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A Comparison of Seasonal Adjustment Methods: Dynamic Linear Models versus TRAMO/SEATS

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Summary

Seasonal adjustment can be done in the state space framework by Dynamic Linear Models. This approach is compared with seasonal adjustment by TRAMO/SEATS. The comparison uses simulated time series and real Swedish foreign trade data, the latter allowing a discussion on the consistency issue in aggregation, i.e. direct versus indirect seasonal adjustment. We start by a simple dynamic model and then increase the model structure using Gibbs sampling to identify coefficients for the state evolution matrix. Our empirical study shows that the simpler state spate approach exaggerates seasonal adjustment while the extended model with sampled coefficients may offer a tool for seasonal adjustment. For simulated data, we find that TRAMO/SEATS is better than the state space approach.

Key Words: Dynamic Linear Models, DLM, seasonal adjustment, consistency, foreign trade

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Disclaimer: As usual. All opinions in this paper are the ones of the author.

1. Introduction

In official statistics, seasonal adjustment is generally done either by TRAMO/SEATS or by X-12-ARIMA. A drawback of these prevalent methods is their complexity since adjustments are based on analysis in the frequency domain. Seasonal adjustment can alternatively be done by state space modeling. Dynamic linear models (Harrison & Stevens, 1976) offer a Bayesian state space approach that can be applied to seasonally patterned time series. Varieties of the state space approach can be found in e.g. Durbin and Koopman (2001), Hyndman et al. (2008) and, for ARIMA models, in Hamilton (1994).

In this paper, DLM is compared to TRAMO/SEATS. Comparisons are made on simulated series and on Swedish foreign trade data, which opens for a comparison between direct and indirect seasonal adjustment. Studies of direct and indirect adjustment of foreign trade data have been done by Maravall (2006) and Hood & Findley (2001), based on TRAMO/SEATS and X-12-ARIMA, respectively, and there seems not to be any comparison of DLM against these methods. In the following, Section 2 presents the background necessary for operating DLM. In Section 3, the problem of consistency is addressed and evaluation measures are presented. Practical shortcomings of DLM are also mentioned. Section 4 contains a simulation study comparing the two methods. An application on foreign trade is presented in Section 5, and the results are evaluated in Section 6. Some conclusions end the paper in Section 7.

2. Dynamic Linear Models

2.1 Setting up the filter

The following notation comes from West & Harrison (1989). Normality is assumed throughout, which, if necessary, may be replaced by assuming Student's t distribution. Let Y_t be the univariate time series observation and let $\boldsymbol{\theta}_t$ be an $(n \times 1)$ vector of unobserved components. To assess the latent model, we assume the following normality properties:

$$(Y_t \mid \boldsymbol{\theta}_t) \sim N(\mathbf{F}_t^{'}\boldsymbol{\theta}_t, V_t) \qquad (2.1a)$$

$$(\boldsymbol{\theta}_t \mid \boldsymbol{\theta}_{t-1}) \sim N(\mathbf{G}_t\boldsymbol{\theta}_{t-1}, \mathbf{W}_t) \qquad (2.1b)$$

The defining quadruple of the model $\{\mathbf{F}_t, \mathbf{G}_t, V_t, \mathbf{W}_t\}$ contains (i) the coefficient vector $\mathbf{F}_t$ of dimension $(n \times 1)$ from the regression of Y_t on $\boldsymbol{\theta}_t$, (ii) the $(n \times n)$ state evolution matrix $\mathbf{G}_t$, (iii) the observational error variance V_t and (iv) the $(n \times n)$ state innovation variance matrix $\mathbf{W}_t$. The structures of the three elements $\mathbf{F}_t$, $\mathbf{G}_t$ and $\mathbf{W}_t$ are given in subsection 2.2. The distributions given in expressions (2.1a, 2.1b) can be restated as a state space system:

Observation equation: $$Y_t = \mathbf{F}_t^{'}\boldsymbol{\theta}_t + v_t,\ v_t \sim \mathrm{N}(0, V_t). \qquad (2.2a)$$

State equation: $$\boldsymbol{\theta}_t = \mathbf{G}_t\boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t,\ \boldsymbol{\omega}_t \sim N(\mathbf{0}, \mathbf{W}_t). \qquad (2.2b)$$

The initial information set D_o contains all information available at start and is updated by additional observations on Y_t; $D_t = (Y_t, D_{t-1})$. When an observation is made, the posterior of the unobserved state will be normally distributed with expected value $\mathbf{m}_{t-1}$ and variance matrix $\mathbf{C}_{t-1}$**:**

a) Posteriors at t-1: $(\boldsymbol{\theta}_{t-1} \mid D_{t-1}) \sim N(\mathbf{m}_{t-1}, \mathbf{C}_{t-1})$

To arrive at *a)* we first use the prior estimate of the state and its variance with the evolution variance in expression (2.1b):

b) Prior at t: $(\boldsymbol{\theta}_t \mid D_{t-1}) \sim N(\mathbf{a}_t, \mathbf{R}_t)$.

By (2.1b), the expected value of the state vector is $\mathbf{a}_t = \mathbf{G}\mathbf{m}_{t-1}$. The prior variance is subject to the same forwarding and added the state innovation variance matrix $\mathbf{W}_t$: $\mathbf{R}_t = \mathbf{G}\mathbf{C}_{t-1}\mathbf{G}' + \mathbf{W}_t$.
The forecast distribution of Y_t based on all the information at *t*-1 follows:

c) One-step ahead forecast: $(Y_t \mid D_{t-1}) \sim N(f_t, Q_t)$, with $f_t = \mathbf{F}'\mathbf{a}_t$ and $Q_t = \mathbf{F}'\mathbf{R}_t\mathbf{F} + V_t$.

When Y_t is observed, the posterior distribution in *a)* is obtained by applying the theory of conditioning in multivariate normal distributions (Subsection 2.3) and by incorporating *b)* and *c)*:

d) Posterior at t: $(\boldsymbol{\theta}_t \mid D_t) \sim N(\mathbf{m}_t, \mathbf{C}_t)$ with $\mathbf{m}_t = \mathbf{a}_t + \mathbf{A}_t\mathbf{e}_t$ and $\mathbf{C}_t = \mathbf{R}_t - \mathbf{A}_t\mathbf{Q}_t\mathbf{A}_t'$, where $\mathbf{A}_t = \mathbf{R}_t\mathbf{F}\mathbf{Q}_t^{-1}$ and $e_t = Y_t - f_t$.

The updating algorithm is started by assuming something (or nothing) about the state vector and its variance:

e) Initial prior: $(\boldsymbol{\theta}_t | D_o) \sim \mathrm{N}(\mathbf{m}_0, \mathbf{C}_0)$

The unobserved system vector $\boldsymbol{\theta}_t$ contains all information about the system at a given time point *t*. In our case $\boldsymbol{\theta}_t$ will collect the season, the trend and the trend evolution estimates, to be explained in Subsection 2.2. The observation error v_t in expression (2.2a) contains not only the ordinary measurement error but also one-period transient model effects, such as extreme weather impacts or labor strikes.

$\mathbf{W}_t$ is related to the system variance $\mathbf{C}_t$ by $\mathbf{W}_t = \mathbf{G}\mathbf{C}_{t-1}\mathbf{G}'(\delta^{-1} - 1)$, where δ is a discount factor $(0 < \delta \le 1)$. The prior variance can be rewritten as $\mathbf{R}_t = \mathbf{G}\mathbf{C}_{t-1}\mathbf{G}' + \mathbf{W}_t = \mathbf{G}\mathbf{C}_{t-1}\mathbf{G}'\delta^{-1}$. The discounting is a subjective choice of the magnitude of impacts of sudden fluctuations, and is perhaps the most difficult choice in DLM. The proportional ($\delta^{-1} - 1$) should be kept low to avoid model instability but may at the same time reflect the seasonal frequency in the data, c.f. monthly or quarterly data. A large δ renders a small innovation variance with a (perhaps excessively) smooth system vector, while a small δ renders a larger innovation variance, implying that the system vector will account for more irregularity during a longer period. Rougier (2003) suggests a δ in the interval [0.9, 0.99] with close attention to presumable bad choices.

The observational variance V_t is generally not known and is assumed to have an inverse gamma prior and equals the inverse precision ϕ^{-1} for the corresponding series. Then precision ϕ will follow a Gamma prior with $(\phi \mid D_{t-1}) \sim Gam(n_{t-1}/2, d_{t-1}/2)$ with n_{t-1} being the number of observations in the prior and with d_{t-1} the prior variance estimate. As observations accrue, $n_t = n_{t-1} + 1$ which adjusts the degrees of freedom for the Gamma distribution and

$d_t = d_{t-1} + S_{t-1}e_t^2 / Q_t$, and $(\phi \mid D_t) \sim N(n_t / 2, d_t / 2)$. This updating may be carried on to the update of the system variance and similarly, the covariance matrix is updated

$$\mathbf{C}_t = (S_t / S_{t-1})[\mathbf{R}_t - \mathbf{A}_t \mathbf{A}_t^{'} \mathbf{Q}_t],\ S_t = d_t / n_t .$$

The forecast variance is updated with S_t: $Q_t = \mathbf{F}^{'} \mathbf{R}_t \mathbf{F} + S_{t-1}$, see *c)* on page 3.

