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Comovements of the Dow-Jones Stock Index and US GDP

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Abstract

This paper explores the connection between Dow-Jones industrial average (DJIA) stock prices and the US GDP growth. The analysis is mainly done in the frequency domain but relevant time domain results are also reported. Prior to such studies, time series are often rather mechanically detrended using i.e. the Hodrick-Prescott filter. Detrending is however a complicated task, which might lead to distortions, especially in the often neglected presence of heteroscedasticity.

Keywords: Spectral analysis, detrending filters, heteroscedasticity, the connection between stock prices and economic growth.

1. Introduction

Numerous time domain studies have described the relationship between economic variables. Some other studies have investigated the relationship between, say economic growth, and non-economic variables such as current age distribution in a country or energy consumption, see e.g. Lee (2005). In the frequency domain similar studies are also quite common. Öller (1990) used a frequency domain approach to investigate the fit and comovements of business survey data and industrial production data in Finland. The exchange rate comovements of 12 countries were studied by Orlov (2009).

National product series, such as GDP, typically contain a unit root (Granger, 1966). Trends and unit roots show up as low or infinite frequency variations in the spectral density. Standard analysis requires stationarity and hence economic time series are detrended prior to further analysis. Done properly, detrending eliminates an infinite peak at zero frequency. Given a finite time series, it is impossible to design an ideal filter, and one has to make a good approximation. Filters may distort the frequency content of the cyclical part. Simple first-differencing, for instance, amplifies the higher frequencies at the expense of lower frequencies. Moreover, in the case of short series, abrupt variations in the frequency response give rise to Gibbs´ phenomenon, see e.g. Priestley (1981, pp. 561).

The most widely used detrending filters are the ones suggested by Hodrick and Prescott (HP) (1997), Beveridge and Nelson (BN) (1981) and Baxter and King (BK) (1999). Ma and Park (2004) used the HP filter in a comovement study of the interest rates in US, Japan and Korea. The HP filter was also applied in Uebele and Ritschl (2009), prior to a comovement study of stock markets and business cycles in Germany before World War I. The BK filter has been used by i.a. Stock and Watson (1999). Most studies focus solely on business cycle frequencies, typically on the business cycle band-pass suggested by Burns and Mitchell (1946) of between 6 and 32 quarters. But much information may be extracted also outside this frequency band. This is further discussed in Section 4.

To the author´s best knowledge no detrending filter exists, which takes the highly possible event of heteroscedasticity into consideration. This is surprising because in spectral analysis contributions to the variance at specific frequencies are of prime interest. Neglecting heteroscedasticity will distort frequency domain results, see the discussion in e.g. Engle (1974). Because of this the heteroscedasticity removing filter of Öller and Stockhammar (2007) will be considered here. The univariate and comovement frequency domain results from the filter proposed in ibid. will be compared with the results from the ones that do not take heteroscedasticity into account. The detrending filters will be further discussed in Section 3.

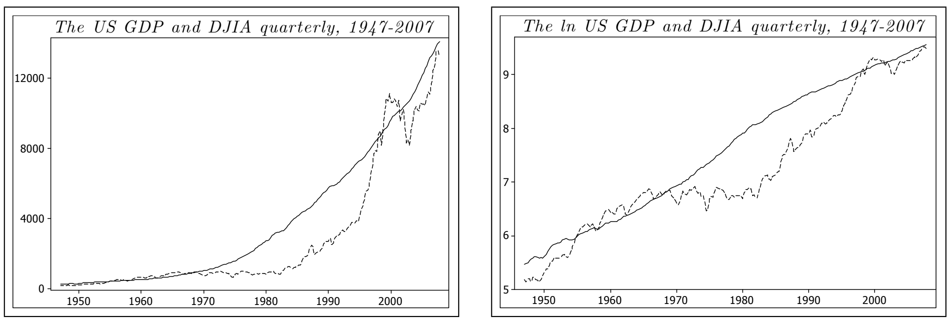
In Stockhammar and Öller (2008) it was shown that a Normal-Asymmetric Laplace (NAL) mixture distribution accurately describes the frequency distributions of US, UK and Australian GDP quarterly series. Interestingly, in Stockhammar and Öller (2010) the same distribution was found also to work well for Dow-Jones industrial average (DJIA) daily closing prices. This fact encouraged a closer look at the movements of stock indexes and GDP series. This will be pursued here using the above filters to detrend the series prior to spectral analysis of their relations. Comovements of two series are in the time domain studied using cross correlation coefficients. Frequency domain techniques allow us to study correlation differentiated by frequency (coherency), and thus to concentrate on cycles. If the two series are related we suspect the strongest coherency to occur on the business cycle frequencies, typically 6 to 32 quarters. As a check of our results, we also report the relevant time domain estimates.

