Stockholms universitet

Research Report

Department of Statistics

No. 2013:6

A Note on the Trade-off between Direct and Indirect Seasonal Adjustments

Can Tongur

Department of Statistics, Stockholm University, SE-106 91 Stockholm, Sweden

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Can Tongur*

Stockholm University
&
Statistics Sweden

Abstract

Direct and indirect seasonal adjustments can be viewed as opposite formulations of an error minimization problem that occurs when seasonally adjusting a system of time series. In this study, a loss function is formulated that is a weighted combination of the errors of the input time series and the aggregate error. Holt-Winters' exponential smoothing methods on squared error loss functions or robust Huber loss functions are applied to quarterly Swedish GDP and monthly foreign trade data. All input series are seasonally adjusted jointly but still univariately and trade-off point between direct and indirect seasonal adjustments are estimated. The quadratic loss function is found to cause larger differences between direct and indirect seasonal adjustments than the Huber loss function does. Results indicate that pure indirect seasonal adjustment should be avoided for GDP and pure direct seasonal adjustment should be avoided for foreign trade. Adjustments in between with a combined loss function seem to work well for all purposes.

Keywords: direct/indirect seasonal adjustment, Huber loss function, exponential smoothing, trade-off

*Statistics Sweden, BOX 24300, SE-104 51 Stockholm, Sweden.
E-mail: can.tongur (at) scb.se

1 Introduction

Seasonal adjustment of time series often raises the issue of direct or indirect methods. This occurs when systems of time series are considered, for instance time series of gross domestic product – total and for all industries, unemployment – overall and by gender and age, or foreign trade – balance of trade and exports and imports. By *direct* estimation, it is meant that the aggregate time series is seasonally adjusted independent of the seasonal adjustments of the individual series, while *indirect* estimation implies that all time series constituting the aggregate are seasonally adjusted separately so that the aggregate seasonal adjustment is obtained implicitly from their combination.

In this paper, the problem of seasonal adjustment order is attacked by applying a target function in estimations. A trade-off frontier between adjustments is stated for the time series system such that a loss function reaching from indirect to direct estimation is formulated. A Holt-Winters method is used and the loss function is expressed as the method's ability to predict future observations.

The note is organized as follows. In the next section, a brief review of related work is given. Section 3 introduces a basic exponential smoothing method and its state space formulation. A discussion on how to assess the aggregation order is found in Section 4. The data and the procedure are presented in Section 5. Some further findings and conclusions are discussed in Sections 6 and 7.

2 Earlier work

Among the earliest studies, Geweke (1978) considered the order of aggregation and estimation in terms of a multivariate (optimal) model, to which he compared direct and indirect univariate estimation. Using frequency domain arguments, he found that multivariate estimation prior to aggregation was to prefer. As critique of this, Maravall (2005) pointed out that these results were "of limited interest" in practice since the proposed kind of estimation method did not exist in practice and referred e.g. to a study by Ghysels (1997) which indicated contrasting results. Maravall (2005, 2006) advocated direct seasonal adjustment in most cases because of several reasons, among which that less irregularities remain in aggregate series. Planas & Campolongo (2000) stressed that only when series had similar spectral densities, direct estimation was preferable while for series with differing spectral densities, indirect estimation was superior. They also pointed out the advantages of multivariate estimation. Birrell et al. (2008) and Thorburn & Tongur (2013) discussed the issue in terms of state space models. They found that the solution depends on the covariance structures of the involved series. Both studies favored a multivariate, i.e. optimal, approach. However, although multivariate estimation is theoretically possible and desirable, it practically leaves out much of the technical sophistication available in standard seasonal adjustment packages.

3 A simple seasonal adjustment method

3.1 Univariate representation of the algorithm

Assume that a time series Y_t with period *P* (i.e. *P=4* for quarterly data) can be written as a sum of three components: a trend T_t, a seasonal S_t and an irregular effect E_t; $Y_t = T_t + S_t + E_t$. These components, which are unobserved, are assumed to consist of an expected level τ_{t-1} with a growth m_{t-1}, and a seasonal component $s_{t-1,p(t)}$ for the coming period $p(t) \in (1,2,..P)$. Assuming that all these terms were known, the prediction/innovation error would be given by

$$\varepsilon_t = (Y_t - \tau_{t-1} - m_{t-1} - s_{t-1,p(t)})\,. \tag{3.1a}$$

Let $L(\varepsilon) = \sum_{t=1}^{T} L(\varepsilon_t)$ be a loss function measuring how big the loss will be when the innovation errors ε_t are observed. The components ε_t, τ_{t-1}, m_{t-1} and $s_{t-1,p(t)}$ can be estimated by minimizing the total loss under some constraints. The exact procedure will be described in more detail in Sections 4 and 5. The following updating procedure is used, inspired by the Holt-Winters technique and uses exponential decay parameters a^*, b^* and c^*:

$$\tau_t = \tau_{t-1} + m_{t-1} + a^* \varepsilon_t\,, \tag{3.1b}$$

$$m_t = m_{t-1} + b^* \varepsilon_t\,. \tag{3.1c}$$

Following Roberts (1982), the seasonal updating mechanism gives the seasonal effect

$$s_{t,p(t)} = w s_{t-1,p(t)} + c^* \varepsilon_t\,, \tag{3.1d}$$

when $p(t)$ is the concurrent season at time point *t*, whereas the non-concurrent seasons $p \neq p(t)$ are updated with a zero summation constraint

$$s_{t,j} = w s_{t-1,j} - (c^* /(P-1))\varepsilon_t\,, \tag{3.1e}$$

with $w \in (0,1]$. This seasonal damping (or persistence) parameter is set to unity, i.e. $w = 1$, implying that seasonality is assumed non-stationary. Since the model is used only for estimation through one-step predictions, this choice is natural. The seasonally adjusted time series SA(Y_t) can be obtained by $Y_t - s_{t,p(t)}$, the seasonally adjusted and smoothed series (the trend) will be τ_t and the irregular component is by definition $Y_t - \tau_t - s_{t,p(t)}$. This way, components are estimated on all available observations at each time point *t*, i.e. the seasonal adjustment can be made to the last observed time point.

3.2 Univariate representation of a state space counterpart

This estimation algorithm may also be expressed by a presumably underlying state space form. Let $\boldsymbol{\theta}_t$ be the state vector which comprises the components $\{\tau_t, m_t, s_{t,1}, s_{t,2}, ...s_{t,P}\}$ and let $\mathbf{F} = (1 \quad 0 \quad 1 \quad \mathbf{0}_{P-1})'$ be a vector projecting these on Y_t:

$$Y_t = \mathbf{F}'\mathbf{G}\boldsymbol{\theta}_{t-1} + \varepsilon_t, \tag{3.2}$$

where $\mathbf{G}$ is the $(P+2)\times(P+2)$ state transition matrix for seasonal effects, identical to the one found in West & Harrison (1989, Chapter 8) for Dynamic Linear Models (DLM). Let $\mathbf{g}$ be the innovation partitioning vector from (3.1a-e). Updating of the state vector is

$$\boldsymbol{\theta}_t = \mathbf{G}\boldsymbol{\theta}_{t-1} + \mathbf{g}\varepsilon_t, \tag{3.3}$$

with

$$\boldsymbol{\theta}_t = \begin{bmatrix} \tau_t \\ m_t \\ s_{t,1} \\ s_{t,2} \\ \vdots \\ s_{t,P} \end{bmatrix}, \; \mathbf{G} = \begin{bmatrix} 1 & 1 & \mathbf{0}'_{P-1} & 0 \\ 0 & 1 & \mathbf{0}'_{P-1} & 0 \\ \mathbf{0}_{P-1} & \mathbf{0}_{P-1} & \mathbf{0}_{P-1} & \mathbf{I}_{P-1} \\ 0 & 0 & 1 & \mathbf{0}_{P-1} \end{bmatrix} \text{ and } \mathbf{g}' = \begin{bmatrix} a^* \\ b^* \\ c^* \\ -c^*/(P-1) \\ \vdots \\ -c^*/(P-1) \end{bmatrix}.$$

This representation is an extension of the one used by Roberts & Harrison (1984), who proposed a common seasonal matrix between the Holt-Winters method and DLM. This seasonality structure implies estimating a rotating seasonal component, with focus on the concurrent season at each time, as can be realized from the general seasonal updating mechanism. The pitfall of this approach is that noise is equally distributed among non-concurrent seasons, which may cause instability in some cases when seasons are of markedly different magnitude. However, as should be clear, this exponential smoothing formulation does not require any distribution theory, whereas the DLM, which is a Kalman filter, uses normality (or Student-t) of innovation errors.