2.2 Seasonal models

The two time-domain models used here are seasonal effects models and differ in the order of their autoregressive component. The first order model (Model 1) is a local trend model, having a trend and a trend evolution estimate. The second order model (Model 2) has an additional autoregressive component. The trend in Model 1 is

$$\alpha_t = \alpha_{t-1} + \tau_t + \omega_{t,1}^{(\alpha)} \tag{2.3a}$$

and in Model 2 it is

$$\alpha_t = \beta_1 \alpha_{t-1} + \beta_2 \alpha_{t-2} + \tau_{t-1} + \omega_{t,1}^{(\alpha)} . \tag{2.3b}$$

The evolution term is characterized by a random walk:

$$\tau_t = \tau_{t-1} + \omega_{t,2}^{(\alpha)} . \tag{2.3c}$$

The seasonal effects are modeled as random walks:

$$\begin{aligned} s_{t,1} &= s_{t-1,2} + \omega_{t,1}^{(s)} , \\ s_{t,2} &= s_{t-1,3} + \omega_{t,2}^{(s)} , \\ &\vdots \\ s_{t,p} &= s_{t-1,1} + \omega_{t,p}^{(s)} . \end{aligned} \tag{2.3d}$$

Model 1 has state vector

$$\boldsymbol{\theta}_t = (\alpha_t \quad \tau_t \quad s_{t,1} \quad s_{t,2} \quad \cdots \quad s_{t,p})^{'} ,$$

and Model 2 has state vector

$$\boldsymbol{\theta}_t = (\alpha_t \quad \alpha_{t-1} \quad \tau_t \quad s_{t,1} \quad s_{t,2} \quad \cdots \quad s_{t,p})^{'} .$$

Model 1 Model 2

$$G_t = \begin{pmatrix} 1 & 1 & \mathbf{0}'_{p-1} & 0 \\ 0 & 1 & \mathbf{0}'_{p-1} & 0 \\ \mathbf{0}_{p-1} & \mathbf{0}_{p-1} & \mathbf{0}_{p-1} & \mathbf{I}_{p-1} \\ 0 & 0 & 1 & \mathbf{0}_{p-1} \end{pmatrix} \quad (2.4) \quad \text{or} \quad G_t = \begin{pmatrix} \beta_1 & \beta_2 & 1 & \mathbf{0}'_{p-1} & 0 \\ 1 & 0 & 0 & \mathbf{0}'_{p-1} & 0 \\ 0 & 0 & 1 & \mathbf{0}'_{p-1} & 0 \\ \mathbf{0}_{p-1} & \mathbf{0}_{p-1} & \mathbf{0}_{p-1} & \mathbf{0}_{p-1} & \mathbf{I}_{p-1} \\ 0 & 0 & 0 & 1 & \mathbf{0}'_{p-1} \end{pmatrix}. \quad (2.5)$$

$\mathbf{F}_t$ projects the system vector on the observation; $\mathbf{F}_t = (1 \quad 0 \quad 1 \quad \cdots \quad 0)'$ for Model 1 and $\mathbf{F}_t = (1 \quad 0 \quad 0 \quad 1 \quad \cdots \quad 0)'$ for Model 2. As $\mathbf{F}_t$ and $\mathbf{G}_t$ are constant, the time subscripts can be dropped. The seasonal components follow random walks and their development is due to the structure of the covariance matrix $\mathbf{C}_t$, given below.

P is the seasonal dimension (e.g. $p = 4$ for quarterly data), which gives the number of parameters $n = p+2$ or $n = p+3$, for the two models. The seasonal components must sum to zero in order to eliminate the seasonal impacts over one full cycle (e.g. one year). To ensure this, a zero sum constraint, $\sum_{i=1}^{p} \theta_{i|0} = 0$, is imposed on the initial prior of the seasonal block in $\boldsymbol{\theta}_0$. The covariance matrix of $\boldsymbol{\theta}_t$, $E[(\boldsymbol{\theta}_t - E(\boldsymbol{\theta}))(\boldsymbol{\theta}_t - E(\boldsymbol{\theta}))'] = V(\boldsymbol{\theta} \mid D) = \mathbf{C}$ will ensure that the zero-sum constraint is always achieved for the seasonal components:

$$C_o = \begin{pmatrix} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix} & \\ & \kappa_3 \begin{pmatrix} \frac{p-1}{p} & -\frac{1}{p} & \cdots & -\frac{1}{p} \\ -\frac{1}{p} & \frac{p-1}{p} & -\frac{1}{p} & \cdots \\ \vdots & & \ddots & \vdots \\ -\frac{1}{p} & \cdots & & \frac{p-1}{p} \end{pmatrix} \end{pmatrix} \quad \text{and} \quad C_o = \begin{pmatrix} \begin{pmatrix} \kappa_1 & \kappa_4 & 0 \\ 0 & \kappa_1 & 0 \\ 0 & 0 & \kappa_2 \end{pmatrix} & \\ & \kappa_3 \begin{pmatrix} \frac{p-1}{p} & -\frac{1}{p} & \cdots & -\frac{1}{p} \\ -\frac{1}{p} & \frac{p-1}{p} & -\frac{1}{p} & \cdots \\ \vdots & & \ddots & \vdots \\ -\frac{1}{p} & \cdots & & \frac{p-1}{p} \end{pmatrix} \end{pmatrix}$$

Choosing κ_1, κ_2, κ_3 and κ_4 implies using different priors on component variances.

2.3 Revision analysis – smoothing

Standard methods like TRAMO/SEATS and X-12-ARIMA redo seasonal adjustment of previous time points as new data become available. The Markov property of DLM implies that backward estimation, i.e. a smoothing, is required to obtain revised estimates. The best possible estimate of the previous seasonal vector, given the state at the current time point $(\boldsymbol{\theta}_{t-1} \mid D_t)$, is obtained through the Kalman smoother. For any two vectors of normal variables $\mathbf{X}$ and $\mathbf{Y}$, the conditional expectation of $\mathbf{X}$ given $\mathbf{Y}$ takes the form

$$\mathbf{E}(\mathbf{X} \mid \mathbf{Y}) = \mathbf{E}(\mathbf{X}) + \mathbf{V_{XY}} \mathbf{V_{YY}^{-1}} (\mathbf{Y} - \mathbf{E}(\mathbf{Y})), \quad (2.6)$$

and the variance of $\mathbf{X}$ given $\mathbf{Y}$ is

$$\mathbf{V(X \mid Y)} = \mathbf{V_{XX}} - \mathbf{V_{XY}} \mathbf{V_{YY}^{-1}} \mathbf{V_{YX}}. \tag{2.7}$$

The conditional distribution $(\boldsymbol{\theta}_{t-1} \mid D_t) = (\boldsymbol{\theta}_{t-1} \mid D_{t-1}, Y_t)$ is obtained by the joint distribution $(\boldsymbol{\theta}_{t-1}, Y_t \mid D_{t-1})$. Recalling that $E(\boldsymbol{\theta}_{t-1} \mid D_{t-1}) = \mathbf{m}_{t-1}$ and $E(Y_t \mid D_{t-1}) = \mathbf{F}'\mathbf{a}_t = \mathbf{F}'\mathbf{G}\mathbf{m}_{t-1}$, the covariance follows by right multiplication:

$$\begin{aligned} Cov(\boldsymbol{\theta}_{t-1}, Y_t \mid D_{t-1}) = \ & Cov(\boldsymbol{\theta}_{t-1}, \mathbf{F}'(\mathbf{G}\boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t) + \nu_t \mid D_{t-1}) = Cov(\boldsymbol{\theta}_{t-1}, \mathbf{F}'\mathbf{G}\boldsymbol{\theta}_{t-1} \mid D_{t-1}) = \\ & \mathbf{C}_{t-1}\mathbf{G}'\mathbf{F} = \mathbf{V}_{\boldsymbol{\theta}_{t-1}, Y_t}. \end{aligned} \tag{2.8}$$

To reconsider step c) in subsection 2.1, the variance of the one-step ahead forecast is by definition the covariance between Y_t and Y_t given the information at time point *t-1*, D_{t-1};

$$\begin{aligned} Cov(Y_t, Y_t \mid D_{t-1}) &= Cov(\mathbf{F}'(\mathbf{G}\boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t) + \nu_t, \mathbf{F}'(\mathbf{G}\boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t) + \nu_t \mid D_{t-1}) \\ &= \mathbf{F}'(\mathbf{G}\mathbf{C}_{t-1}\mathbf{G}' + \mathbf{W}_t)\mathbf{F} + V_t = \mathbf{F}'\mathbf{R}_t\mathbf{F} + V_t = Q_t. \end{aligned} \tag{2.9}$$