This paper is organized as follows. Section 2 presents the data. In Section 3 filters are discussed used to detrend the series prior to the comovement investigation of Dow Jones Stock Index data and US GDP in Section 4. Section 5 concludes.

2. The data

Here, quarterly figures 1947-2007 (244 observations) of the seasonally adjusted DJIA and the US GDP are studied as appearing on www.finance.yahoo.com and the website of the Bureau of Economic Analysis, www.bea.gov, respectively. The DJIA series was converted to quarterly figures from daily closing prices and calender effects have been accounted for. These series together with their logarithms are presented in Figure 2.1.

Figure 2.1:.DJIA (dashed line) and US GDP (solid line), the original series (left panel), logarithmic series (right panel)



The issue of detrending the above series to enable spectral analysis of their relationships is discussed in the next section.

3. Detrending filters

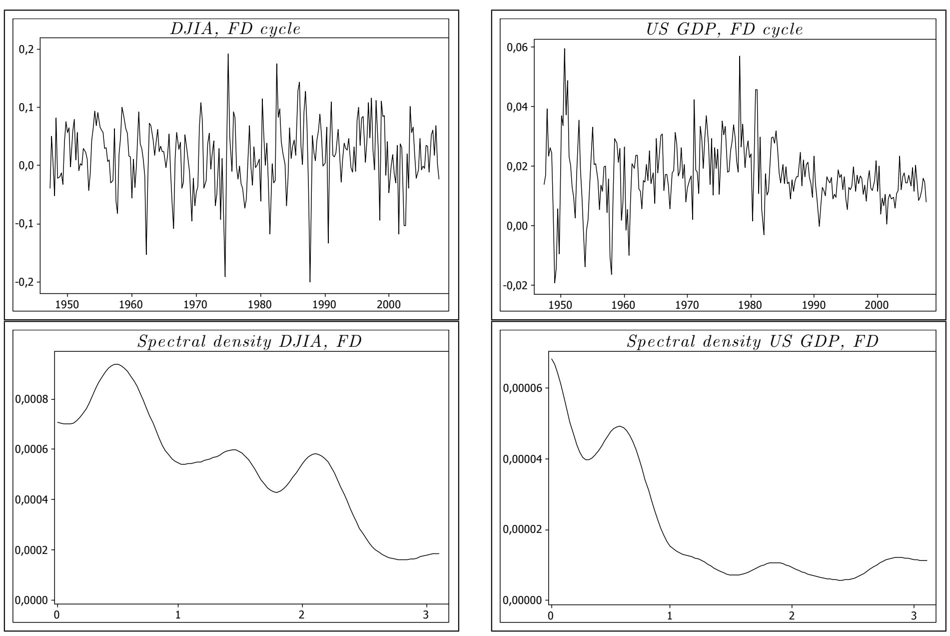
As emphasized by Granger (1966), business cycle peaks in spectral densities are often buried in the massive share of low frequency (trend) variations. Moreover, as noted already by Burns and Mitchell (1946), the business cycle often take the shape of an inverted U followed by a V. These characteristics cannot be described using the harmonic spectral functions in the frequency domain. See e.g. Jenkins and Watts (1968) for a thorough treatment of the spectral and cross spectral functions used in this paper. Several filters have been suggested to reduce the trend domination and to isolate the business cycle. The components of a time series can be defined in at least two ways (Cogley, 2001). One is the filter-design approach and the other is the model-based approach. For completeness, members of both approaches are compared in this study. In the filter-design approaches, the trend and business cycle are defined as components passing through an ideal[1], low, high or band pass filter, whose bands are predetermined according to the assumed variation at specific frequencies.

[1] An ideal filter completely eliminates the frequencies outside the predetermined ones, while passing the remaining ones unchanged.

The approaches are typically ad hoc by nature, in the sense that the statistical properties of the business cycle are not specified. Here, in the presence of finite-length time series, it is impossible to design an ideal filter and a good approximation will have to suffice. The HP and BK filters decribed below are examples of this approach. To overcome some of the criticism mentioned below of the filter-design approach (see e.g. Harvey and Jaeger, 1993), the model-based approach has been suggested. The BN filter described in Section 3.3 is an example of the model-based approach.