3.3 Multivariate representation

It is possible to express the previous formulation in a multivariate form. This is called Vector Exponential Smoothing (see e.g. Athanasopoulos & de Silva, 2010) of *n* series;

$$\mathbf{Y}_t = \boldsymbol{\tau}_{t-1} + \mathbf{m}_{t-1} + \mathbf{s}_{t-P} + \boldsymbol{\varepsilon}_t, \tag{3.4a}$$

$$\boldsymbol{\tau}_t = \boldsymbol{\tau}_{t-1} + \mathbf{m}_{t-1} + \mathbf{a}^*\boldsymbol{\varepsilon}_t, \tag{3.4b}$$

$$\mathbf{m}_t = \mathbf{m}_{t-1} + \mathbf{b}^*\boldsymbol{\varepsilon}_t, \tag{3.4c}$$

$$\mathbf{s}_{t,p(t)} = \mathbf{w}\mathbf{s}_{t-1,p(t-1)} + \mathbf{c}^{*}\boldsymbol{\varepsilon}_{t}\,, \tag{3.4d}$$

$$\mathbf{s}_{t,j} = \mathbf{w}\mathbf{s}_{t,j-1} - (\mathbf{c}^{*}/(P-1))\boldsymbol{\varepsilon}_{t}\,. \tag{3.4e}$$

The innovation of (3.4) is $\boldsymbol{\varepsilon}_t = (\mathbf{Y}_t - \boldsymbol{\tau}_{t-1} - \mathbf{m}_{t-1} - \mathbf{s}_{t-1,p(t-1)})$ and $\mathbf{a}^*, \mathbf{b}^*$ and $\mathbf{c}^*$ are now vectors, so the method in (3.4) is a vector-stacked form of the univariate approach in (3.1a-e), with dimension ($\mathbf{Y}_t$) = $n \times 1$. The multivariate state space representation follows (c.f. 3.2, 3.3):

$$\mathbf{Y}_t = \mathbf{F}'\mathbf{G}\boldsymbol{\Theta}_{t-1} + \boldsymbol{\varepsilon}_t\,, \tag{3.5}$$

$$\boldsymbol{\Theta}_t = \mathbf{G}\boldsymbol{\Theta}_{t-1} + \mathbf{g}\boldsymbol{\varepsilon}_t\,, \tag{3.6}$$

with $\mathbf{G}$ as before, a diagonal innovation matrix $\boldsymbol{\varepsilon}_t = diag(\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)$ of dimension $n \times n$ and

$$\boldsymbol{\Theta}_{t-1} = \begin{pmatrix} \tau_t^{(1)} & \tau_t^{(2)} & \cdots & \tau_t^{(n)} \\ m_t^{(1)} & m_t^{(2)} & \cdots & m_t^{(n)} \\ s_{1,t}^{(1)} & s_{1,t}^{(2)} & \cdots & s_{1,t}^{(n)} \\ s_{2,t}^{(1)} & s_{2,t}^{(2)} & \cdots & s_{2,t}^{(n)} \\ \vdots & \vdots & \cdots & \vdots \\ s_{P,t}^{(1)} & s_{P,t}^{(2)} & \cdots & s_{P,t}^{(n)} \end{pmatrix}, \quad \mathbf{F}' = \begin{pmatrix} 1 & 0 & 1 & \mathbf{0}_{P-1} \\ 1 & 0 & 1 & \mathbf{0}_{P-1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 1 & \mathbf{0}_{P-1} \end{pmatrix}, \quad \mathbf{g} = \begin{pmatrix} a_1^* & a_2^* & \cdots & a_n^* \\ b_1^* & b_2^* & \cdots & b_n^* \\ c_1^* & c_2^* & \cdots & c_n^* \\ -c_1^*\pi & -c_2^*\pi & \cdots & -c_n^*\pi \\ \vdots & \vdots & \cdots & \vdots \\ -c_1^*\pi & -c_2^*\pi & \cdots & -c_n^*\pi \end{pmatrix}.$$

with $\pi = (P-1)^{-1}$ and the state matrix $\boldsymbol{\Theta}_{t-1}$ containing each individual state vector $\boldsymbol{\theta}_{t-1}^{(j)}$ for any one of the $j \le n$ series. This representation can be found for in e.g. West & Harrison (1989, Chapter 15). The diagonal innovation matrix implies a non-assumption of seemingly unrelated time series equations (SUTSE): the series in the system are independently but jointly estimated with uncorrelated errors. This is a justified assumption since standard methods for seasonal adjustment are univariate.

3.4 Admissible parameters

Since $\varepsilon_t = Y_t - \hat{Y}_t$, where $\hat{Y}_t = \mathbf{F}'\mathbf{G}\boldsymbol{\theta}_{t-1}$, the model can be rewritten as

$$\boldsymbol{\theta}_t = \mathbf{G}\boldsymbol{\theta}_{t-1} + \mathbf{g}\varepsilon_t = \mathbf{G}\boldsymbol{\theta}_{t-1} + \mathbf{g}(Y_t - \mathbf{F}'\mathbf{G}\boldsymbol{\theta}_{t-1})\,. \tag{3.7}$$

Back-solving (Hyndman et al., 2008, Chapter 3) until the initial value renders

$$\boldsymbol{\theta}_t = (\mathbf{G} - \mathbf{g}\mathbf{F}'\mathbf{G})^t\boldsymbol{\theta}_0 + \sum_{j=0}^{t-1}(\mathbf{G} - \mathbf{g}\mathbf{F}'\mathbf{G})^j\mathbf{g}Y_{t-j}\,, \tag{3.8}$$

which is a function of the initial state and the data. If $(\mathbf{G} - \mathbf{g}\mathbf{F}'\mathbf{G})$ has all eigenvalues inside the unit circle, the model will be stable in the sense that the importance of past observations decays exponentially at the rate chosen by the modeler. Hence, the parameter spaces for a^*, b^* and c^* (c.f. 3.1b-e) are conventionally restricted to

$$0 < a^* < 1, \qquad 0 < b^* < a^*, \qquad 0 < c^* < 1 - a^*.$$

Should the eigenvalues lie on the unit circle, the model would not be stable but would still render stable forecasts, i.e. the projection of Y_t h periods ahead, $\hat{Y}_{t+h}$, will not be dependent of distant past observations while the projection of the state matrix may be. This forecasting stability, or *forecastability*, allows for wider parameter regions, often bounded at 2 for this model specification; see Hyndman et al. (2008a, Chapter 10) and Hyndman et al. (2008b) for necessary regularity conditions and possible parameter regions. For seasonal adjustment, the mere interest is in one-step ahead predictions, hence forecastability suffices.

4 Assessment of the aggregation order

4.1 An introductory observation

Consider an aggregate of n = 2 series: $Y_t = X_t + Z_t$. Summing the seasonal adjustments of the individual series X_t and Z_t gives the indirect seasonal adjustment of Y_t, while seasonally adjusting Y_t provides a direct seasonal adjustment for this formulation. However, a reformulation by $W_t = -Y_t$ and $V_t = -X_t$ gives the aggregate $V_t = W_t + Z_t$. Now, direct seasonal adjustment calls for adjusting X_t first (Thorburn & Tongur, 2013). The adjustment order is thus in some sense depending on the context and cannot be said to be uniquely determined. Given a specific aggregate formulation, the question may be addressed in terms of minimizing the total innovation of the formulation. This requires a loss function related to both the individual series and the aggregate.

4.2 Robust loss functions

Since our algorithm is not based on a stochastic model, other loss functions than some multiple α of the squared error loss L(α, ε) = $\alpha \sum_{t=1}^{T} \varepsilon_t^2$ may be used, e.g. L(α, ε) = $\alpha \sum_{t}^{T} \varepsilon_t^4$, $\sum_{t}^{T} [1 - \exp(-|\varepsilon_t|^d)]$ or $\alpha \sum_{t=1}^{T} |\varepsilon_t|$, as mentioned by Kalman (1960). Another loss function from robust statistics is the Huber loss criterion (Huber, 1964), which has square penalization until a specific distance δ from the prediction and thereafter an absolute penalization:

$$L(\alpha, \delta) = \begin{cases} \alpha \Sigma_t |\varepsilon_t|^2 / 2, & \text{if } |\varepsilon_t| \le \delta, \\ \alpha(\delta \Sigma_t |\varepsilon_t| - \delta^2 / 2) & \text{elsewhere.} \end{cases} \tag{4.1}$$

The distance δ_j is commonly chosen as a percentile (e.g. the 95th percentile) of the innovation error distribution for each series.