Then, the expected posterior, conditional on additional data, is

$$E(\boldsymbol{\theta}_{t-1} \mid Y_t, D_{t-1}) = \mathbf{m}_{t-1} + \mathbf{C}_{t-1}\mathbf{G}'\mathbf{F}Q_t^{-1}(Y_t - f_t), \tag{2.10}$$

and the variance is

$$V(\boldsymbol{\theta}_{t-1} \mid D_{t-1}, Y_t) = \mathbf{C}_{t-1} - \mathbf{C}_{t-1}\mathbf{G}'\mathbf{F}Q_t^{-1}\mathbf{F}'\mathbf{G}\mathbf{C}_{t-1}. \tag{2.11}$$

3. Diagnostics of seasonally adjusted series

Evaluations of seasonal adjustments are often method-dependent, and there seems to be no standard criteria. Common measures exist, of which some are used here. Additionally, seasonal adjustment of times series that together constitute an aggregate, such as exports and imports, yielding the trade balance, raises the issue of consistency, which will be addressed here.

3.1 Model diagnostics

In the following, some evaluation measures are used for assessing the accuracy of seasonal adjustment and the consistency between aggregations. The measures are either common to the DLM and TRAMO/SEATS or merely adapted to the DLM because of computational limitations of the TRAMO/SEATS output.

3.1.1 Mean squared error

For any estimated parameter ξ, the mean squared error (MSE) is

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{\xi}_t - E(\xi)\right)^2, \tag{3.1}$$

which can be seen is scale-dependent but is still useful when comparing models on single series.

3.1.2 Model fit

Forecasting is not explicitly necessary for seasonal adjustment with DLM because of posterior updating but model fit is still important and is here measured by (3.1) and the scale-free mean absolute percentage error (MAPE) of forecasts f_t:

$$MAPE = \frac{1}{T-p}\sum_{t=p+1}^{T}\left|\frac{y_t - f_t}{y_t}\right|. \qquad (3.2a)$$

This measure is undefined at $y_t = 0$ so an alternative is to use a forecast error measure proposed by Hyndman & Koehler (2008), also scale independent. This is named the mean absolute scaled error (MASE) and targets comparing the model fit by relating it to an in-sample naïve forecast error:

$$MASE = (\sum_{t=p+2}^{T}| y_t - y_{t-1} |)^{-1} \sum_{t=p+2}^{T}| y_t - f_t | \qquad (3.2b)$$

MASE is desired to be below unit value (<1), since unit value means that the forecast model is no better than a naïve approach, i.e. the previous observation.

3.1.3 Residual seasonality in irregular components

A detrended and seasonally adjusted series should be purely white noise. Here, irregular components are checked for autocorrelations. White noise autocorrelations should, under normality, lie within $2/\sqrt{T-p}$ of zero on a 5% level (see Hamilton, 1994). This will be visualized graphically. Autocorrelation coefficients are tested for significance by using the Ljung-Box Q-statistic.

3.1.4 Revision

As new observations are made, seasonally adjusted next-to endpoint observations are revised. In some sense, this revision is the conjectural misjudgment at the previous time point, given today's information. For DLM, the Markov property implies that the prior estimates are unchanged and only the posterior distribution for the final data point is estimated. The mean absolute revision error (MARE) of the system vector estimate can be computed as

$$MARE = 100 \times \frac{1}{J}\sum_{j=1}^{J}\frac{1}{T-(p+j)}\sum_{t=p+j+1}^{T}\left|\frac{\mathbf{F}'\hat{\boldsymbol{\theta}}_{t-j|t} - \mathbf{F}'\hat{\boldsymbol{\theta}}_{t-j|t-j}}{\mathbf{F}'\hat{\boldsymbol{\theta}}_{t-j|t-j}}\right| \quad \text{for } j>0. \qquad (3.3a)$$

Equation (3.3a) should be interpreted as the revision in the entire model at any time point and the concurrent season *k:* $\alpha_{t-j} + s_{t-j,k}$. The smoothed state vector for period *t-j*, given *t*, is compared to the benchmark from time *t-j*, i.e. the first estimate. Revision of each component can be analyzed directly. The mean absolute revision error in the adjusted series can be defined as a simplification of (3.3a):

$$MARE-D = 100 \times \frac{1}{J}\sum_{j=1}^{J}\frac{1}{T-(p+j)}\sum_{t=p+j+1}^{T}\left|\frac{SA_{t-j|t} - SA_{t-j|t-j}}{SA_{t-j|t-j}}\right| \quad \text{for } j>0, \qquad (3.3b)$$

in which $SA_{t-j|t-j}$ is the seasonally adjusted series $(Y_t - s_{t,k})$ for the coinciding season k at the first occurring time point *t-j* while $SA_{t-j|t}$ is the revised seasonally adjusted series , estimated at the later time point *t* .

3.1.5 Roughness/smoothness

Dagum (1979) has proposed two different kinds of roughness measures for seasonally adjusted data. One of the methods is applicable for all adjustment methods and the other being specific to the X-11 methodology. The former is the difference (∇) between consecutive points of seasonally adjusted data, is scale-dependent and is one of two components in the Hodrick-Prescott filter:

$$R = \sum_{t=p+2}^{T}(\mathrm{SA}_{\tau} - SA_{t-1})^2 = \sum_{t=p+2}^{T}(\nabla SA_t)^2 \qquad (3.4)$$

Note to (3.4): Since the DLM has an initial startup period of *p* observations, the first adjustment of interest is *p+1* so *R* should begin at *p+2*.

For simulated series, the benchmark will be the true roughness which is known while applied to real series, this measure lacks objective evaluation criteria.

3.1.6 Signs of growth rates

Direction of the growth of a series is a core question when comparing seasonal adjustment methods or aggregations. Direct and indirect seasonal adjustments should intuitively render the same growth direction. The statistic of interest is thus the difference in sign of monthly and yearly growth rates.

3.1.7 Discrepancy between aggregations

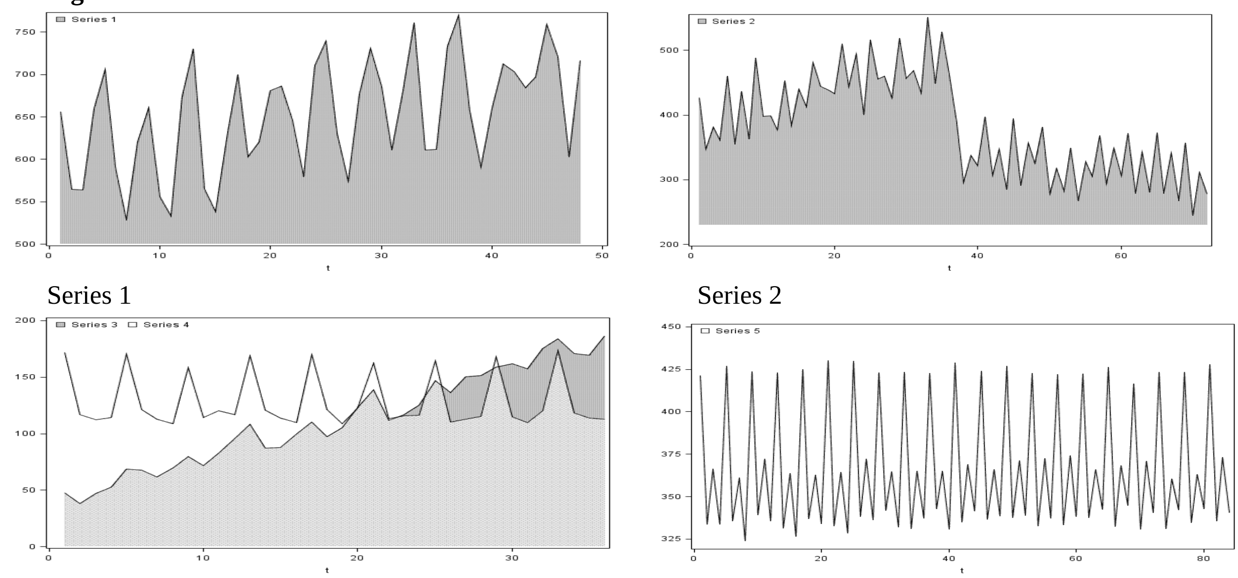
Direct and indirect seasonal adjustments are expected not to differ too much. Penalizing large discrepancies between a direct estimation and the summarized estimation of *k* series is done by the following distance measure:

$$D = \sum\nolimits_{t=p+1}^{T}(SA_t^D - \sum_{k=1}^{K} SA_t^k)^2 \ . \qquad (3.5)$$

4. Application to simulated data

A comparative study between DLM and TRAMO/SEATS should comprise mathematical derivations of the methods, but this would be a tedious task, if even possible. Instead, we apply these methods on a handful of simulated series with components predetermined. Five series, all quarterly for parsimony, with different attributes are analyzed, see Figure 4.1. Each series is simulated just once.

