The studies by the private institutes are based on some version of Random Digit Dialling (RDD). Sample sizes are between 1000 and more than 2000 individuals, except for Statistics Sweden: 7000 individuals. All institutes use weighing to decrease the variance. Statistics Sweden calibrates with known register variables. Most institutes ask about the voting at the previous election and weigh with e.g. age, sex and the outcome at the previous election. This means that there are probably sampling design effects δ_k for each of the *k* institutes, such that the variances becomes smaller than the binomial distribution variance

$$V_t = Var(X_t - P_t \mid P_t) = \delta_k P_t (1 - P_t) / n \qquad (9)$$

Note that the design effects need to be estimated from the data, also containing other than just sampling design effects, such as interviewer effects and non-optimal estimation. If the sampling schemes are constructed to estimate the majority proportion efficiently, they may be less appropriate for estimating some individual parties' votes. In practice, we will not be able to introduce any design effects since we lack the detailed prior information. We will thus assume that the design effects are the same for all institutes. Additionally, the institutes have different ways of formulating the interview questions, so we allow them to have different biases

$$E(X_t - P_t \mid P_t, k) = \theta_k, \quad k=1,2,..5.$$

depending on which of the *k* institutes has made the poll X_t. As can be seen from Table 1, the institutes treat uncertain persons in different ways. The figures are thus not possible to compare or combine. Some of the institutes use different types of probing of respondents that are reluctant to answer, like for instance asking about just party block preferences instead of party preferences or which parties she or he hesitates between. Since all institutes give party preference estimates that sum to 1 across all parties, we have used these, excluding the non response and the no opinion and/or uncertain voters.

4.4 Parameter estimation

Our model requires estimating the parameters α, β, γ, δ and the institute effects θ_k. It also requires starting values for the level and trend, $E(P_0)$ and $E(T_0)$, and their variances $C(P_0, P_0)$, $C(P_0, T_0)$ and $C(P_0, T_0)$. We have used fairly uninformative starting values and large variances, independent of the sample. Consequently, the uncertainty intervals become wider during the first months of the data set due to this start-up. Parameters are estimated by maximising the likelihood function from observed data. Since each observation of $X_t - E(P_t)$, given the history (i.e. all observations before time *t*) is assumed to be approximately normally distributed and the likelihood function is

$$L(\alpha, \beta, \gamma, \delta) = \prod_t \frac{1}{2\pi (V_t + CPP_t)^{1/2}} \exp\left(\frac{(X_t - EP_t)^2}{2(V_t + CPP_t)}\right),$$

where the product is over all opinion polls. Taking logarithms and omitting the constants, this becomes

$$2l(\alpha, \beta, \gamma, \delta) = \sum_t \left(\frac{(X_t - EP_t)^2}{V_t + CPP_t} + \ln(V_t + CPP_t)\right). \qquad (10)$$

This expression is used for estimating parameters and for testing hypotheses concerning them. The standard deviations are derived from the second derivatives of (10).

4.5 The logistic transformation

Whenever the proportion of voters for a certain party or party block is relatively large, it is reasonable to assume that its development can be described by the processes described above. However, when the proportions are close to zero (or one) the process may reach zero (or one) and the forecasted proportions may become negative (or exceed one). To cope with these situations, which are likely to occur in an election system having several smaller parties, we use a logistic transformation

$$Q_t = \ln(P_t) - \ln(1 - P_t)$$

and assume that Q_t will follow the above processes. $Y_t = \ln(X_t) - \ln(1 - X_t)$ is the corresponding data transformation, with approximate mean Q_t and approximate variance $V_t / (P_t(1 - P_t))^2$. If the variance is given by (9), this simplifies to $Var(Y_t - Q_t \mid P_t) = \delta / (nP_t(1 - P_t))$. Thus we can use the same formulas as above for this process, but insert this variance expression in the innovation formulas.

The logistic transformation has the advantage that the process will never become larger than unity. It will also result in more stable variances. Furthermore, this is a symmetric transformation in the sense that it can be used both for the party size and for the complementary event of not voting for the party (with opposite sign).

5 Results

5.1 Model diagnostics and parameter estimation

Estimation results (Table 2) show that the added trend estimate is not significantly different from zero. This result goes against common belief. More sophisticated methods might detect a trend, but this would most likely have only a marginal effect on the other estimates and it is most likely a local trend, if it exists at all.

As mentioned in Subsection 3.1, γ measures how stable the party's support is. The Left Party and the Christian Democrats seem to have a stable core of voters as compared to other parties. This is in accordance with what political scientists say. The design effect (δ) is absent for most parties. The estimation and weighing methods seem to work well for the big parties and for the Alliance (government coalition), since the design effect is here below unity. For the small and extreme parties, they do not work. The reason is probably that the institutes design their methods so as to estimate the majority as accurately as possible. This is particularly conspicuous for the new party, the Sweden Democrats, where there are very few observations yet.

The θ_k –values and η measure the differences between the institutes (k: 1=SIFO, 2=Temo, 3=SKOP, 4=Statistics Sweden, 5=Novus). The figures are not comparable between the party columns and the Alliance column due to the logistic transformation. But the η-value for the Alliance says that the estimates for SKOP were 3.44 percent units higher before their change of the weighing method. Apart from this, there were only small institute biases. One should primarily compare the θ_k-values in the same column and not with zero since it is difficult to estimate the true bias with only two elections. Bias comparisons between institutes are easier since there are multiple observations for each institute. For the Alliance, all biases were less than 1 % and the standard error (last column) was much smaller than the estimated biases. Hence, there were significant differences between the institutes.