Since the aggregation problem only occurs when there is an aggregate of $n \geq 2$ series, there will be *n+1* single loss functions to minimize simultaneously. A total loss function $D(\alpha, \beta)$ is defined as the sum of *d*-normed loses for some weights α and β:

$$D(\alpha, \beta) = \frac{\alpha}{n} \Sigma_t \mid \varepsilon_{t,1} \mid^d + \frac{\alpha}{n} \Sigma_t \mid \varepsilon_{t,2} \mid^d + \ldots + \frac{\alpha}{n} \Sigma_t \mid \varepsilon_{t,n} \mid^d + \beta \Sigma_t \mid \varepsilon_{t,Agg} \mid^d , \qquad (4.2)$$

where $\varepsilon_{t,j}$ *(j=1,2,..n)* is the error for each *j* series, and the aggregate error of the *n* series summarized at every time point is $\varepsilon_{t,Agg} = \sum_{j=1}^{n} \varepsilon_{t,J}$. For the Huber loss, the total loss function $D(\alpha, \beta)$ is the sum of the individual series' losses and the aggregate loss:

$$D(\alpha, \beta) = \sum_{j=1}^{n} L_j(\alpha, \delta_j) + L_{Agg}(\beta, \delta_{Agg}) .$$

A quadratic loss (*d*=2) implies high sensitivity to outliers/irregular observations and results in a non-robust model fit (Rousseeuw, 1991, and Hastie et al., 2009), so an absolute loss (*d*=1) may then be more appropriate. The Huber loss function is a mixture of these two cases.

4.3 A continuum of adjustments between direct and indirect approaches

The weights α and β, $0 \leq \alpha, \beta \leq 1$, reflect the preferential trade-off between the errors of the individual series and the aggregate error, and without loss of generality, $\beta = 1 - \alpha$. A minimization of the total loss at α =1 over all series and time points implies that the aggregate series does not matter and that mere focus is on finding the best possible estimate of each series. The aggregate series is then *indirectly* seasonally adjusted. Minimizing the loss function at α =0 over all series and time point implies that the aggregate sum of errors is minimized at each time point, which puts emphasis on the aggregate formulation, i.e. best possible estimate of *direct* seasonal adjustment. A weight between these extremes can be viewed as the trade-off frontier in assessing the importance of the two approaches, as visualized in Figure 1. This continuum of adjustments, from α =1 to α =0, can be studied by normalizing the loss function through

$$C = D(\alpha) / (\alpha D(\alpha = 1) + (1 - \alpha) D(\alpha = 0))$$

in order to eliminate the impact of the linear increase in α .

$$\min\left(\sum_{j=1}^{n}\left(\sum_{t=1}^{T} L_j(\varepsilon_{t,j})\right)\right) \qquad\qquad \min\left(\sum_{t=1}^{T} L_{Agg}\left(\sum_{j=1}^{n} \varepsilon_{t,j}\right)\right)$$

1 α 0

Figure 1. Trade-off frontier between indirect estimation ($\alpha = 1$) with *n* individual losses L_j and direct estimation ($\alpha = 0$) with aggregate loss $L_{Agg.}$

The aggregate error is obtained indirectly in our formulation, so explicit estimation of the aggregate series $Y_{t,Agg} = \sum_{t=1}^{n} Y_{t,j}$ is considered ancillary with respect to the estimation of the loss function since the aggregate decomposition should be the sum of the individual series' decompositions, e.g. the aggregate trend based on two series is by definition $\tau_{t,1} + \tau_{t,2} = \tau_{t,Agg}$. This reflects the fundamental motive for this paper: direct estimation of the aggregate series regardless of the subseries is rarely an isolated question since the subseries still require seasonal adjustment, independent of the aggregate series. Birrell et al. (2008) assess the estimation problem in another fashion: by making a data transformation, they achieve a state space representation of *n* series in which the aggregate is the *n*:th series, a necessary mathematical maneuver since it is a linear combination of the others.

4.4 Finding a setting that works both as direct and indirect seasonal adjustment

Each estimate $\mathbf{g}$ of the parameter vectors $\mathbf{a}^*, \mathbf{b}^* \text{ and } \mathbf{c}^*$ is the point estimate that minimizes the total loss conditional on the trade-off weight α. Dual to this, one may instead study the influence on the total loss function from choosing another α conditional on the best estimate $\mathbf{g}$. Such a variability function, henceforth VF, implies evaluating the total loss function at different weights α, now denoted α^*, conditional on the weight α for which the best setting $\mathbf{g}$ was estimated: $VF(\alpha^* \mid \alpha, \mathbf{g})$.

The VF is interpreted as the incurring total loss from a non-optimal estimation of the time series system given another desirable trade-off. It can be expressed for direct ($\alpha = 1$) and indirect ($\alpha = 0$) seasonal adjustment in normalized form as

$$\frac{VF(\alpha^* = 0 \mid \alpha = 1)}{VF(\alpha^* = 0 \mid \alpha = 0)} = VF_{10} \quad \text{and} \quad \frac{VF(\alpha^* = 1 \mid \alpha = 0)}{VF(\alpha^* = 1 \mid \alpha = 1)} = VF_{01}. \qquad (4.3\text{a, b})$$

The ratio VF_{10} in (4.3a) is the error from using direct seasonal adjustment given that we know the best possible estimate at indirect seasonal adjustment, normalized at the best direct seasonal adjustment estimate. VF_{01} (4.3b) is the error of applying indirect seasonal adjustment given that we have the best possible estimate at direct seasonal adjustment, normalized at indirect seasonal adjustment. The margins of the variability function, with base value at either direct or indirect seasonal adjustments, are denoted $VF(\alpha \mid \alpha^* = 0) / D(\alpha^* = 0)$ and $VF(\alpha \mid \alpha^* = 1) / D(\alpha^* = 1)$, respectively, and they represent the relative total loss in the continuum of best possible seasonal adjustments (c.f. 4.3a-b).

5 Estimation and data

5.1 Estimation

The approach of Section 3 requires that the initial state vector $\boldsymbol{\theta}_0 = \{\tau_0, m_0, s_{0,1}, s_{0,2}, \ldots s_{0,P}\}'$ and the exponential decay parameters a^*, b^* and c^* are determined simultaneously for each series at each weight α in the loss function. This can be an intractable optimization problem since the state vector variances can be quite large. The problem may be reduced stepwise through the following.

Initial states are first *seeded* according to the following heuristic scheme proposed by Hyndman et al. (2002).

- A *P+1* moving average SMA_t is computed through the first 4*P* observations from *t=P/2+ j*, *j>0*. Then taking $y_t - SMA_t$ from *t=P/2+1* to *t= 7P/2* gives a detrended series. The average value over each season is then used as initial seasonal component after being normalized to zero-summation over all seasons during this initial period.

- The level component is estimated by seasonally adjusting the series through the preceding step and then fitting a linear regression of the first 10 observations on a time indicator *t*=1,2,..,10. The intercept from this regression will be the initial level and the slope will be the initial level growth component.

This seeding is done independently for each series and used for finding a_j^*, b_j^* and c_j^* by minimizing the (single) loss function for each series. This is carried out through a Newton-Raphson procedure included in the SAS ® software.

Estimating the *3n* parameters a_j^*, b_j^* and c_j^* simultaneously can be difficult for large *n*. These elements in the parameter vectors $\mathbf{a}^*, \mathbf{b}^*$ and $\mathbf{c}^*$ can be written as $a_j^* = a_o a_j$, $b_j^* = b_o b_j$ and $c_j^* = c_0 c_j$, respectively. Then, a_0, b_0 and c_0 are the common factors of all input series' parameters. These common factors can be found by first processing the *n* series independently by the Newton-Raphson procedure with the obtained seed states. By taking logarithms of the obtained parameter estimates, e.g. $\log(a_0 a_j) = \log(a_0) + \log(a_j)$ and subtracting the mean logarithmic estimate over all series, the variable parts a_j, b_j and c_j are obtained after the necessary anti-logging. Finally, the variable parts are kept fixed and the three common factors a_0, b_0 and c_0 are estimated by applying the Newton-Raphson procedure to minimize the total loss function for desired weights α in the uniform interval [0,1], inclusively. Now having a complete set of *3n* parameters and seed states, a full Newton-Raphson optimization of the total loss function can be done based on these precise start values.