Figure 4.1 Illustration of the simulated series



Series 1 Series 2

Series 3 (in bold) and 4 Series 5

Seen in Figure 4.1, Series 2 has a level shift and in Series 3, the seasonal components are hard to distinguish, i.e. weak seasonality (the growing line, in bold). Series 4 and 5 have a predictable seasonal structure unlike Series 1 which has a perturbed seasonal pattern (see specifications in Appendix A.2). The target of the simulations was to study how DLM and TRAMO/SEATS captured the latent components. Since TRAMO has a head start in modeling the data generating process (due to back-/forecasting), informative priors were obtained for DLM by two consecutive runs. First, the filter was run with non-informative seasonal priors from t=1 to t=T. The posterior seasonal estimates at the last time point (t=T) were then used as priors (after necessary reordering) for the second run starting at t=1.

In Table 4.1, MSE of seasonal components, irregular component and the trend are given for TRAMO/SEATS, DLM with and without informative priors based on a single run for each series.

Table 4.1 Mean squared error (MSE) of estimated components by TRAMO/SEATS and DLM

Series	Seasonal MSE			Irregular MSE			Trend MSE		
	TRAMO/ SEATS	DLM+	DLM++	TRAMO/ SEATS	DLM+	DLM++	TRAMO/ SEATS	DLM+	DLM++
1	96.4*	369.8	106.6	196.9*	658.7	382.1	793.7	624.7	605.8*
2	§	181.9	63	§	660.4	586.2	192.2*	874.6	1277
3	2.9	10.9	2.8	5.8*	25.3	18.8	15.6*	21.6	21.6
4	2.9*	117.2	3.7	10.4	76.3	8.6*	10.3*	27.7	13.9
5	2.5*	121.6	3.3	4*	87.1	7.3	10.8*	28.3	18.3

(+) Non-informative priors. (++) Informative priors. First p=4 observations are omitted in MSE computations**.** (§) Measure not obtained accurately. * indicate the best measure.

TRAMO/SEATS seems to be generally more accurate than DLM in capturing the latent components (10 cases of 15). With informative priors, DLM is better than TRAMO/SEATS in just two cases, two cases are inconclusive and one case, seasonal MSE for Series 3, is a tie. This may indicate that DLM is inadequately specified in this study, or perhaps assessing the problem in the time domain is inferior to the frequency domain approach.

Table 4.2 MAPE and MASE of forecast errors and Ljung-Box Q.

Series	MAPE		MASE		LB-Q		
	DLM +	DLM ++	DLM+	DLM++	DLM +	DLM ++	TRAMO/SEATS
1	0.06	0.04	0.56	0.41	61.0 *	33.8	9.6
2	0.08	0.08	0.47	0.42	85.8 *	111 *	19.7 *
3	0.07	0.06	0.67	0.58	28.5	33.6	11.0
4	0.08	0.04	0.39	0.18	62.1 *	24.6	13.8
5	0.02	0.01	0.16	0.07	230 *	21.2	13.8

Note: MAPE not possible to obtain from the standard output of TRAMO/SEATS. Critical value 36.41 for DLM, TRAMO/SEATS critical value is depending on model.

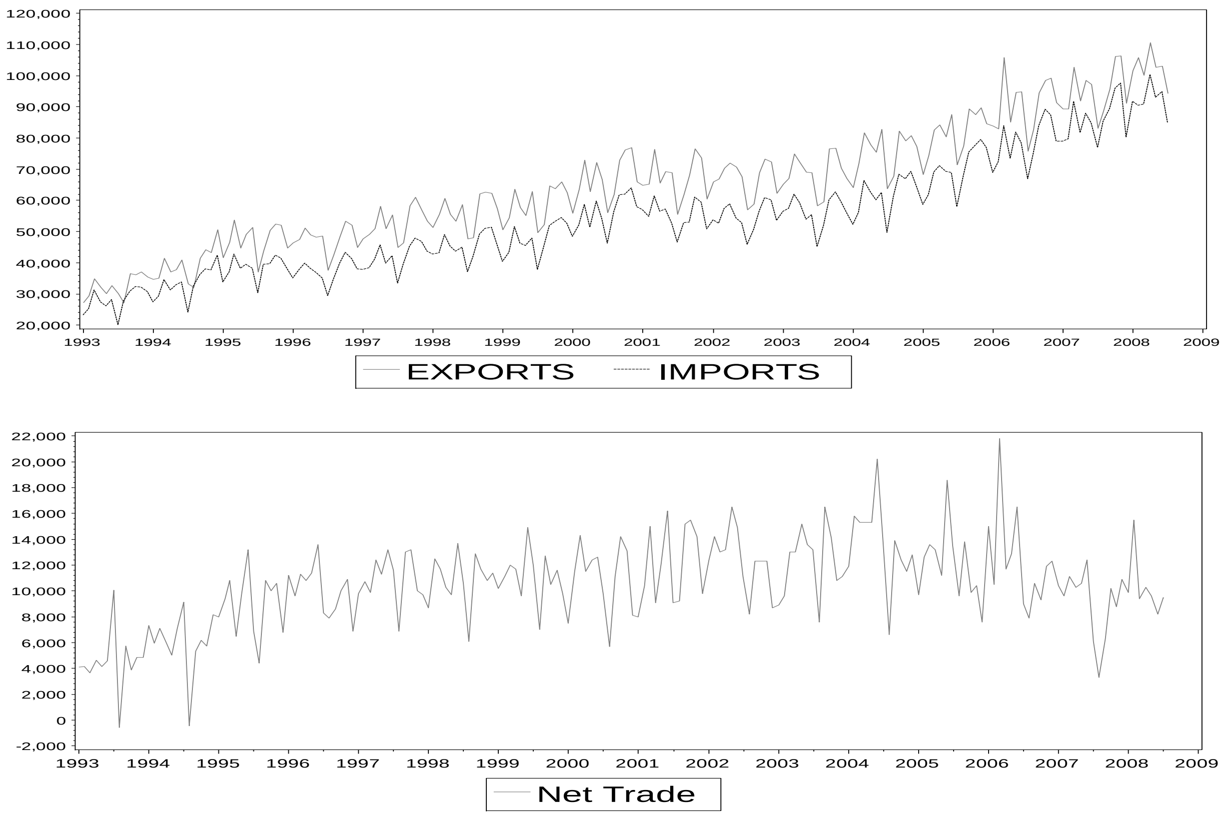
Seen in Table 4.2, informative priors are necessary when it comes to forecasting accuracy (MAPE, MASE). The Ljung Box Q-statistic for squared residuals signals remaining seasonality in residuals in all but one case of uninformative priors while using informative priors compares well with TRAMO/SEATS.

5. Data and model specification

5.1 Data of the study

We use monthly Swedish trade data from January 1993 to July 2008, consisting of 187 observations, obtained from Statistics Sweden. The Intra-European trade is collected by the statistical agency as a cut-off survey while the Extra-European trade is collected by the Swedish Customs office as a census. Data of total exports and imports are known to be of high quality and values reported here are in current prices, Million Swedish Kronor (SEK)

Figure 5.1 Exports and imports and Net trade. Original series, in Million SEK.



Seen in Figure 5.1, imports and exports have a quite similar behavior with log-linear trends and are most likely log-difference stationary.

5.2 Specification of Model 1

The most stable models were found for δ in the neighborhood of 0.85, 0.9, 0.95 or 0.99, the last being the default setting in BATS software (**B**ayesian **A**nalysis of **T**ime **S**eries, see Pole, West and Harrison, 1994). Smaller discount factors rendered either large or non-positive system error covariance matrices, a situation also occurring in BATS.

The level prior was set to be the first observation (January 1993) and the trend variance prior set to 10 % of the squared mean of the series. Prior variances for the trend evolution, seasonal components and observation variances $\mathbf{V}_0$ were set to one tenth (10%) of the prior trend variance. This precaution of setting initial priors was due to the observed problems when specifying the variance matrix – it was soon found that the definitional range of the variance matrix was quite narrow, which also affected the choice of discount factors.