Table 2. Parameter estimates (when β is estimated as 0 there is no trend and thus α is meaningless). The Alliance is estimated with the untransformed model and the parameters are not comparable to those for the individual parties. The last column is the estimated standard errors of the bias compared to the mean bias of the other institutes for the Alliance.

	M	FP	KD	C	S	V	MP	SD	Alliance	
α			0.0034						0.0042	Standard
β	0	0	0.0001	0	0	0	0	0	0.0007	Errors
γ	0.0124	0.0110	0.0053	0.0042	00156	0.0050	0.0098	0.0103	0.197	0.026
δ	0.74	0.89	1.22	0.82	0.70	1.33	1.01	1.56	0.70	0.08
η	0.044	0.128	0.195	0.171	-0.106	-0.015	0.005	-0.008	3.442	0.264
θ_1	-0.036	-0.016	-0.055	0.005	-0.025	0.370	0.049	0.079	0.941	0.166
θ_2	-0.033	-0.030	-0.095	-0.021	0.012	0.411	-0.030	-0.030	0.430	0.016
θ_3	-0.021	-0.010	-0.107	-0.062	-0.047	0.417	0.051	-0.056	0.701	0.022
θ_4	-0.080	0.081	-0.023	-0.027	0.062	0.356	-0.050	-0.038	-0.567	0.027
θ_5	-0.060	-0.044	-0.109	-0.040	-0.027	0.439	0.036	0.083	0.111	0.039

5.2 Estimates of proportions

5.2.1 Party block level

In Table 2, the estimated proportion of voters supporting the incumbents (currently the Alliance proportion) at the end of our observation period (November 1, 2012) is 47.1. The variance has decreased to 1/3 of the variance of only one poll. If the proportion is estimated for an earlier date than the concurrent one, then the additional use of opinion polls occurring after the specific date renders an estimate with higher precision. The variance of such an estimate will be almost 50% lower than the one that uses opinion polls only from one side.

Remaining results are illustrated graphically. Figures 1-4 all describe how large support the present government has. Model fit can be seen in Figure 1. Here, 95 % prediction intervals and actual outcome of opinion polls are shown. Estimation is based on all previous polls before the prediction point for the inter-election period September 2006 – September 2010). When two studies have the same reference date, only the interval for the last study is shown. Exactly seven poll outcomes of the 175 polls are not covered by the interval, which is exactly 5 %, supporting the normality assumption. The raggedness of the boundaries in the figure depends on several factors. The main reason is that the studies have different sizes, and the prediction intervals are shorter for studies with larger sample sizes since the measurement errors are included in the precision. Another reason is that new information is incorporated in the prediction/forecast after each opinion poll is made, thus at each time the predicted level changes with a small jump.

Figure 1. Predictions of the present proportions of the Alliance and 95 % prediction intervals for the outcomes of all polls in the interelection period given all previous polls. Inter-election period September 2006-September 2010.

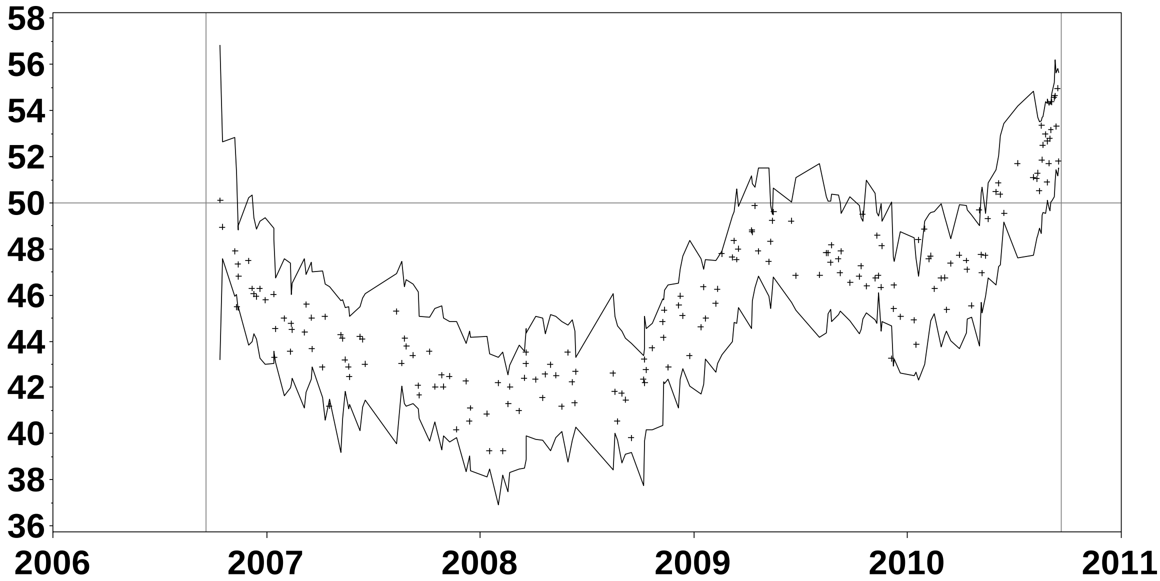


Figure 2 shows the same period as above but now we estimate the true level for the period using polls up until that date. The intervals here are much narrower since the uncertainties of the individual polls are not included and the curves are not quite as ragged as in Figure 1. Even shorter intervals are seen in Figure 4, where all polls are used, i.e. also those done after the indicated date (ex ante). In that case, we have combined the estimates of our model run from both ends of the data period, hence the wide intervals in the left end have disappeared. Figure 3 is the continuation of Figure 2, with estimates for the period until October 2012. The effect of the party leader change for the Social Democrats in January 2012 is clearly seen.