The exponential decay parameters a^*, b^* and c^* are allowed to exceed one but are constrained to two. The break-point δ_j in the Huber loss function is set to the 95th percentile of innovation errors for each series. The total loss function is estimated by starting at $\alpha=0$ and moving in 199 steps, $\left[\frac{0}{199},\frac{1}{199},...,\frac{199}{199}\right]$, to $\alpha=1$, making 200 estimates, i.e. 200 different seasonal adjustments of each series.

5.2 Data

Two different time series systems are used here. The first data set is monthly Swedish foreign trade in goods, exports and imports, spanning from January 1975 to December 2012 in current prices, seen in Figure 5. The aggregate in this case is the balance of trade, i.e. exports minus imports. These are high quality data with generally high precision and are collected by Statistics Sweden and the Swedish Customs Office monthly as a cut-off census with very low undercoverage.

The second data set is quarterly Swedish Gross Domestic Product (GDP) in fixed prices with reference year 2011, computed by Statistics Sweden. The GDP series span from the first quarter 1993 to the third quarter 2012 and consists of combinations of series according to SNI (NACE) 2007 classification. GDP is studied both as an aggregate of 9 series of similar size, denoted GDP-9, (Figure 6) and in more detailed form of 30 series of varying sizes, denoted GDP-30. An extract of some series is given in Figure 7.

These two time series systems exemplify the reformulation principle of Section 4: the foreign trade aggregate, i.e. the balance of trade, is the sum of series with different signs (exports minus imports), and aggregate GDP is a grand total of series with positive signs.

6 Results

Seasonal adjustments of the foreign trade and the GDP-9 systems and some of the series in the GDP-30 system are given in Figures 5, 6 and 7. The foreign trade series, which cover almost 40 years, are split into two time periods since their levels are steadily increasing over time.

Relative losses at endpoints are given in Table 1. For the foreign trade, the relative loss incurring from indirect estimation given a direct specification (VF_{01}) is markedly greater than the opposite case of direct estimation given an indirect specification (VF_{10}): 1.42 against 1.13 with Huber loss, and 1.50 against 1.14 with quadratic loss. For the GDP-9 system, the relative differences are quite small; 1.03 for VF_{01} against 1.02 for VF_{10} with the Huber loss and somewhat larger with squared loss. For the larger GDP-30 system, the overall relative discrepancies increase (as was pointed out by Maravall, 2005) and the relative differences between approaches grow. The GDP-9 system seems to be the most homogenous system with respect to specification, i.e. insensitive to direct or indirect seasonal adjustment, whereas the foreign trade is the most heterogeneous system, most sensitive to the choice of estimation order.

Measure\Series	Foreign Trade	GDP-9	GDP-30
Huber VF_{10}	1.13	1.02	1.10
Huber VF_{01}	1.42	1.03	1.15
Quadratic VF_{10}	1.14	1.05	1.16
Quadratic VF_{01}	1.50	1.11	1.21

Table 1. Relative measures (expression 4.3) of differences between direct and indirect estimation.

Figures 2a-c show that total losses are not linear in α. The skewed areas indicate regions where the total loss decays more slowly and are thus indicative of what trade-off weights not to choose. For foreign trade, it is seen that the largest loss appears when α is low, implying that estimations tending at direct seasonal adjustment will be least favorable with respect to total system discrepancies. This is an expected result since the balance of trade is a volatile series that may be negative.

For GDP, the opposite situation occurs: the heavier loss regions appear with larger α, indicating that estimations tending at indirect seasonal adjustment will be most undesirable with respect to total loss in the system, although also depending on the choice of loss function, which was seen from Table 1. The Huber losses are, by construction, smaller than squared losses, and for the GDP systems they also tend to be more skewed towards indirect adjustments.

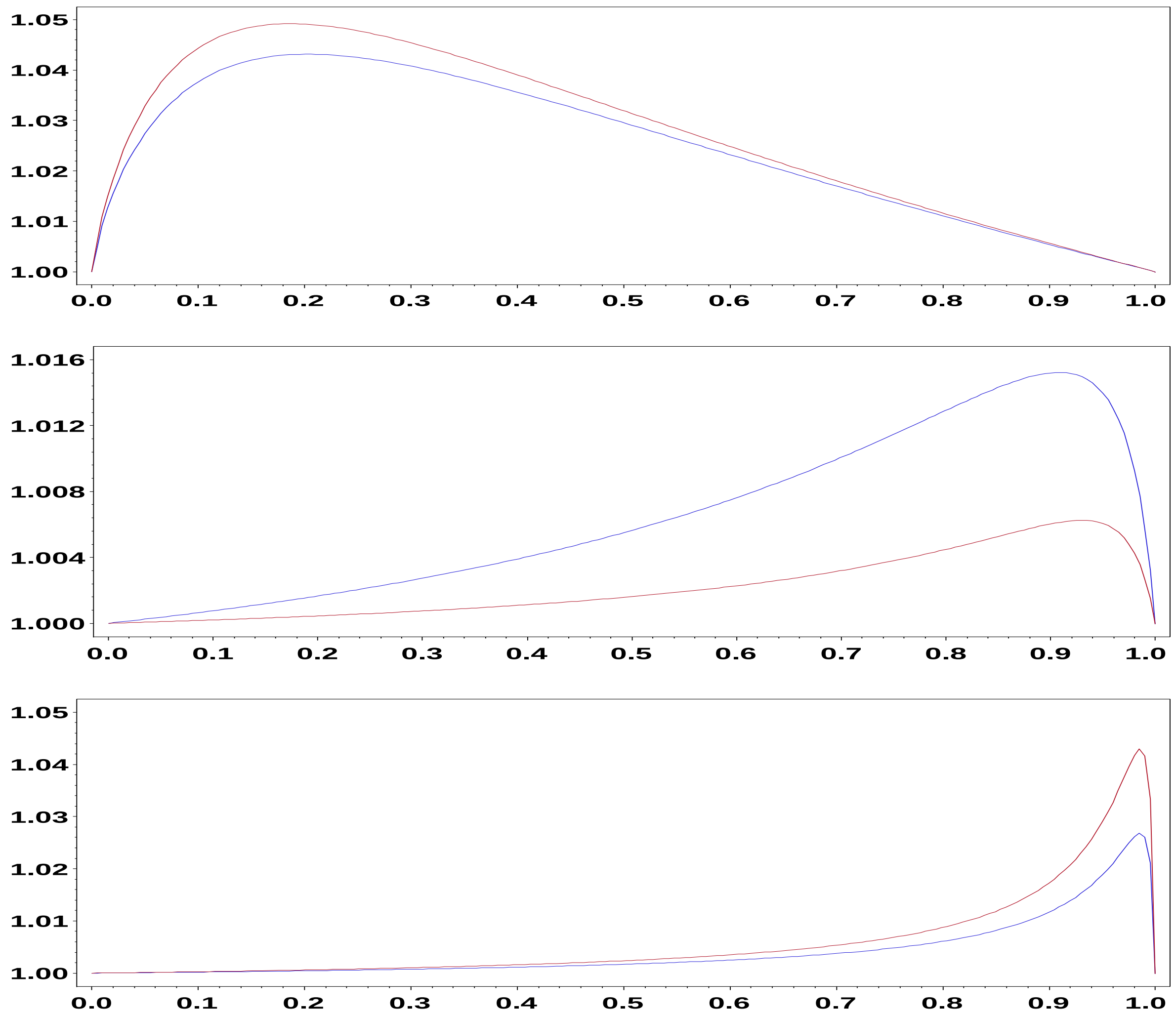

Figure 2a-c. Normalized loss functions for α in the unit interval: **a)** foreign trade, **b)** GDP-9, and **c)** GDP-30. The larger (outer) functions are the quadratic loss and the smaller (inner) functions are the Huber loss.

The variability function margins for the GDP-30 system with Huber loss are given in Figures 3a-b. The first graph shows the relative increase in loss from applying seasonal adjustment for any tradeoff other when parameters are estimated α =0. The second graph shows the opposite case of α =1. These pictures confirm the interpretation of Figure 2c: the largest relative changes in total loss occur in the neighborhood of indirect seasonal adjustment, α =1. The losses are flat until close to indirect estimation. For the foreign trade system (Figures 4a-b), it is seen that the incurred losses change continuously when moved away from the original estimate, and their shapes confirm the endpoint losses in Table 1.