5.3 Specification of Model 2

Model 2 was specified similar to Model 1. The autoregressive parameters of the trend, which is a latent variable, had to be estimated. The problem resembles parameter estimation in regression analysis, also applicable to the state space setting. In order to estimate the trend persistency coefficients β_1 and β_2, a Markov Chain Monte Carlo procedure was used by applying a Gibbs sampler for autoregression, see Lenk (2001). The latent component α_t in expression (2.3b) follows by construction a normal distribution and assuming that regressors (components) have no covariance (i.e. conditional maximum likelihood) will imply

$$\alpha_t \sim N(\mathbf{A}\boldsymbol{\beta}, \sigma^2 I_n), \qquad (5.2)$$

where n is the number of observations until t , inclusive. E(α_t) is given by $\mathbf{A}\boldsymbol{\beta}$ where $\mathbf{A}$ by definition is the data matrix obtained from the DLM recursion and $\boldsymbol{\beta} = (\beta_1 \;\; \beta_2)'$. The parameters are $\boldsymbol{\beta}$ and σ^2. Let the hypothetical likelihood of (5.2) be

$$p(\boldsymbol{\alpha} \mid \boldsymbol{\beta}, \mathbf{A}) = (2\pi)^{-n/2} (\sigma^2)^{-n/2} \exp\left(-(2\sigma^2)^{-1}\left((\boldsymbol{\alpha} - \mathbf{A}\boldsymbol{\beta})'(\boldsymbol{\alpha} - \mathbf{A}\boldsymbol{\beta})\right)\right. \qquad (5.3)$$

This is a strong assumption, postulating (5.3) to be a real likelihood, while it only in fact exists by construction. **A** is not obtained until the end of the recursion, consisting of all coherent trend estimates. In that sense, this is a proper likelihood but obtained from a recursion.

The set of priors are:

$$p(\boldsymbol{\beta}, \sigma^2) = p(\boldsymbol{\beta}) p(\sigma^2) \qquad (5.4)$$

$$\boldsymbol{\beta} \sim N(\boldsymbol{\mu}_0, \mathbf{V}_0) \qquad (5.5)$$

$$\sigma^2 \sim InverseGamma\left(\frac{r_0}{2}, \frac{s_0}{2}\right) \qquad (5.6)$$

The posterior conditional distribution for $\boldsymbol{\beta}$ is

$$p(\boldsymbol{\beta} \mid \mathbf{A}, \boldsymbol{\alpha}, \sigma^2) \sim N(\boldsymbol{\mu}_n, \mathbf{V}_n) \tag{5.7}$$

where the variance $\mathbf{V}_n = (\mathbf{V}_0^{-1} + \frac{1}{\sigma_n^2}\mathbf{A}'\mathbf{A})^{-1}$ and the mean $\boldsymbol{\mu}_n = \mathbf{V}_n(\mathbf{V}_0^{-1}\boldsymbol{\mu}_0 + \frac{1}{\sigma_n^2}\mathbf{A}'\boldsymbol{\alpha})$. $\mathbf{V}_0$ and $\boldsymbol{\mu}_0$ are initial priors. The a posteriori conditional distribution for σ^2 is inverse Gamma

$$p(\sigma^2 \mid \boldsymbol{\beta}, \mathbf{A}, \boldsymbol{\alpha}) \sim IG\left(\frac{r_n}{2}, \frac{s_n}{2}\right), \tag{5.8}$$

where $r_n = r_o + n$ and $s_n = s_0 + RSS$ (RSS = residual sum of squares). Having priors and expressions for the conditional posteriors, and using Bayes' rule to obtain the joint distribution given the likelihood of data, a simplified Gibbs sampling can be used to get the parameters of interest. The estimation of $\boldsymbol{\beta}$ is done by starting the Markov chain with an initial set of priors $\boldsymbol{\beta}^{(0)}$ and $\sigma^{(0)}$, obtained from draws from expressions (5.5) and (5.6). The prior of $\boldsymbol{\beta}^{(j)}$ is plugged into the DLM which is run through from t=1 to t=T. Residuals are obtained from the irregular component, of which the sum of squared is taken as if it were a RSS from a linear regression. $\sigma^{(j)}$ is drawn from the inverse Gamma with shape parameter $s_n/2$ and degrees of freedom $r_n/2$ and used in the normal distribution from which the $\boldsymbol{\beta}^{(j)}$ are drawn. This procedure is repeated 1000 times (= chain length) after 200 burn-in observations and the average of the estimates of $\beta^{(i)}$ and $\sigma^{(i)}$ from overall 25 chains are used since the averages approximate a Monte Carlo integration of the joint posterior.

5.4 TRAMO/SEATS specification

TRAMO/SEATS was operated in automatic mode here. The default model in the TRAMO is the Airline model ARIMA(0,1,1)(0,1,1) against which other models are compared in estimation. No calendar effects were used (RSA=3) in order to achieve comparability with DLM. Outliers were treated by default, but not in DLM.

5.5 Software

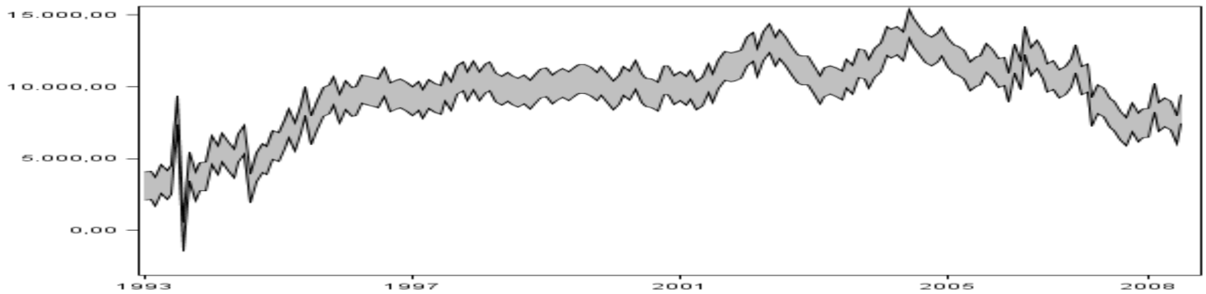
The programming for this study considering DLM was done in Gauss ®. The algorithm was verified by shadow programming in IML in SAS ®. TRAMO/SEATS ® was used in the Windows version R12.6.

6. Estimation Results

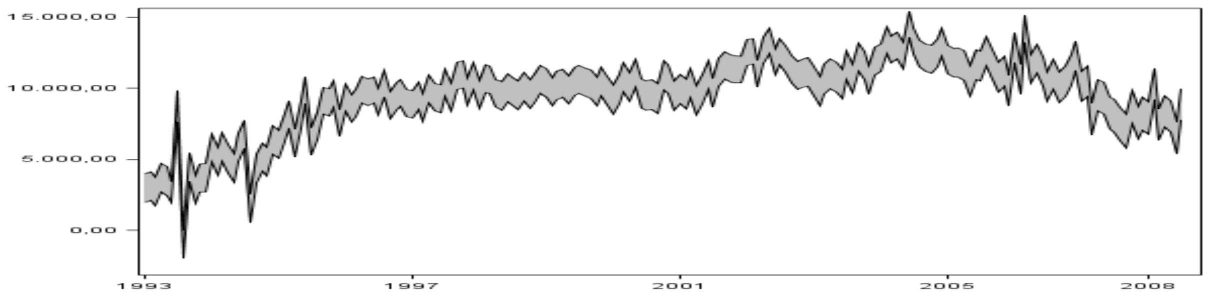
Tables A.1 and A.2 show model estimates for four different discount factors. With respect to mean squared error (MSE) of the irregular components and mean absolute percentage error (MAPE) of forecasting, Model 1 with δ = 0.85 and Model 2 with δ = 0.9 were chosen for seasonal adjustment. The coefficient sampling for Model 2 rendered practically same coefficients for all discount factors.

Estimates from TRAMO/SEATS are given in table A.3. For both exports and imports, an ARIMA (2,1,0)(0,1,1) model was fit with correction for one and two outliers, respectively. The Airline model was fit for the net trade series, with two additive outliers. Seasons were modeled as ARMA(11,11) where each seasonal parameter was a function of the eleven preceding; $s = (1 + B + B^2 + ... + B^{11})$ with unit value for all AR-coefficients, similar to Maravall (2006). This parametrization is discussed in Roberts & Harrison (1984). The trend-cycles were integrated moving average, IMA(2,2). For both imports and exports, the TRAMO suggested an ARMA(2,2) transitory component, meaning that the irregular components had a identifiable pattern for a specific interval in the series.