Figure 2. Best estimate of the Alliance's proportion given all previous polls. Inter-election period September 2006- September 2010.

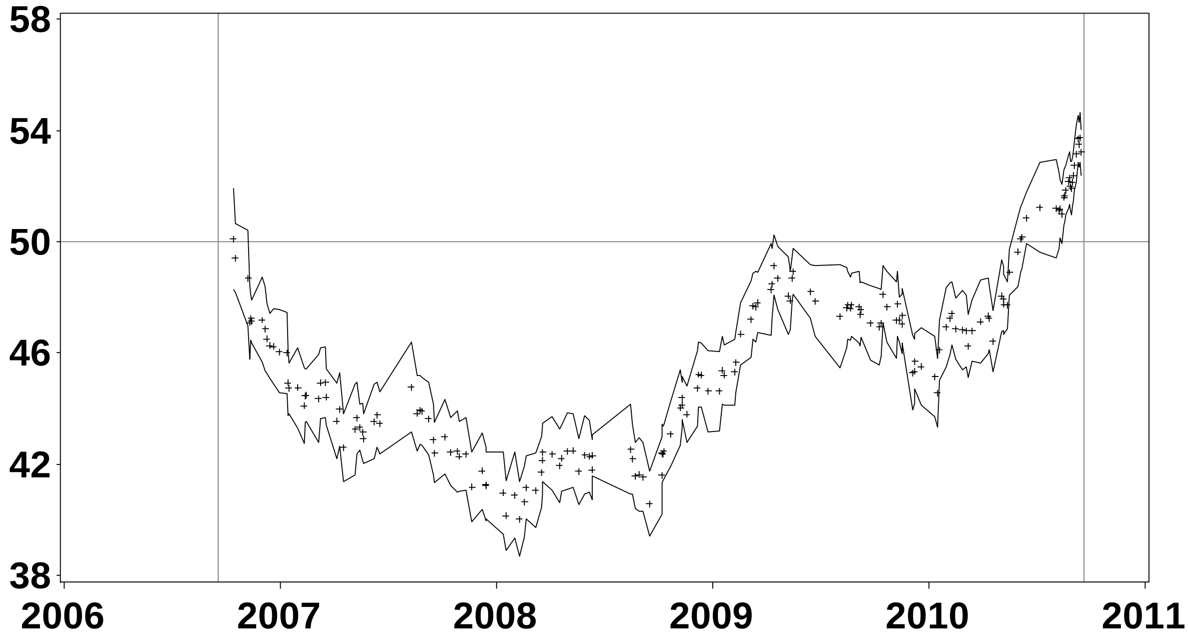
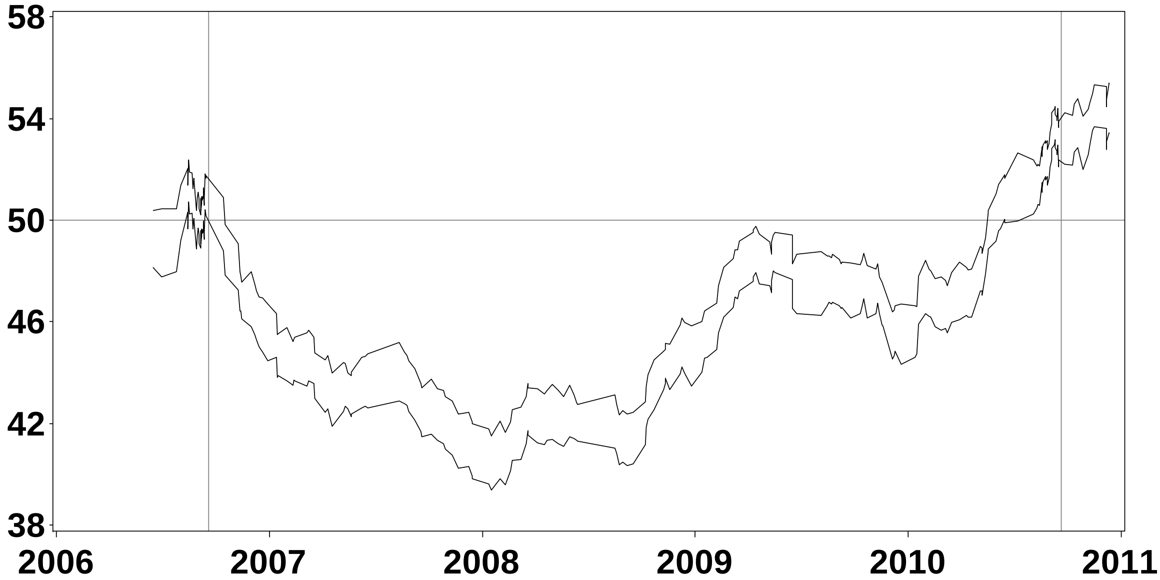


Figure 3. Best estimate of the Alliance's proportion given all previous polls. Post-election period September 2010 – November 2012.