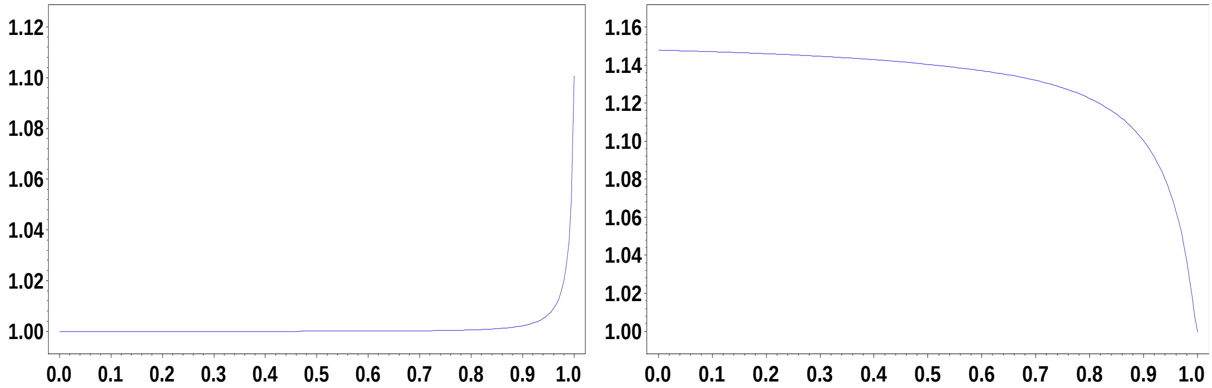

Figure 3a-b. Variability function margins relative to base value for GDP-30 with Huber loss. **a)** Direct estimation with base value $\alpha^* = 0$. **b)** Indirect estimation with base value $\alpha^* = 1$. Horizontal axes show α and vertical axes show relative loss.

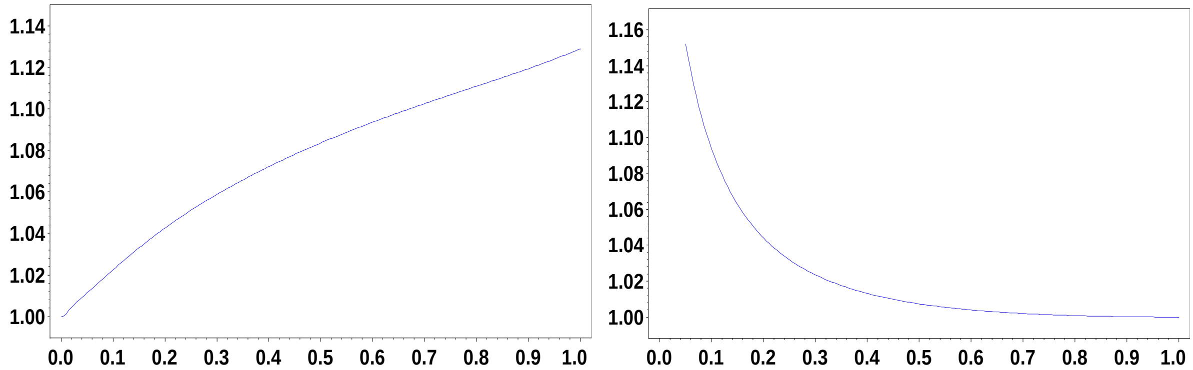

Figure 4a-b. Variability function margins relative to base value for foreign trade with Huber loss. **a)** Direct estimation with base value $\alpha^* = 0$. **b)** Indirect estimation with base value $\alpha^* = 1$. Horizontal axes show α and vertical axes show relative loss at base point.

Some illustrations of the variability function are given in Figures 8-10. It is clearly seen from Figures 8 and 9 that the largest loss for the GDP-30 system incurs from applying direct seasonal adjustment (α^*=0) when parameters are estimated for indirect seasonal adjustment (α=1). Figure 10 shows the opposite case for the foreign trade: the largest loss is incurred when estimations are direct α=0 and applied at indirect adjustment (α^*=1). These results mirror the endpoint losses given in Table 1.

The complete continuum of seasonal adjustments of the aggregate series $SA(Y_{Agg})$ or any other series can be illustrated through the following relative measure. By taking each of the estimated 200 seasonal adjustments of the series at each time point and then subtracting the mean of these (200) adjustments renders a value that is demeaned across the seasonal adjustments. This value is then put in relation to the range of the 200 adjustments, i.e. maximum seasonally adjusted value minus minimum seasonally adjusted value of the 200 adjustments:

$$\frac{SA(Y_{Agg,i,t}) - \sum_{\iota=1}^{200} SA(Y_{Agg,i,t})/200}{\max_i SA(Y_{Agg,i,t}) - \min_i SA(Y_{Agg,i,t})} \qquad \text{for } i=1,2,...,200,\ \forall\, t.$$

In Figure 11, the continuum of seasonal adjustments of total production, i.e. the aggregate series, is obtained by applying the Huber loss to GDP-30. It is clearly seen that for a large region from zero to almost unity of α, there are small differences between adjustments, while close to unity, i.e. when close to indirect seasonal adjustment, the seasonal adjustments tend to differ markedly from the rest of the α-interval.

7 Concluding remarks

The notion of this study has been to consider whether direct and indirect seasonal adjustments can be traded off for some mixture in between. We have seen that either approach may be undesirable, whereas an estimation targeting both may be the least harmful with respect to total error remaining in the time series system.

A conclusion drawn from these results is that the choice of loss function impacts the choice of aggregation order. Since standard estimators are Minimum Mean Squared Errors (MMSE) estimators due to normal theory, they will always penalize large errors more than e.g. the Huber loss, and therefore increase the rift between direct and indirect seasonal adjustments more than appears to be necessary.

The seasonal adjustment problem formulation used here requires all series be available contemporaneously. Should this not be the case, the proposed weighing approach is still applicable on a complete data set at the latest available time point. Once parameters are estimated for the formulation, they may be used in forthcoming seasonal adjustments of the individual series, rendering their desired aggregate.

A simple Holt-Winters algorithmic approach has been used here, but the methods remain to be considered for use in other modeling approaches such as ARIMA or Kalman filtering.

Acknowledgments

The author is grateful to Daniel Thorburn, Stockholm University, for the idea behind this paper and all the guidance he has given for its completion, and to Lars-Erik Öller, Stockholm University, for his valuable remarks.

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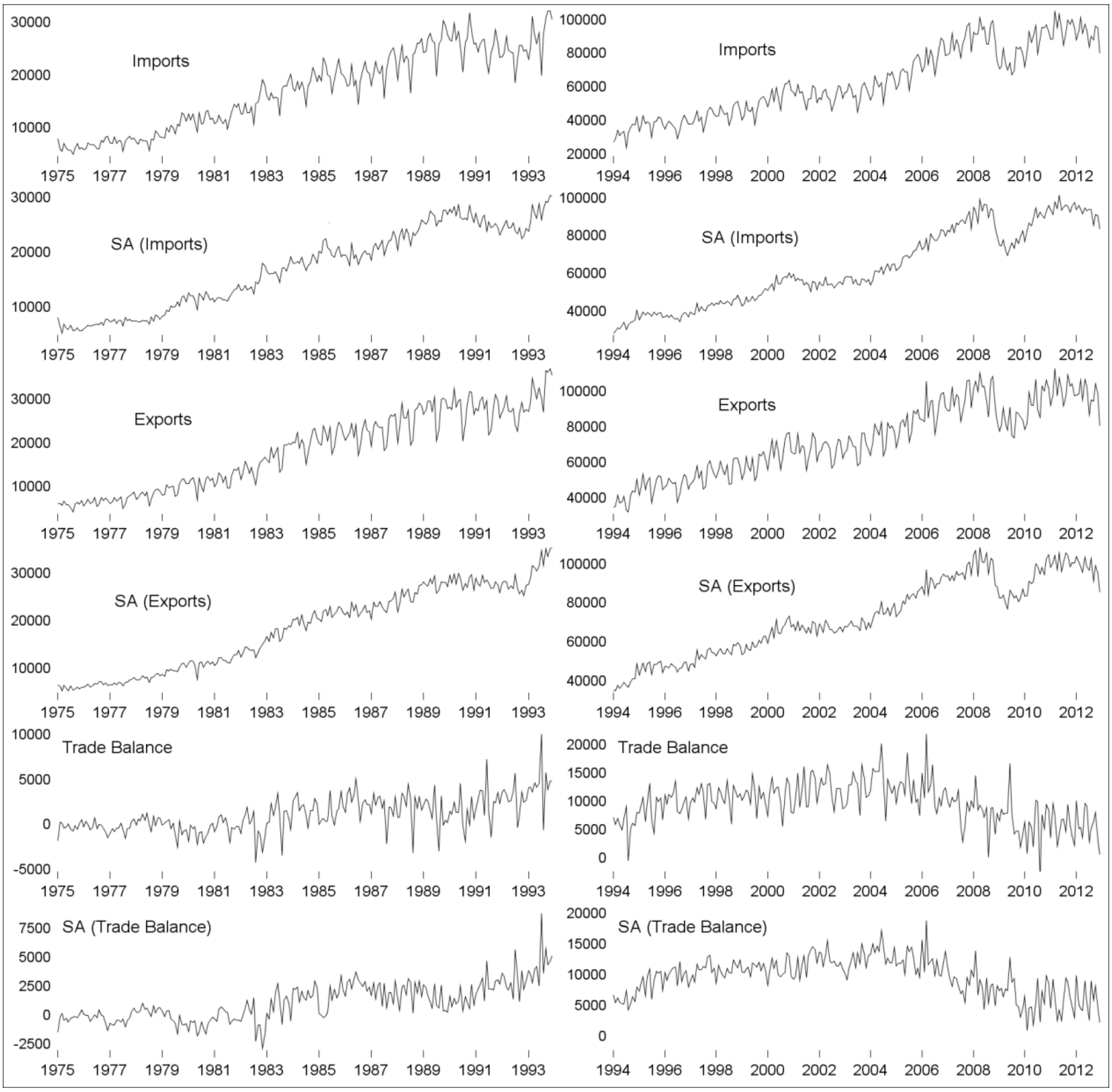