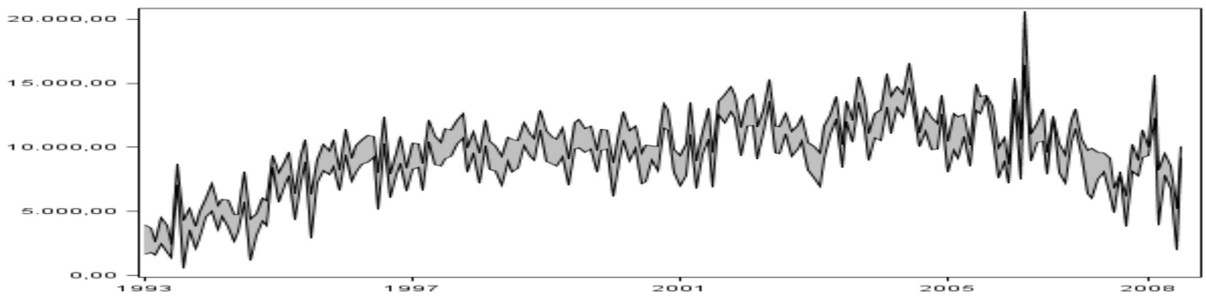
Figure 6.1 (a-c): Seasonal adjustments of net trade, directly and indirectly obtained through Model 1, Model 2 and TRAMO/SEATS. Indirect adjustment shown on -2 000 units (= - 2 000 Million Swedish Kronor (SEK))below actual value. The bold area is the difference between direct and indirect adjustments.



(a) DLM Model 1



(b) DLM Model 2



(c) TRAMO/SEATS

Figure 6.1 (a-c) show that the largest asymmetry between direct and indirect seasonal adjustment is from TRAMO/SEATS, while The DLM adjustments are more coherent with the nominal difference of -2 000 Million Swedish Kronor (SEK). Seasonal adjustment of imports and exports (Figure A.1) show that DLM smoothens to such an extent that the series resemble trends rather than seasonally adjusted series.

6.1 Diagnostics

6.1.1. Mean Squared Error of forecasts

Seen in Table A.1, the lowest discount factor ($\delta = 0.85$) for Model 1 yielded the smallest MSE for all series so there was little gain from increasing the structure of DLM in Model 2 from a forecasting point of view.

6.1.2 Model fit

In Tables A.1 and A.2, MAPE and MASE are given for both models. The two measures both indicated that discount factors 0.85 and 0.9 were preferable for Model 1 and Model 2, respectively. The MAPE behaved similarly in most cases for the two models but for the net trade, which is a difficult and volatile series, Model 1 failed strongly with respect to MAPE.

6.1.3 Residual seasonality in irregular components

The irregular components are seen in Figure A.2 (a-f). The irregulars from Model 1 alternate with smaller magnitude, closely related to the oversmoothing, while the irregulars from Model 2 are larger and with a slightly more random appearance (scaling is different in figures). The autocorrelation functions (ACF) are displayed in Figure A.3 (a-f) and the Ljung-Box Q statistic (Table 6.1) indicate that residuals from Model 1 appear to be non-random with significant autocorrelations while Model 2 has lower values than TRAMO/SEATS in two of three cases.

Table 6.1 Ljung-Box Q for autocorrelations of squared residuals.

Series	Model 1	Model 2	TRAMO/SEATS
Exports	90.66 *	13.88	21.39
Imports	72.60 *	17.1	21.05
Net Trade	50.07 *	31.73	26.55

Critical value 36.41 (5 %).

6.1.4 Revisions of estimates

Revisions of the system vector (MARE) and the adjusted series (MARE-D) are small for both specifications, Table A.1 and A.2, columns 5 and 6. This measure could not be tried for TRAMO/SEATS, but empirical knowledge tells us that revisions can be large.

6.1.5 Roughness

Roughness ratios are given in Table 6.2, with TRAMO/SEATS as benchmark. The adjustments from Model 2 were twice as rough as from Model 1, but both were markedly smaller than the benchmark.

Table 6.2 Roughness ratios of DLM against TRAMO/SEATS based on the roughness measure in expression (3.4)

Adjusted Series	Ratio Model 1 / TRAMO/SEATS	Ratio Model 2 / TRAMO/SEATS
Imports	0.22	0.46
Exports	0.15	0.33
Net trade (direct)	0.10	0.23
Net trade (indirect)	0.09	0.21

6.1.6 Signs of growth rates

The direction of growth rates were controlled for when comparing the direct and indirect approaches. Model 1 showed a consistent growth rate sign for both monthly and yearly rates, and for Model 2, just 1 of 163 observations of the yearly rates differed. For TRAMO/SEATS, 5 of 163 yearly growth rates differed in sign between direct and indirect adjustments and 14 of 174 monthly rates differed.

6.1.7 Discrepancy between aggregations

In Figure 6.1, the distances between adjustments are displayed. Numerically, it was 21 291 for Model 1, 2 217 773 for Model 2 and 38 716 262 for TRAMO/SEATS, implying that for this empirical case, the distance measure favored DLM. One obvious reason for the excellent non-discrepancy in DLM may be the excessive smoothing that occurs in our cases.

6.2 Informative variance priors

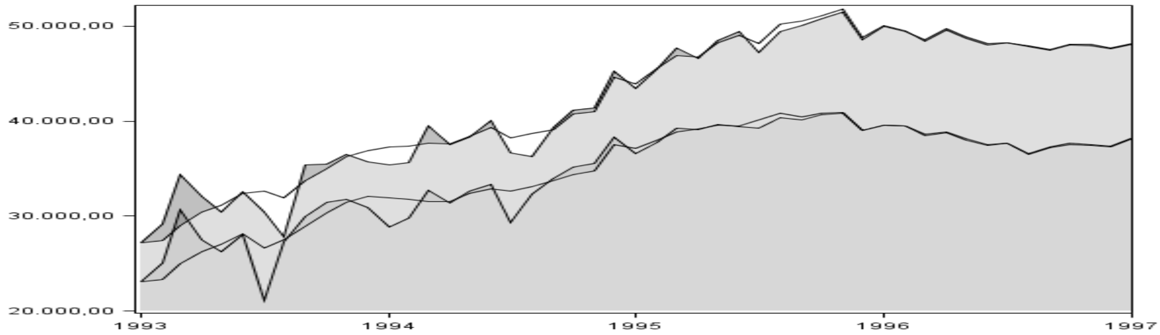
Variance priors for Model 1 were obtained by considering the series during 12 months prior to start in January 1993, i.e. entire 1992. Variance degrees of freedom was set to $n=12$. The trend was assumed stable so the standard errors of the trend in exports and imports were set to 500 units (each unit being 1 Million SEK) and 250 units for the net trade trend. The trend evolution is considered more volatile, thus set to three times the trend standard error; 1 500 units for the two trade series and 750 for the net trade series. Seasonal fluctuations can be a major part of the volatility so standard errors for seasonal component were set to 5 000 units. Diagnostics are found in Table 6.3.

Table 6.3 Model 1 Informative variance prior for imports, exports and net trade. $\delta = 0.85$. MSE and MASE of forecast MAPE. Revision errors MARE and MARE-D.

Series	Delta	MSE	MAPE	MASE	MARE	MARE-D
Imports	0.85	96 939	4.68	0.54	0.07	0.60
Exports	0.85	151 263	4.77	0.53	0.07	0.64
Net trade	0.85	55 683	19.58	0.74	0.30	2.51

Seen in Figure 6.2, informative variance priors stabilize adjustments in the startup and convergence between informative and uninformative priors is observed after some 40 observations (i.e. 3-4 observations each season). The system variance showed a decrease of 20-25 % and overall results indicate that back-casting, or two sequential runs (as with the simulated series), is necessary for precision in the initial period.

Figure 6.2 Informative and non-informative priors in Model 1 for exports and imports. First 50 observations. The upper lines are the seasonal adjustments for exports and the lower lines are the seasonal adjustment for imports obtained from informative/uninformative priors. The smoother lines are due to informative priors.



7. Conclusions

This paper indicates that a state-space approach by DLM could be elaborated to a practical alternative to TRAMO/SEATS. Lacking comparison criteria complicates proper numeric evaluations but, given the sophistication of TRAMO/SEATS, the criteria given in this paper show that this issue is worth studying. Topics that also should be addressed are outlier treatment and calendar day adjustments, which would make DLM more complete for comparisons.
The most obvious downside with the state space approach is the overachievement of eliminating seasonal fluctuations since series are shown to become too smooth. This may be due to the model specification in this study, so the topic should be investigated further with alternative specifications of the system evolution matrix. In conclusion, DLM needs more elaboration and definitely informative priors in order to be comparable with TRAMO/SEATS. As for consistency between direct and indirect seasonal adjustment, DLM proved to be more consistent between aggregations, but this result is to some extent contaminated by the excessive smoothing that DLM introduces.

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Appendix

A.1 Estimation results

Table A.1 Model 1 diagnostics for imports, exports and net trade. Discount factor Delta, MSE of prior/forecast, MAPE and MASE of forecast, revision errors MARE and MARE-D.