Figure 4. Best estimate of the Alliance's proportion given all polls during the inter-election period and three months prior to and three months after the elections (June 2006- December 2010).



5.2.2 Individual parties

Table 3 contains estimates of the party preferences on November 1, 2012, one day after our total data period. Estimated precisions are given and forecasts for the elections in September 2014, including standard errors, which are almost half of those given by ordinary polls. It is worth noting that the standard error of the Sweden Democrats (SD) is higher than that of the Green Party (MP) and the Christian Democrats (KD). This is an effect of the higher design effect (δ) for the Sweden Democrats. The same difference is seen for the Christian Democrats and the Centre Party (C).

Estimated supports for the eight parties are given in Figure 5. In these analyses, the use of the logistic transformation is important due to the differences in size over time for some of the parties. One may see the decrease during the second half of 2008 and the first half of 2009 for the Social Democrats (S), mirrored by an increase for the Moderates (M). In the second half of 2009, the Green Party captured voters from many of the other parties. Around Christmas 2009, the Sweden Democrats got a level shift.

Table 3. Estimates of the party preferences and forecasts for the election in September 2014 based on all observations until November 1, 2012. The final row shows the probability of passing the election threshold.

	M	FP	KD	C	S	V	MP	SD
Estimate	29.55	5.73	3.52	4.36	34.1	5.80	8.66	6.74
(S.E)	0.60	0.25	0.16	0.14	0.67	0.21	0.32	0.34
Forecast	29.55	5.72	3.50	4.36	34.1	5.80	8.60	6.74
(S.E)	7.18	1.80	0.75	0.50	9.71	0.78	2.28	1.94
Prob.(>4%)	1.00	0.90	0.25	0.79	1.00	0.998	0.999	0.98

5.3 Forecasts

5.3.1 Percentages of the voters

The primary goal of this study was to combine the opinion polls in order to estimate true party support. But as seen above, the model can also be used for forecasting elections, given the assumption that the parameters stay constant during the forecast horizon. When estimating the level at dates close to the opinion polls, the model is fairly robust against errors in the assumptions, whereas longer term forecasts imply extrapolation and thus are more sensitive to false assumptions.

Figure 6 shows an ex ante forecast of the election in September 2010. Immediately after the previous election in September 2006 the intervals are quite wide. As the election comes closer in time the intervals become shorter. One month before the election the whole interval is above 50 % for the Alliance, i.e. it predicted the majority correctly with more than 95 % confidence.

The same analysis is seen in Figure 7 but for the outcome of the future 2014 elections. The intervals are still quite wide in October 2012. It should be mentioned that there is a considerable risk that one or more of the parties in the Alliance will not exceed the 4 % threshold for the parliament. This is not accounted for here, but can be done by applying a multivariate prediction technique.

Figure 5. Best estimates of support for the eight Swedish parties represented in the parliament after the 2010 elections. Inter-election period September 2006 - September 2010 based on all previous polls.