Figure 5. Monthly Swedish Exports and Imports & Balance of Trade during 1975-2012 and seasonal adjustments estimated at $\alpha = 0.5$ with Huber loss. Years 1975-1993 in the first column and 1994–2012 in the second column.

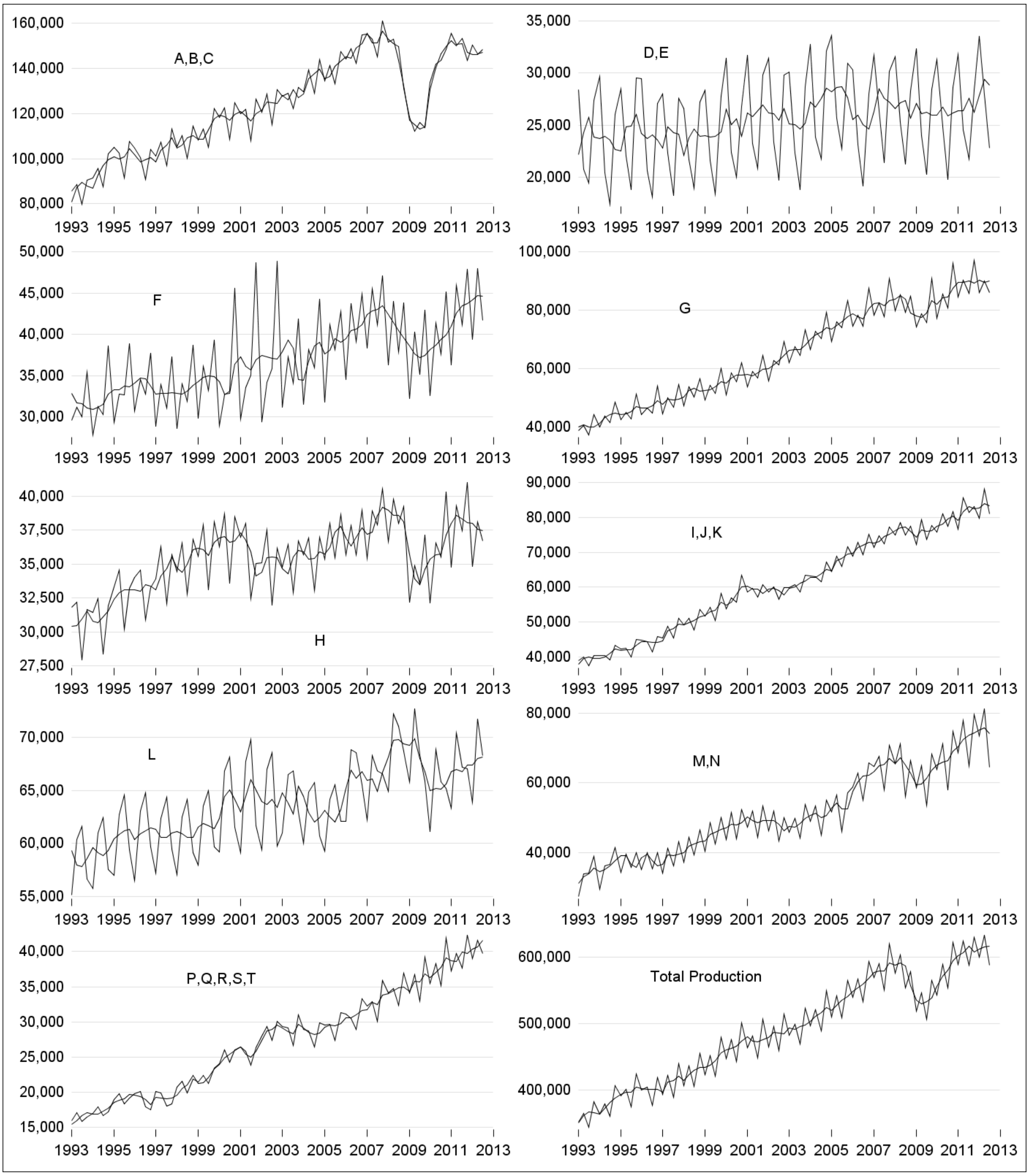

Figure 6. Quarterly Swedish GDP series during 1993-2012 and seasonal adjustments estimated at α =0.5 with Huber loss. 9 aggregate groupings of NACE 2007 and total GDP.

Codes given in Figure 6 follow: A) Agriculture, forestry, fishing. B) Mineral extract. C) Manufacturing industry. D,E) Electricity, gas, steam and air conditioning, water supply, waste. F) Construction. G) Wholesale and retail trade. H) Transport and storage. I) Hotels and restaurants. J) Information and communication. K) Financial services and insurance. L) Real estate activities. M,N) Professional, scientific, technical and administrative activities. P,Q) Education, health and social work activities. R) Personal and art services. S,T) Other service activities and activities of households as employers.