Series	Delta	MSE	MAPE	MASE	MARE	MARE-D
Imports	0.85	174 212	4.92	0.55	0.12	0.59
Exports	0.85	246 315	4.97	0.54	0.13	0.61
Net trade	0.85	88 183	26.66	0.74	0.43	2.27
Imports	0.90	934 175	5.29	0.59	0.21	0.43
Exports	0.90	1 345 568	5.16	0.56	0.21	0.43
Net trade	0.90	431 023	26.88	0.71	0.71	1.48
Imports	0.95	4 876 662	6.13	0.71	0.29	0.25
Exports	0.95	6 037 190	5.68	0.61	0.30	0.23
Net trade	0.95	1 617 775	27.93	0.71	0.88	0.74
Imports	0.99	22 497 135	7.62	0.92	0.35	0.08
Exports	0.99	21 595 272	6.90	0.74	0.34	0.07
Net trade	0.99	5 195 928	31.96	0.80	0.91	0.22

Table A.2 Model 2 diagnostics for imports, exports and net trade. Discount factor Delta, estimated coefficient for trend, MSE of prior/forecast, MAPE and MASE of forecast, revision errors MARE and MARE-D.

Series	Delta	Coefficients (β_1 β_2)	MSE	MAPE (%)	MASE	MARE (%)	MARE-D (%)
Imports	0.85	0.782, 0.222	19 257 450	15.29	1.40	4.88	6.20
Exports	0.85	0.767, 0.234	388 670	5.35	0.58	0.15	0.68
Net trade	0.85	0.571, 0.400	92 568	10.26	0.74	0.41	2.22
Imports	0.90	0.782, 0.222	922 099	5.19	0.59	0.24	0.46
Exports	0.90	0.767, 0.234	1 349 663	5.13	0.56	0.24	0.46
Net trade	0.90	0.571, 0.400	454 264	9.51	0.72	0.62	1.42
Imports	0.95	0.782, 0.222	4 536 495	5.94	0.69	0.34	0.26
Exports	0.95	0.767, 0.234	5 908 991	5.61	0.61	0.33	0.25
Net trade	0.95	0.570, 0.401	1 711 259	9.41	0.70	0.72	0.71
Imports	0.99	0.782, 0.222	18 864 465	7.34	0.87	0.41	0.08
Exports	0.99	0.767, 0.234	20 435 229	6.79	0.73	0.40	0.08
Net trade	0.99	0.570, 0.401	4 495 395	10.94	0.73	0.76	0.20

Note that coefficients coincided for all settings of the discount factor.

Table A.3: Estimates yielded by fully automatic procedure in Tramo-Seats for Windows

	Imports	Exports	Net trade
Model	(2,1,0) (0,1,1)	(2,1,0),(0,1,1)	(0,1,1)(0,1,1)
Transformation:	Logs	Logs	
AR coefficients	0.8657 (13)	0.9383 (15)	**
	0.5341 (8.2)	0.5722 (9.0)	
SAR coefficients	**	**	**
MA coefficients	**	**	-0.8099 (-17.)
SMA coefficients	-0.6856 (-9.6)	-0.8507 (-10.)	-0.7470 (-10.)
Decomposition			
Seasonal	ARMA (11,11)	ARMA(11,11)	ARMA(11,11)
AR coefficients	$\sum_{j=0}^{11} B^j$ (unit value)	$\sum_{j=0}^{11} B^j$ (unit value)	$\sum_{j=0}^{11} B^j$ (unit value)
MA coefficients	0.4215, 0.6069, 0.9499, 0.5036, 0.4764, 0.4410, 0.1630, 0.0281, -0.2132, -0.3521, -0.2060	0.3817, 0.6625, 0.9269, 0.4173, 0.4520, 0.3794, 0.1270 , 0.0802, -0.1566, -0.3047, -0.1433	0.6299, 0.3253, 0.0850, -0.0947, -0.2192, -0.2950, -0.3295, -0.3303, -0.3051, -0.2612, -0.2052
Trend-Cycle	IMA(2,2)	IMA(2,2)	IMA(2,2)
AR coefficients	**	**	**
MA coefficients	0.0309, -0.9691	0.0134 , -0.9866	-1.7861, 0.7907
Transitory Component	ARMA(2,2)	ARMA(2,2)	**
AR coefficients	0.8657, 0.5341	0.9383, 0.5722	**
MA coefficients	-0.5812, -0.4188	-0.5996, -0.4004	**

Numbers within parentheses are t-values for coefficients. Note that all seasonal component estimates are non-stationary with unit AR coefficients, hence the backshift polynomial.

A.2 The data generating process for simulated series

Define the season as the previous observed value and a normally distributed random component Z_1 with level k_1:

$$s_t = s_{t-p} + k_1 \times Z_{1,t}, \qquad Z_{1,t} \sim N(0,1). \tag{4.1}$$

Working with quarterly data (*P=4*) and to ensure zero-summation, the fourth quarter (*p=4*) is set to a linear combination of the preceding quarters:

$$s_t^{p=P} = -(s_{t-1}^{p=P-1} + s_{t-2}^{p=P-2} + s_{t-3}^{p=P-3}). \tag{4.2}$$

Using a linear restriction in (4.2) could be bypassed by adding a random component but that would not be necessary for our purpose. Alternatively, expression (4.1) may be written as

$$(1-\phi L^P)s_t = k_1 \times Z_{1,t}$$

where the autoregressive parameter $\phi = 1$ and L^P is the seasonal lag operator $L^P y_t = y_{t-P}$ and rather than using unit value for ϕ and making the correction in (4.2), zero summation could alternatively be achieved by a more stringent specification of ϕ, see Cleveland (2002). The trend is time-driven by a factor k_2 and a multiple k_3 of s_0^1 to control the level and additionally incorporates a uniformly distributed random component $k_4 \times U_t$ where $U_t \sim Uniform(0,1)$:

$$T_t = k_2 t + k_3 s_0^1 + k_4 U_t . \quad (4.3)$$

The irregular component is set to follow a normal distribution:

$$I_t = k_5 Z_{2,t} , \qquad Z_{2,t} \sim N(0,1) \quad (4.4)$$

The simulated series are seasonally adjusted by Model 1 and by TRAMO/SEATS with automatic model selection, presuming no calendar effects (parameter RSA=3).

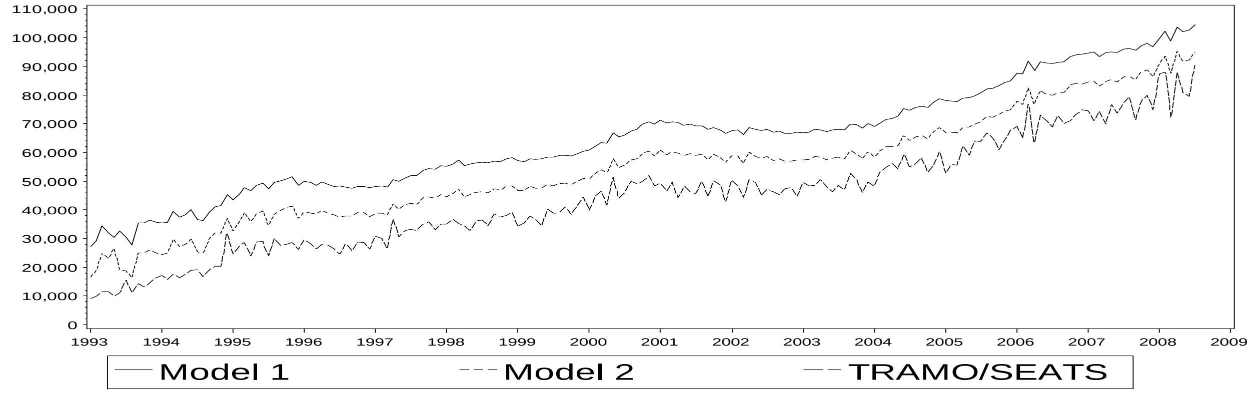
Table A.4 Specifications of simulated series. Discount set to 0.9

Series	**T**	k_1	k_2	k_3	k_4	k_5	**Initial seasons** $s_0^1, s_0^2, s_0^3, s_0^4$
1	48	k_1=1	k_2= 2	k_3=8	k_4=100	k_5=3	70,-30,-60,20
2	72	k_1=2	k_2=3 if t<37, k_2= -1 if t>36	k_3=7	k_4=70 if t<37, k_4= 50 if t>36	k_5=3	50,-30, 10,-30
3	36	k_1=3	k_2=4	k_3=3	k_4=20	k_5=3	10,-5,-8,3
4	36	k_1=1	k_2=3	k_3=3	k_4=10	k_5=3	40,-13,-16,-11
5	84	k_1=1	k_2=log(t)*1/t	k_3=6	k_4=5	k_5=3	60,-30,0,-30

Table A.4 gives the specifications of the simulated series. The first series has a slowly growing level ($k_2 = 2$) with large positive errors ($k_4 = 100$) and seasonal components with small noise ($k_1 = 1$). The fifth series has a slowly decaying trend ($k_2 = \log(t) * 1/t$, i.e. $f(x) * f'(x)$, $f(x) = \log(x)$) with rather even and distinct seasonal components.