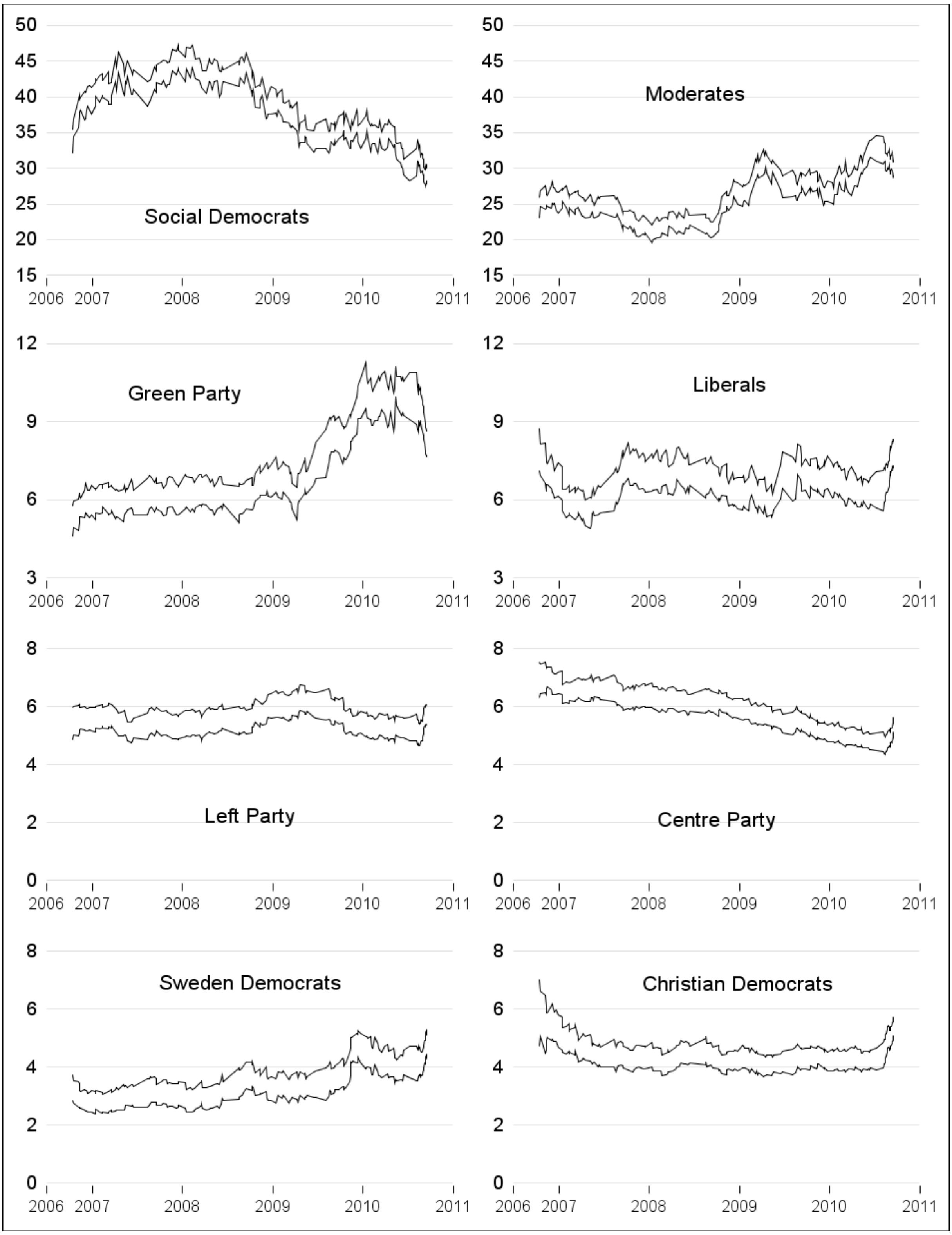


Figure 6. Forecast intervals (95%) for the outcome of the election in September 2010. Alliance's proportion estimated on all previous polls. Observed opinion polls are marked.

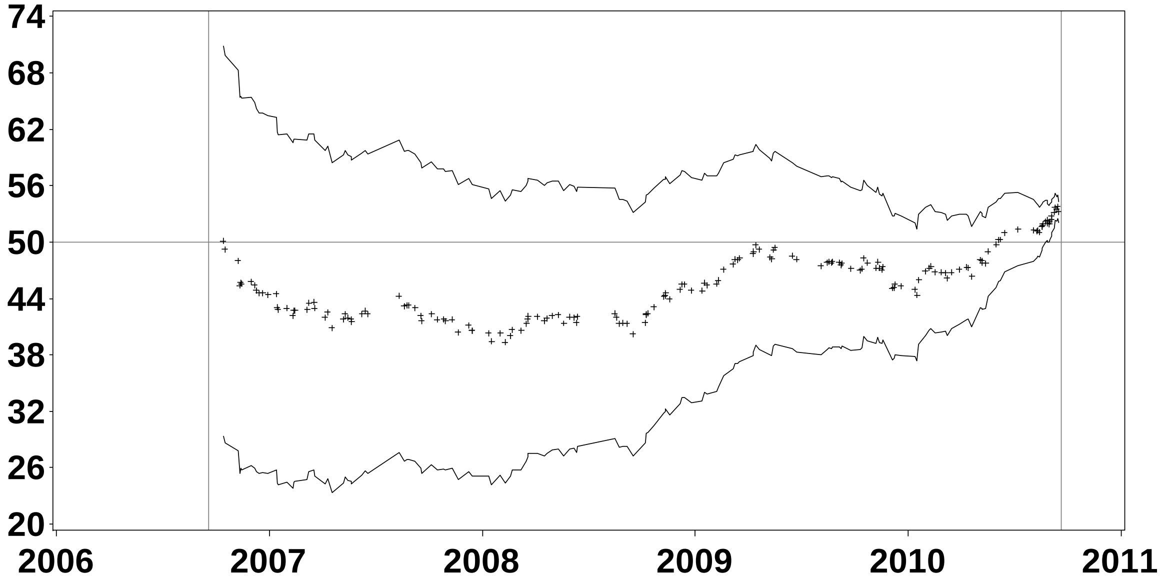


Figure 7. Forecasts for the coming election outcome in September 2014. Percentage for the Alliance using all polls available at the time of forecast. Data until November 2012. Expected outcome and 95 % forecast intervals. The possibility of the parties within the Alliance not passing the parliamentary threshold is not taken into account.