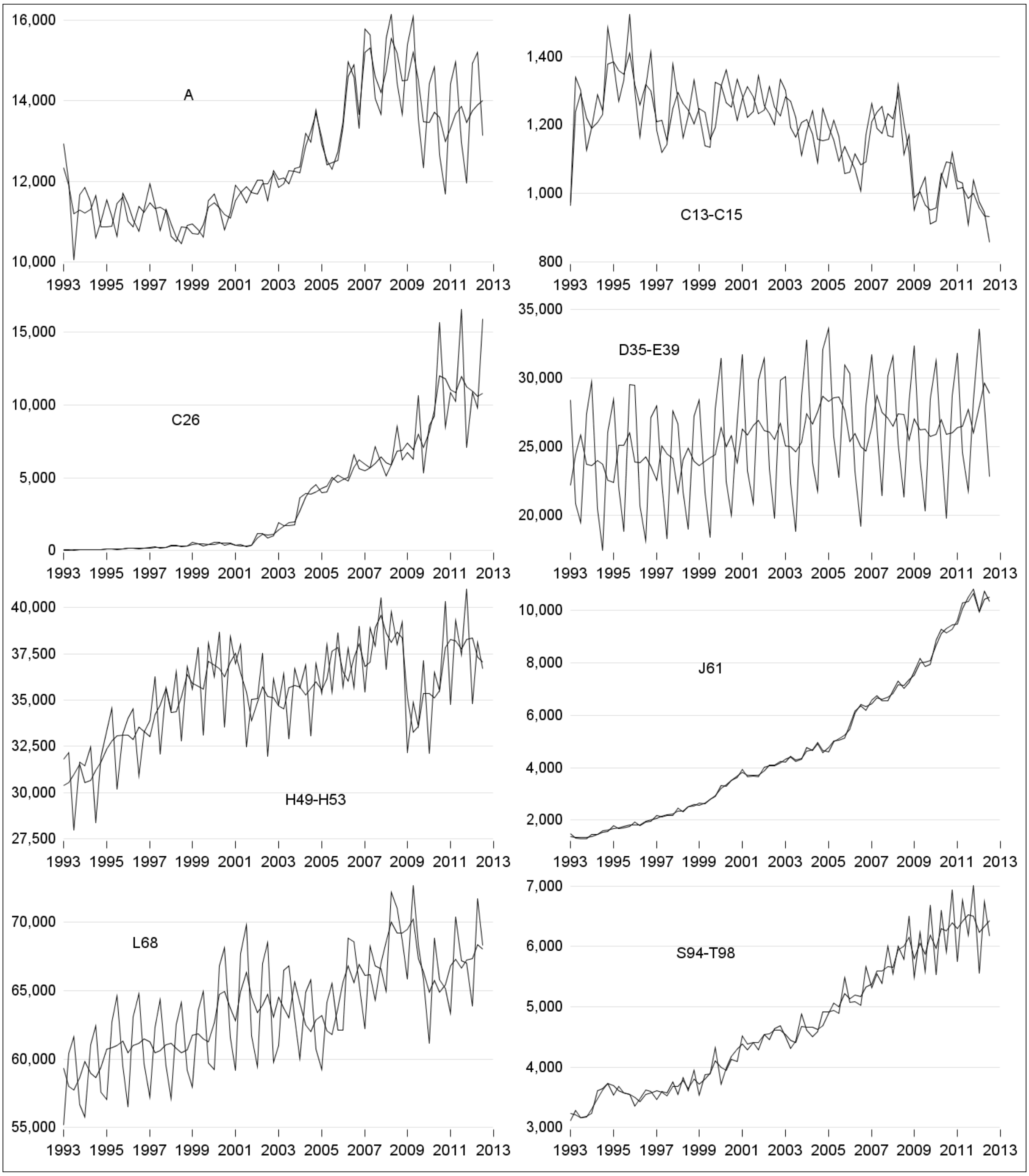

Figure 7. Selected series from GDP-30 indicated by their NACE code and seasonal adjustments estimated at α =0.5 with Huber loss.

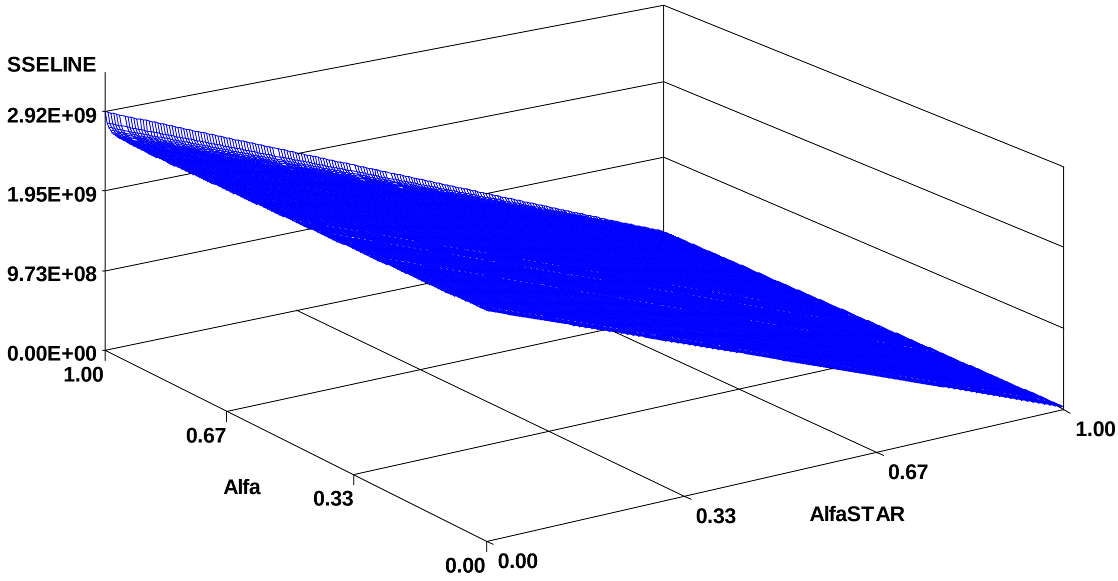

Figure 8. Variability function for GDP-30 with Huber loss. Non-normalized. The left front axis is α and the right front axis is α^*. Nearest point of view is (α =0, α^* =0).

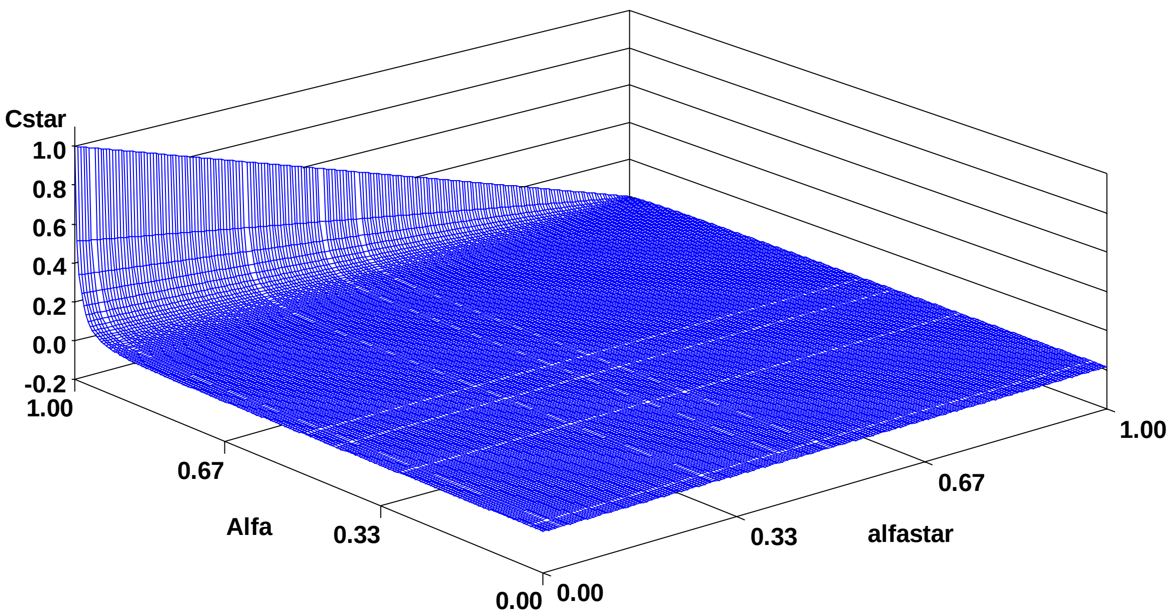

Figure 9. Variability function for GDP-30, Huber loss. Leveled down by subtraction of minimum values of loss at (α, α^*) = (0,0) and (1,1) and thereafter scaled to unity in height (vertical axis labeled *Cstar*). The left front axis is α and the right front axis is α^*, intersecting at nearest point of view (0,0).

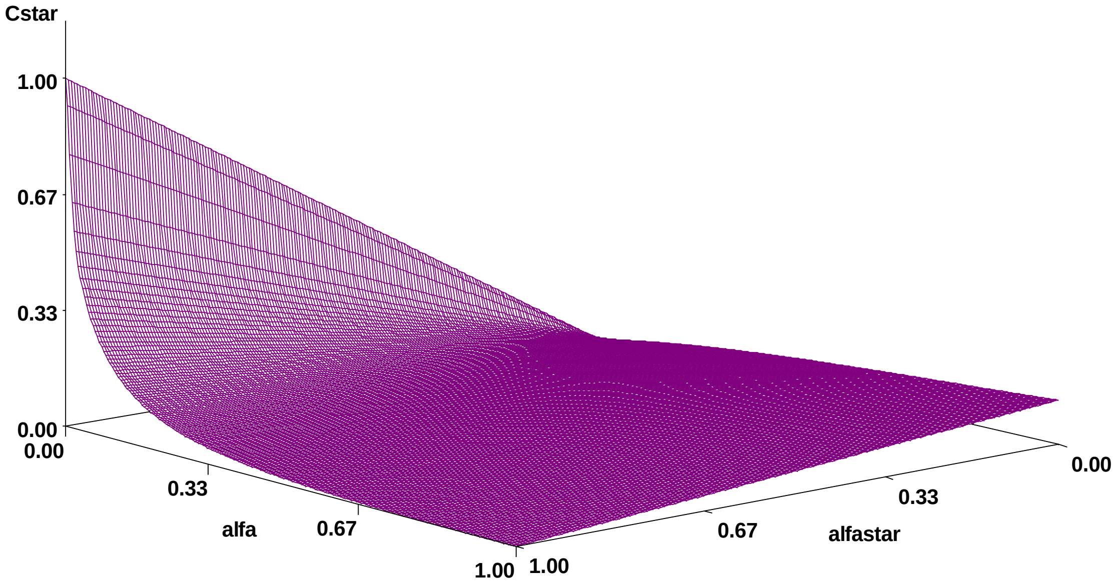

Figure 10. Variability function for foreign trade, Huber loss. Leveled down by subtraction of minimum values of (α ,α^*) at (0,0) and (1,1). Height axis scaled to unity. The left front axis is α and the right front axis is α^*, intersecting at nearest point of view (1,1).