The simplest way to detrend a time series y_t is to calculate the *first differences (FD)*[2], $y_t^{FD} = \Delta y_t$ where y_t^{FD} is the detrended series from the FD filter, $\Delta = (1 - B)$ where B is the backshift operator such as $B^k y_t = y_{t-k}$. The FD of the logarithmic (Diff ln) DJIA and US GDP are shown in Figure 3.1 (upper panel) together with their spectral densities (lower panel).

Figure 3.1: The Diff ln series (upper panel) and the corresponding spectral densities (lower panel)

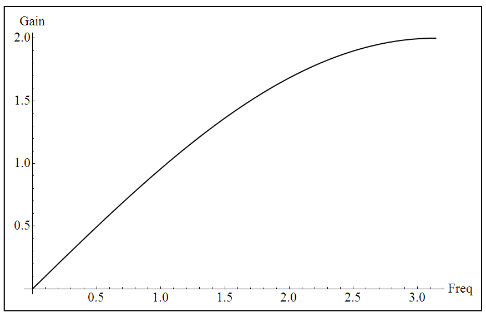


Heteroscedasticity is evident in the upper panels in Figure 3.1, especially in the filtered US GDP. There are some drawbacks using first differences. First, it is not a symmetric filter. This is however of no consequence when applying cross-spectral functions on two FD detrended series. As indicated by the lower panels

[2] Another way to get rid of unit roots in bivariate studies is to model in error correction form, given that the root is present in both series.

in Figure 3.1, the densities of the FD series are still dominated by low frequency variations. This is because the true integrating order of the series is somewhere between one and one and a half, see the discussion in Öller and Stockhammar (2007), ÖS(2007) hereafter. Candelon and Gil-Alana (2004) concluded that the US GDP series is integrated of order $I(1.4)$. That is, an additional fractional difference of order 0.4 will eliminate the spectral density domination at low frequencies. Also, the FD filter reweights the densities towards higher frequencies as indicated by Figure 3.2.

Figure 3.2: The gain function of the FD filter[3]



Despite the drawbacks with FDs they have been used in comovement studies like this one, e.g. by Wilson and Okunev (1999), and recently by Orlov (2009). Also, Knif et. al (1995) used the FD filter prior to cross-spectral analysis of the Finnish and Swedish stock markets.

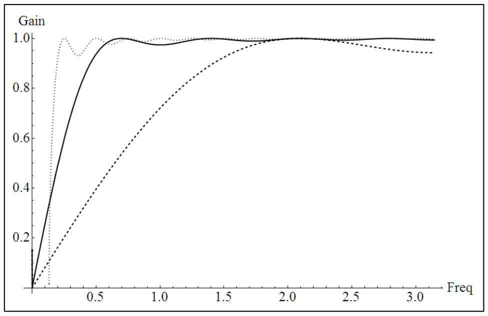
Calculating deviations from *centered moving averages (MA)* generated as

$$y_t^{c,MA} = y_t - \frac{1}{2p+1} \sum_{i=-p}^{p} y_{t+i},$$

where $k = 2p+1$ is the window length and $y_t^{c,MA}$ is the cyclical component calculated from the MA filter, is another method of detrending time series sometimes used in practice. Figure 3.3 shows the gain of various values for k.