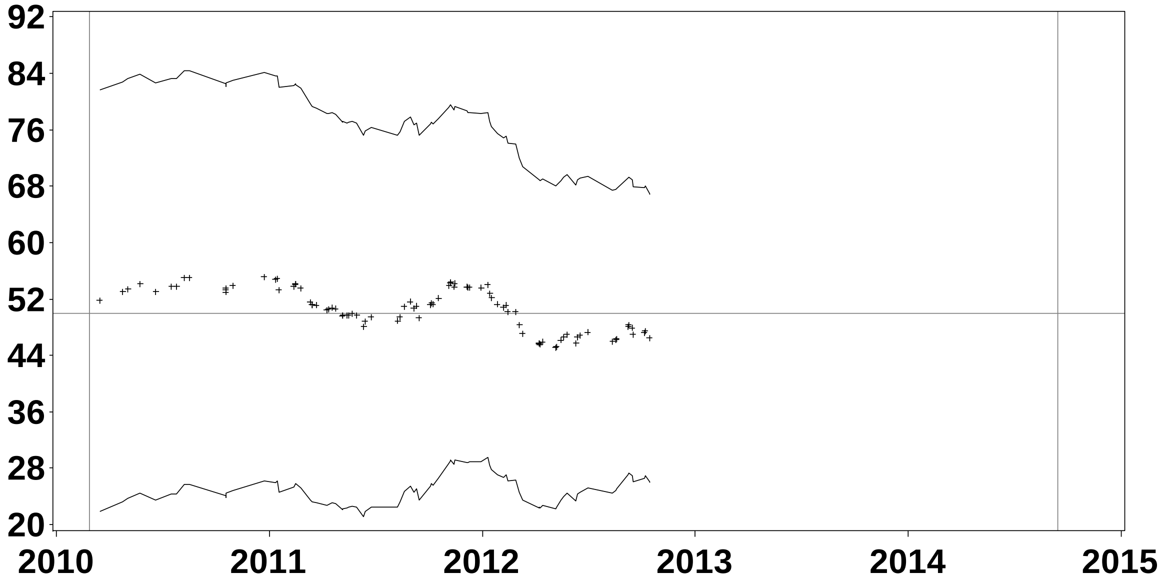
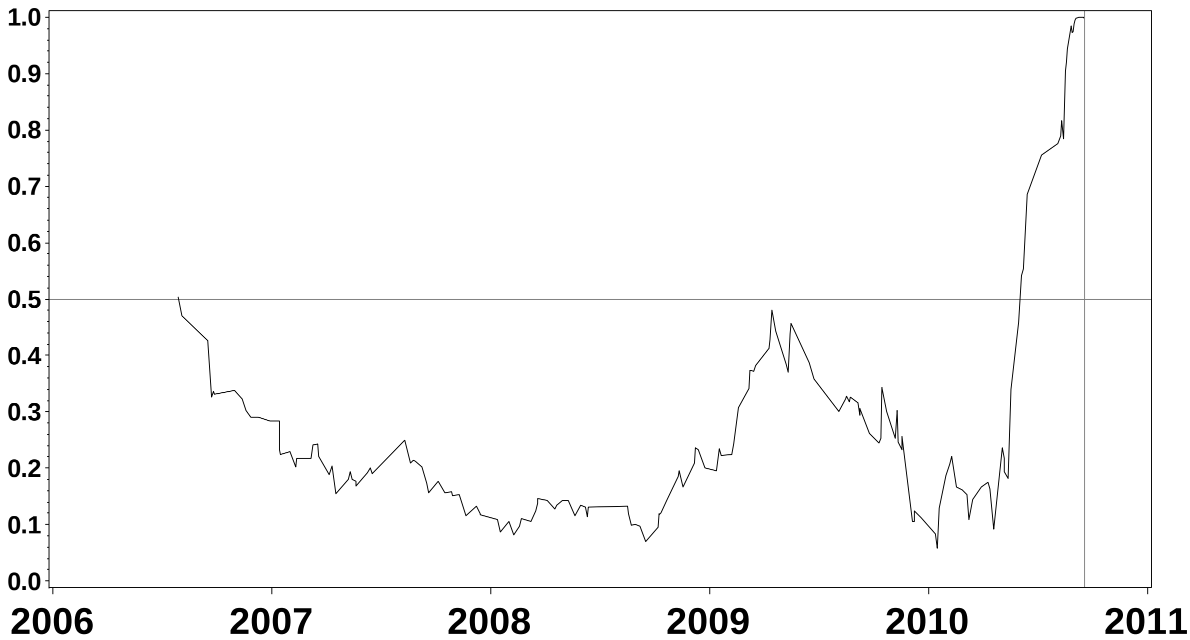


Table 3 shows forecasted outcomes of the elections in 2014. There were no detectable trends so the forecasts are the same as the best concurrent estimate for all parties but the Christian Democrats: α_{KD} = 0.0034, β_{KD} = 0.0001. The estimated γ –values (volatility) for the Christian Democrats, the Centre Party and the Left Party were only half as large as for the other parties according to Table 2. This indicates stability, as reflected in smaller standard errors of the forecasts.