Figure A.1 (a-b): Seasonal adjustments of imports and exports. Estimation through Model 1 (on top), Model 2 on minus 10 000 Million Swedish Kronor SEK of its actual value (in the middle) and TRAMO/SEATS on minus 20 000 Million Swedish Kronor SEK of its actual value (lowest line).

(a) Exports



(b) Imports

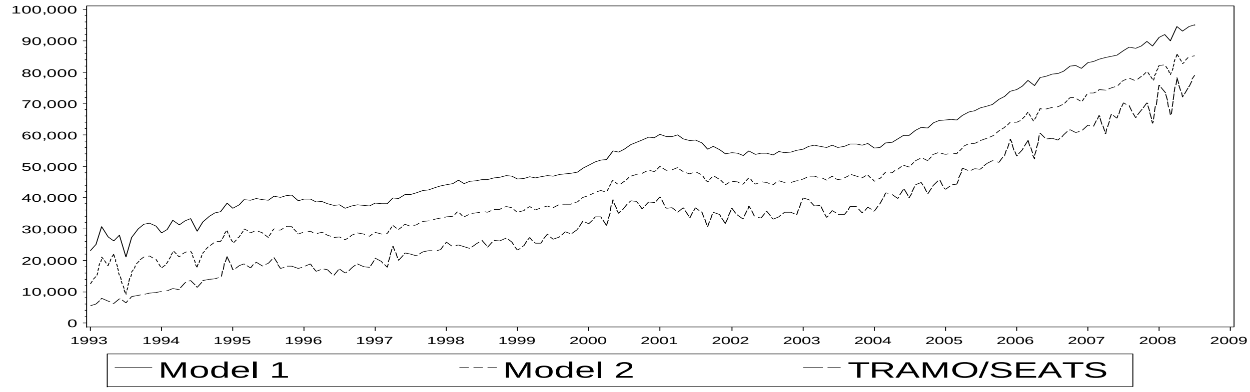
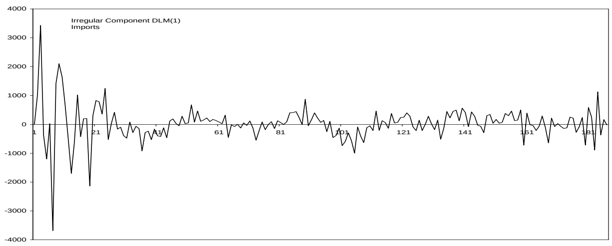
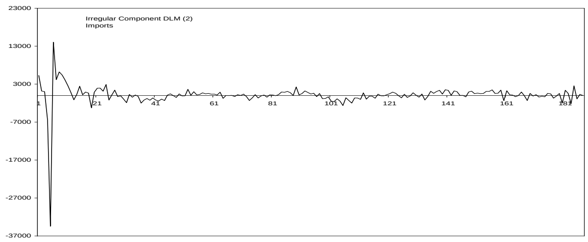


Figure A.2 (a-f): Irregular components of the two DLM models. Beware of scaling differences.

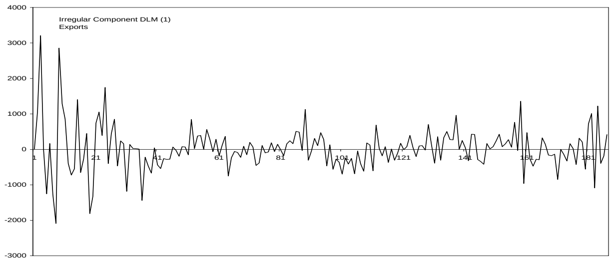
(a) Imports Model 1



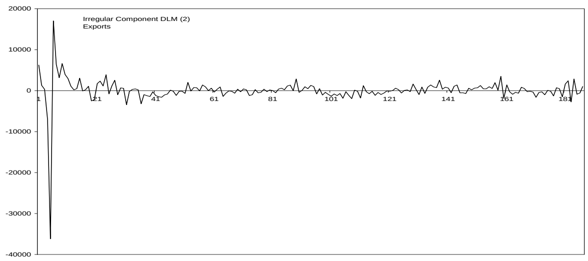
(d) Imports Model 2



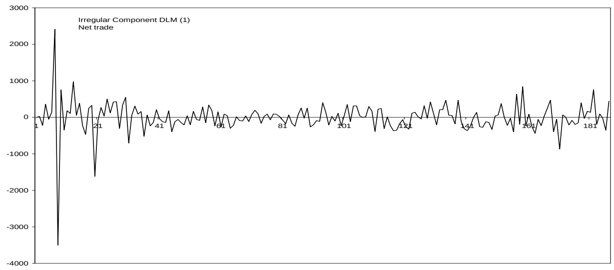
(b) Exports Model 1



(e) Exports Model 2



(c) Net trade Model 1



(f) Net trade Model 2

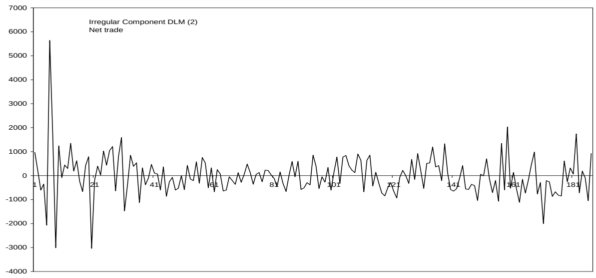
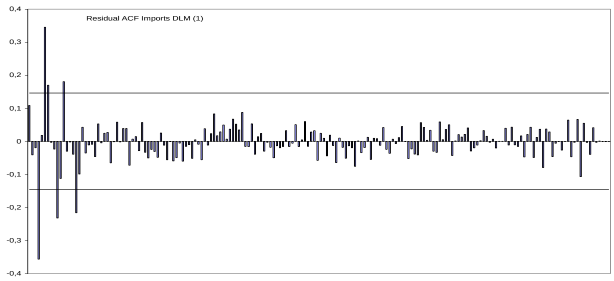
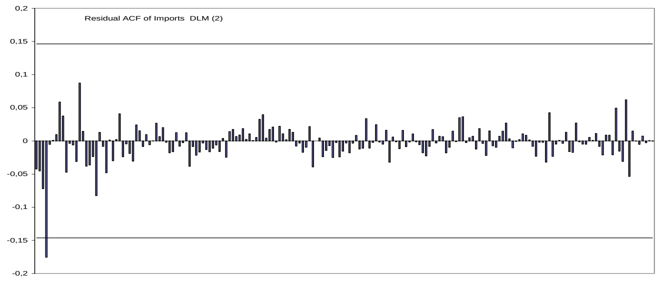


Figure A.3 (a-f): Autocorrelations of irregular components for Model 1 and Model 2 with a 95 % confidence bound.

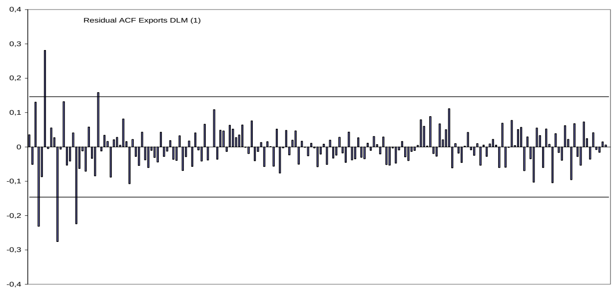
(a) Imports Model 1



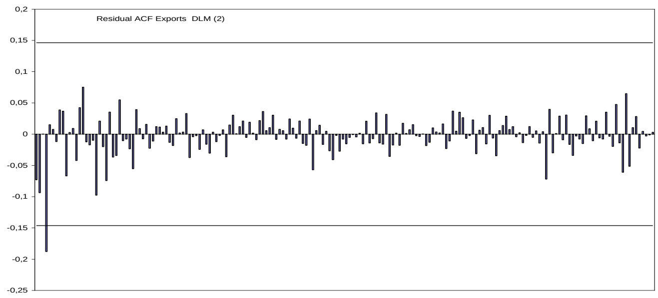
(d) Imports Model 2



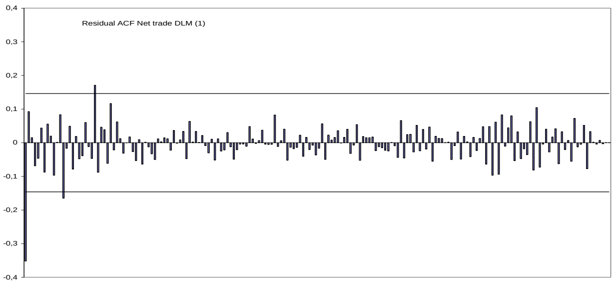
(b) Exports Model 1



(e) Exports Model 2



(c) Net trade Model 1



(f) Net trade Model 2