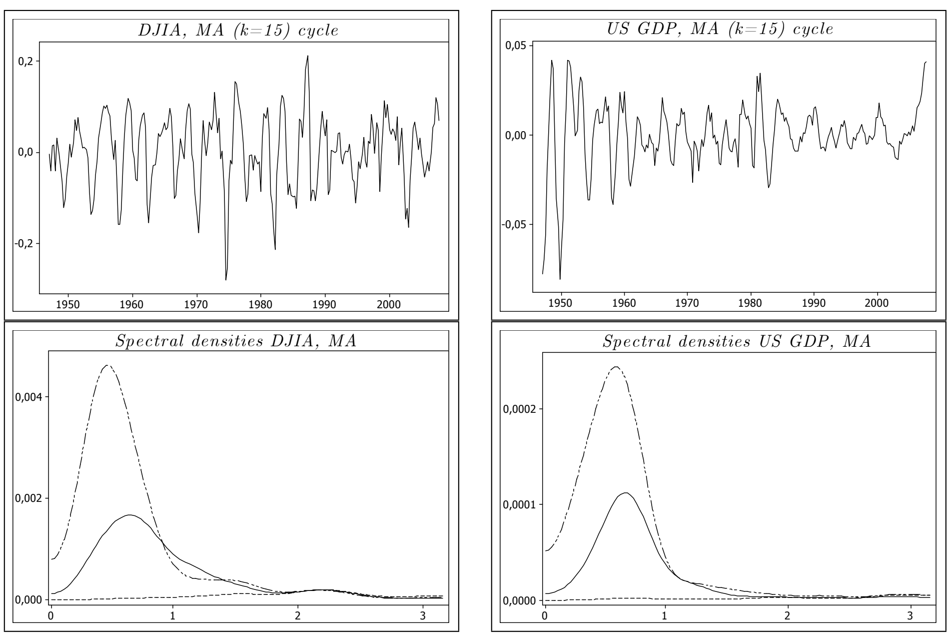
[3] The gain function of the first difference filter is $G(w) = \sqrt{2(1-\cos w)}$, where w is the frequency.

Figure 3.3: The gain functions of $y_t^{c,MA}$ *using* $k = 3$ *(dashed line),* $k = 9$ *(solid line) and* $k = 15$ *(dotted line)*[4]



The MA is a symmetric filter. Applying the MA($k = 15$) filter on the DJIA and US GDP series yields the filtered series in Figure 3.4. The corresponding spectral density and the spectral densities of the MA($k = 3$) and the MA($k = 9$) filter are also included (lower panel).

Figure 3.4: The MA($k = 15$*) detrended DJIA and US GDP (upper panel). The lower panel shows the spectral densities of the cyclical components calculated from the MA(*$k = 3$*) (dashed line), MA(*$k = 9$*) (solid line) and MA(*$k = 15$*) (dashed/dotted line)*



[4] The gain function of the moving average filter that estimates the cycle is $G(w, k) = \sqrt{1 - \frac{1 - \cos kw}{k^2(1 - \cos w)}}$.

Comparing with the FD filter, the MA filter does a better job removing the trend in the series. As the window lengths get wider, the spectral peaks are shifted to the left. Both the FD and MA filter produce series found to be stationary using the Phillips-Perron (PP) or the augmented Dickey-Fuller (ADF) tests. But they are also significantly heteroscedastic according to the ARCH-LM test.

The most widely used detrending filters are described in subsections 3.1-3.3.[5]

3.1 The Hodrick-Prescott filter

Perhaps the most commonly used filter to detrend economic time series is the one suggested by Hodrick and Prescott (1997). The HP-filter was designed to decompose a macroeconomic time series into a nonstationary trend component and a stationary cyclical component. Given a non-seasonal time series y_t, the decomposition into unobserved components is

$$y_t = g_t + c_t,$$

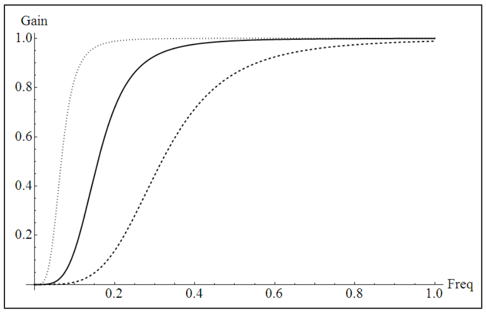
where g_t denotes the unobserved trend component at time t, and c_t the unobserved cyclical component at time t. Estimates of the trend and cyclical components are obtained as the solution to the following minimization problem

$$\min_{[g_t]_{t=1}^{N}} \left\{ \sum_{t=1}^{N} c_t^2 + \lambda \sum_{t=3}^{N} (\triangle^2 g_t)^2 \right\}, \qquad (3.1)$$

where $\triangle g_t = g_t - g_{t-1}$ and $g_{\min}$ is the HP-filter and the cyclical component is calculated as: $y_t^{c,HP} = y_t - g_t$. The first sum of (3.1) accounts for the accuracy of the estimation, while the second sum represents the smoothness of the trend. The positive smoothing parameter λ controls the weight between the two components. As λ increases, the HP trend becomes smoother and vice versa. Note that the second sum, $(\triangle^2 g_t)$, is an approximation to the second derivative of g at time t. The HP-filter is symmetric and can eliminate up to four unit roots in the data, see e.g. Cogley and Nason (1995). For quarterly data (the frequency used in most business-cycle studies) there seems to be a consensus in employing the value $\lambda = 1600$. The gain of deviations from the HP trend using various values on λ is presented in Figure 3.5.