5.3.2 Predicting specific events

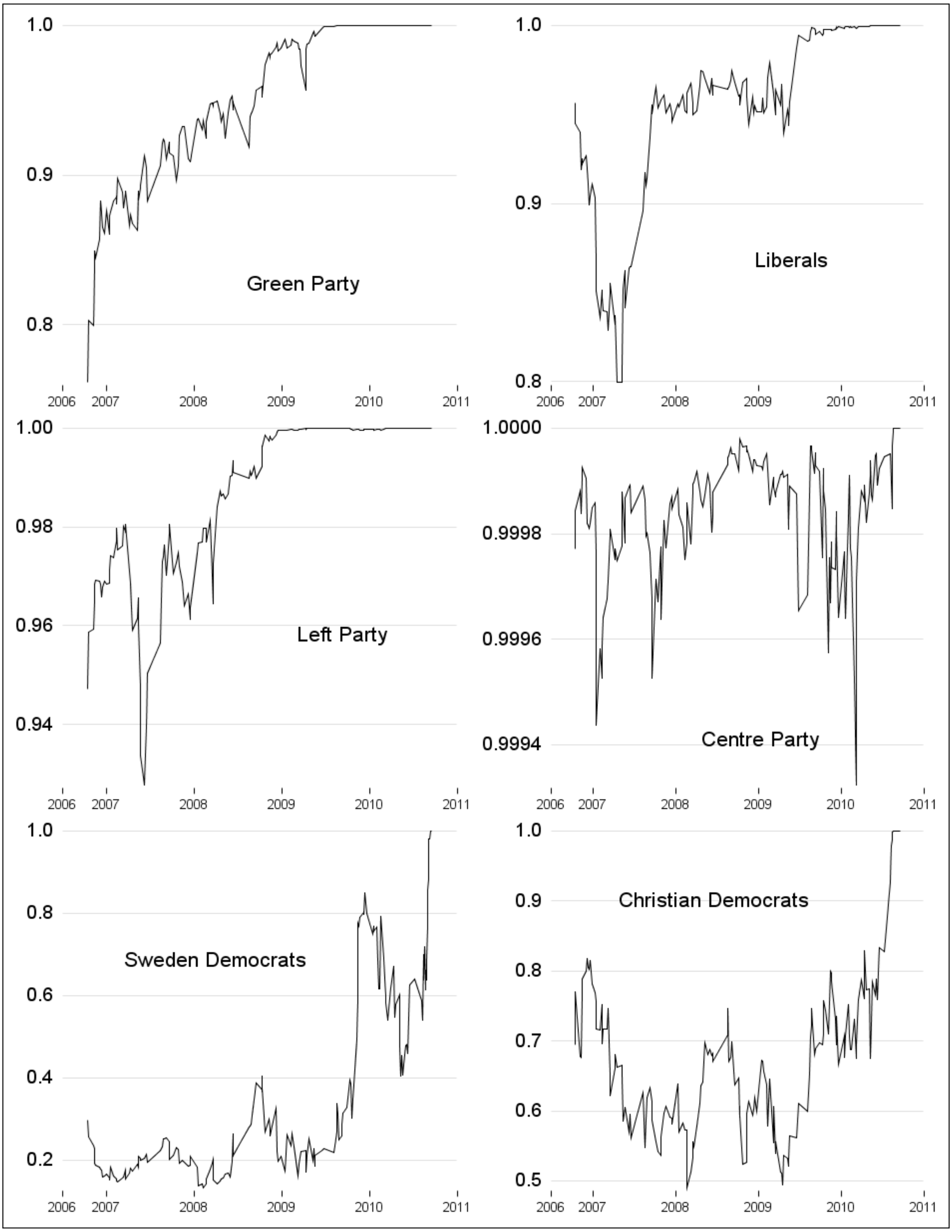
In Figure 8, the probability of victory for the Alliance against the red-green opposition is computed for all time points in the inter-election period, given the opinion polls up until the concurrent time.

Figure 8. The probability of the Alliance getting more votes than the red-green opposition in the elections 2010. Estimated on all previous polls.



With eight parties in parliament, some of them will usually risk not passing the threshold in the forthcoming election. The risks for not entering the parliament in the elections 2014 are shown in Table 3. As is seen, all the small parties in the Alliance run a risk of not passing the election threshold, with highest probability for the Christian Democrats. These probabilities do not take the possibility of tactical voting into account. In Figure 9, this risk is illustrated for the six smaller parties during the inter-election period. For instance, the chances for the Sweden Democrats to enter the parliament were around 20 % until the middle of 2009. The chances then increased rapidly and during the whole year 2010, the chances were above 40 %. The Social Democrats and the Moderates had zero risk of not entering the parliament.

Figure 9. Probability of exceeding the parliamentary threshold of 4 % given all previous polls for all parties except the Social Democrats and the Moderates which have probability one. Note that the scales on the y-axes are different between the parties.



6 Discussion

A model based on Wiener processes and diffusion models seems to fit the voter behaviour quite well (on an aggregate level). The model can be motivated by the central limit theorem for processes since the number of voters is the sum of a large number of individual voters. This kind of model is frequently used in physics and financial mathematics.

The model showed a good fit for forecasting polls. Exactly 5 % of the predicted poll results happened to be outside the prediction intervals during the inter-election period and slightly more after the last election. This may be explained by the political turbulence around the Social Democrats after the elections in 2010.

By combining the opinion polls in the way proposed here, the precision is improved in two ways. Several polls can be combined in an optimal way, determined by the data, and the measurement biases of the different institutes can be estimated and eliminated. Secondly, the Wiener process approach does not have the rigidity and arbitrariness of the methods referred to as polls of polls (Silver, 2008), and it is more robust than many regression approaches used in other studies.

Estimates of the voter shares within the observation period are fairly robust against misspecifications of the model. The method invites the user to predict future developments, but forecasts are not quite as robust, although they are reasonable. The method also allows us to estimate quantities which are not addressed by ordinary polls such as the probability that a party exceeds a specific threshold or the probability that a certain coalition will get a majority of the votes, which is important in a multiparty political system. Another feature is that specific properties of the different institutes can be assessed directly from the data, such as design effects and measurement biases (relative to the other measurements).

It must be stressed that the forecasts are mechanical, i.e. the approach does not and is not intended to take other information than polls explicitly into account. The idea is that most information should already have influenced the individual persons in the sample, similar to financial markets where all information is capitalised in prices. Information will thus be included implicitly and automatically into the estimates and forecasts.