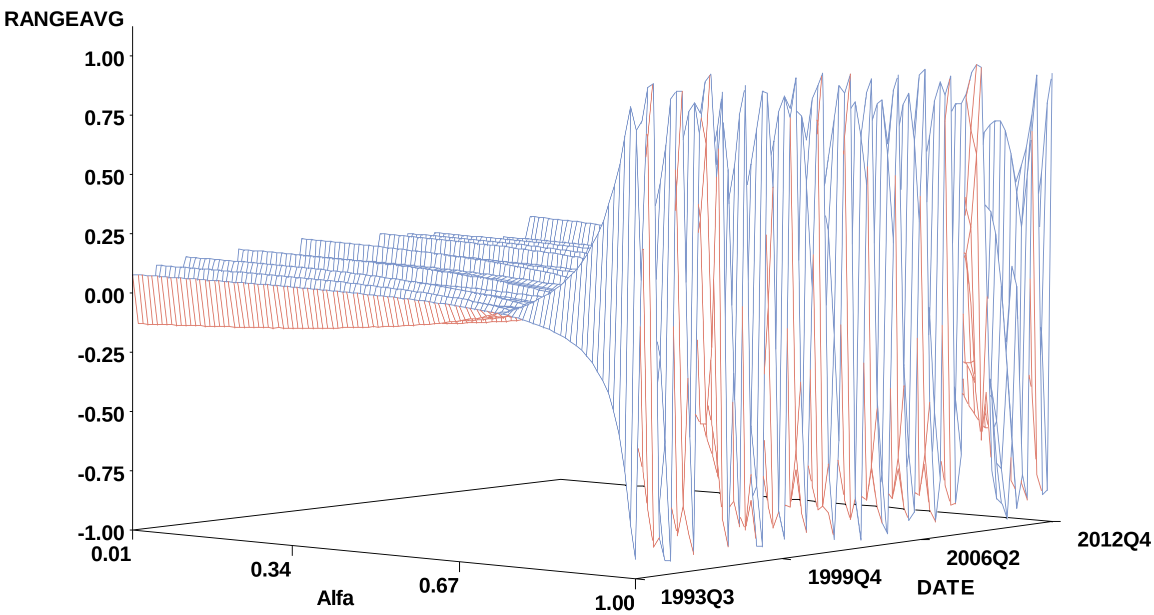

Figure 11. Seasonal adjustment of total production obtained from GDP-30, Huber loss. Relative differences between seasonal adjustments in a continuum from α =1 to α =0. Computation explained in Section 6. The left front axis is α with α =1 in nearest view and the right front axis is the time span, with 1993 in nearest view.

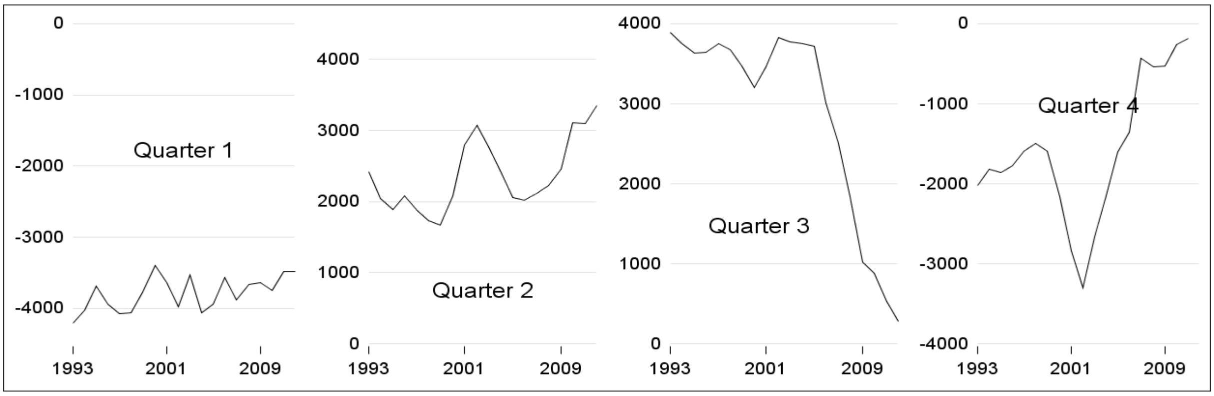

Figure 12. Estimated quarterly seasonal components for *real estate activities*, SNI (NACE) L68.

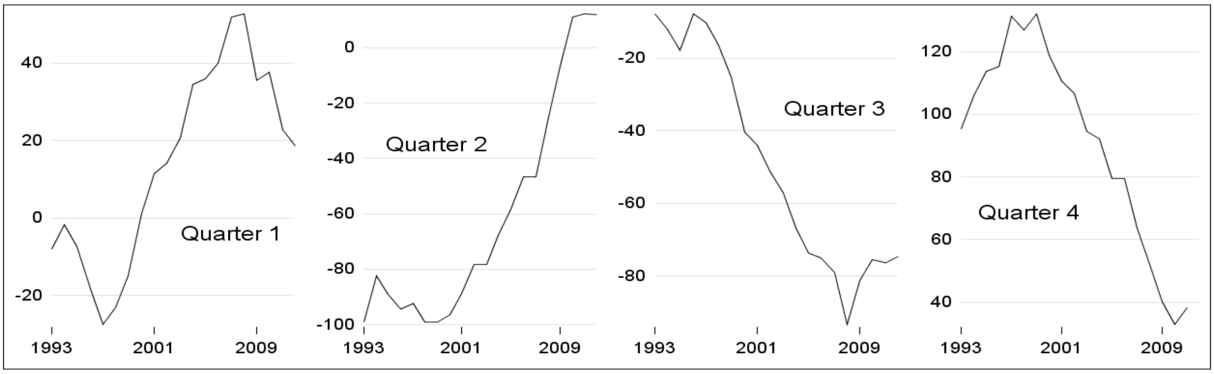

Figure 13.Estimated quarterly seasonal component for *manufacturing of textiles, clothing and leather products* SNI (NACE) C13-C15.

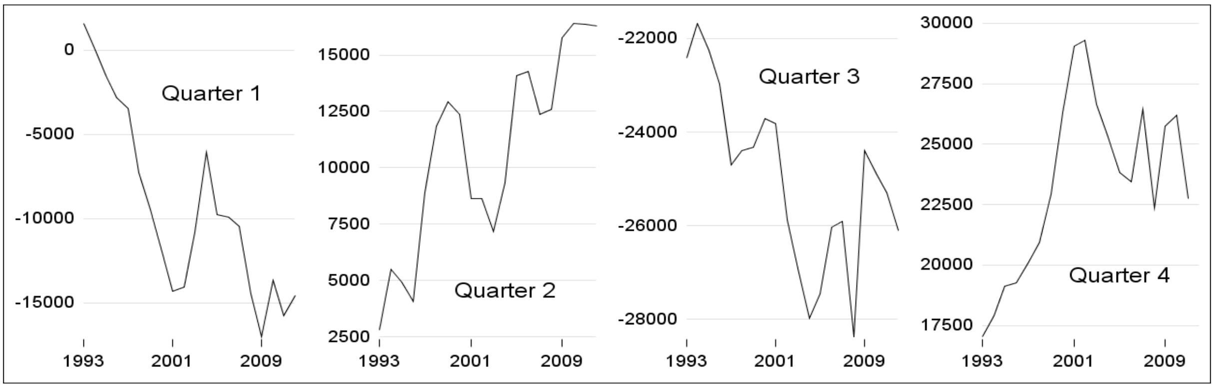

Figure 14. Estimated quarterly seasonal component for total GDP of industrial production.