In this paper we have only studied one party (or party group) at a time. It is possible to enhance the model to a multivariate setting in order to study the simultaneous behaviour of several parties. In that case, it should be possible to study both the total share of the parties in a coalition and the individual parties at the same time to answer questions like: What is the probability that a group gets a majority and none of the parties falls under the threshold?, or: Which parties lose votes if another specific party gains votes? This extension will however require the use of more mathematics, it will imply estimation of more parameters, and given the amount of data available, this implies less accuracy.

The methods in this paper can be applied to other situations, where there is a series of measurements of the same quantity at different time points, for example the Labour Force Surveys (LFS) and for repeated market surveys.

Acknowledgement

The authors would like to thank Lars-Erik Öller, Stockholm University, for all his valuable comments that have improved the paper.

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Appendix. Expected values, variances and covariances

In this appendix the two stochastic differential equations (4) and (5) are solved. First T_{t+s} and P_{t+s} are expressed in terms of previous values of T_t and P_t and the two white noise processes. Solving equation (4) and expressing T_{t+s} in terms of T_t gives

$$T_{t+s} = \exp(-\alpha s)T_t + \beta\int_t^{t+s} \exp(-\alpha(t+s-u))dW_1(u)\,. \tag{A1}$$

Solving (5) and expressing the future party sympathy in terms of the situation at time *t* gives

$$P_{t+s} = P_t + \int_t^{t+s} T_u du + \gamma\int_t^{t+s} dW_2(u). \tag{A2}$$

If the expression (A1) for T_{t+s} is inserted we get

$$P_{t+s} = P_t + \int_t^{t+s}\left[\exp(-\alpha(u-t))T_t + \beta\int_t^{u}\exp(-\alpha(u-v))dW_1(v)\right]du + \gamma\int_t^{t+s} dW_2(u)\,. \tag{A3}$$

The first part of the middle term is easily computed. For the second part we change the order of integration and after that compute the (new) inner integral. After some calculation one finds that

$$P_{t+s} = P_t + \frac{1-\exp(-\alpha s)}{\alpha}T_t + \beta\int_t^{t+s}\frac{1-\exp(-\alpha(t+s-v))}{\alpha}dW_1(v) + \gamma\int_t^{t+s} dW_2(u)\,. \tag{A4}$$

The expected values of T_{t+s} and P_{t+s}, conditional on T_t and P_t, are easily derived from (A1) and (A4):

$$E(T_{t+s} \mid T_t) = \exp(-\alpha s)T_t\,,$$

and

$$E(P_{t+s} \mid P_t, T_t) = P_t + \int_t^{t+s} E(T_u \mid T_t)du$$

$$= P_t + \frac{1-\exp(\alpha s)}{\alpha}T_t\,.$$

The variances and covariances are a little more complicated but we give an outline of the proofs. The variance of T_{t+s} in terms of the variance of T_t is also easily found using (4):

$$Var(T_{t+s}) = Var(\exp(-\alpha s)T_t) + Var(\beta\int_t^{t+s}\exp(-\alpha(t+s-u))dW_1(u)$$
$$= \exp(-2\alpha s)Var(T_t) + \frac{\beta^2}{2\alpha}\int_t^{t+s}\exp(-2\alpha(t+s-u))du$$
$$= \exp(-2\alpha s)Var(T_t) + \frac{\beta^2}{2\alpha}(1-\exp(2\alpha s)).$$

The variance of P_{t+s} is a little more complicated since, according to (5), it depends on four terms. The last two of these terms are independent of each other and of the two first terms:

$$Var(P_{t+s}) = Var(P_t) + \frac{(1-\exp(-\alpha s))^2}{\alpha^2}Var(T_t) + \frac{2(1-\exp(-\alpha s))}{\alpha}Cov(P_t, T_t) +$$
$$+\beta^2\int_t^{t+s}(\frac{1-\exp(-\alpha(t+s-v))}{\alpha})^2 dv + \gamma^2\int_t^{t+s}du$$
$$= Var(P_t) + \frac{(1-\exp(-\alpha s))^2}{\alpha^2}Var(T_t) + \frac{2(1-\exp(-\alpha s))}{\alpha}Cov(P_t, T_t) +$$
$$+\frac{\beta^2}{\alpha^2}(s + \frac{1-\exp(-2\alpha s)}{2\alpha} - 2\frac{1-\exp(-\alpha s)}{\alpha}) + \gamma^2 s.$$

The covariance between T_{t+s} and P_{t+s} can be derived from (4) and (6) :

$$Cov(T_{t+s}, P_{t+s}) = \exp(-\alpha s)Cov(T_t, P_t) + \frac{\exp(-\alpha s)(1-\exp(-\alpha s))}{2\alpha}Var(T_t) +$$
$$+\beta^2\int_t^{t+s}\frac{\exp(\alpha(t+s-u))(1-\exp(\alpha(t+s-u))}{\alpha}du$$
$$= \exp(-\alpha s)Cov(T_t, P_t) + \frac{\exp(-\alpha s)(1-\exp(-\alpha s))}{2\alpha}Var(T_t) +$$
$$+\frac{\beta^2}{2\alpha^2}(1-\exp(-\alpha s))^2.$$