[5] The filter proposed by Christiano and Fitzgerald (2003) is here omitted. The asymmetric and time-varying features of this filter generate phase shifts, and nothing can be said about the stationarity of the output (even if the input is stationary).

Figure 3.5: The gain of the HP filter using $\lambda = 100$ (dashed line), $\lambda = 1\ 600$ (solid line) and $\lambda = 50\ 000$ (dotted line)[6]

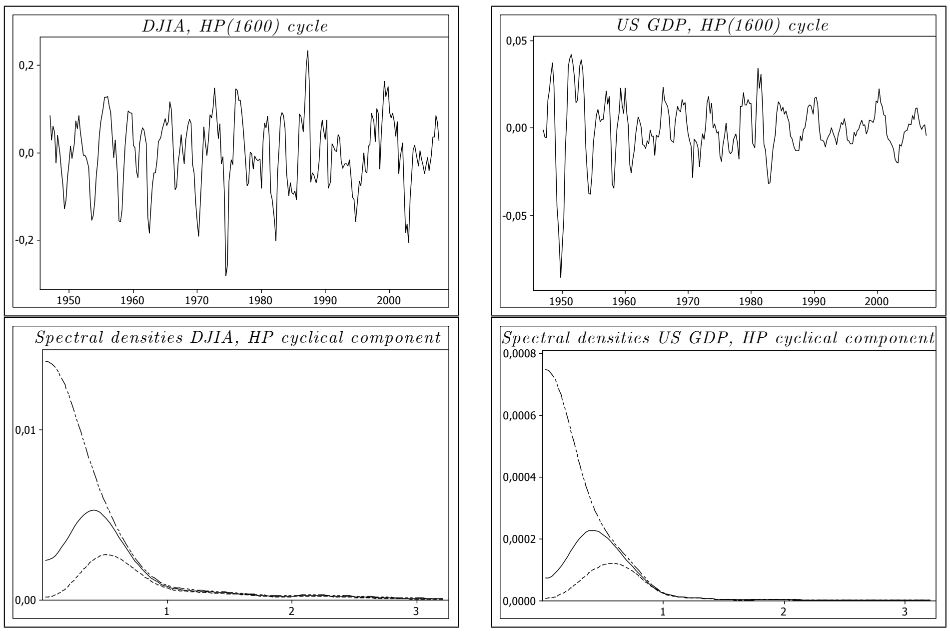


As with first differences and deviations from moving averages, the HP-filter places zero weight at zero frequency. King and Rebelo (1993) criticized the HP-filter and provided examples of how it alters measures of persistence, variability, and comovement when it is applied to observed time series. In addition, Harvey and Jaeger (1993) and Cogley and Nason (1995) showed that the HP filter induces spurious cycles when applied to the level of a random walk process. Criticism are also found in Maravall (1995) and Canova (1998). Because of this, Kaiser and Maravall (1999) provided a computationally convenient modification of the HP filter by including two model based features.

The upper panel in Figure 3.6 shows the HP(1 600) detrended series and the corresponding spectral density (lower panel). The spectral densities from the HP(100) and HP(50 000) filters are also included.

[6] The gain function of the HP filter that estimates the cycle is $G(w, \lambda) = 1 - \frac{1}{1+4\lambda(1-\cos w)^2}$.

*Figure 3.6: The HP(*1 600*) detrended DJIA and US GDP (upper panel). The lower panel shows the spectral densities of the cyclical components calculated from the HP filter using* $\lambda = 100$ *(dashed line),* $\lambda = 1\ 600$ *(solid line) and* $\lambda = 50\ 000$ *(dashed/dotted line)*



As shown in Figure 3.6, detrending using the HP($\lambda = 50\ 000$) failed. That is, much of the trend remains in the HP($\lambda = 50\ 000$) cyclical component. Using $\lambda = 100$, the cyclical components have almost no density at zero frequency and peaks at 12 quarters both for DJIA and US GDP. Using the standard value for quarterly data $\lambda = 1\ 600$, the spectral densities are larger at zero frequency with the effect that the peaks are shifted towards lower frequencies and peaks at 16.5 quarters both for DJIA and US GDP (equivalent of frequency $w = 0.38$ in the above cross-spectral formulas). All series were found to be stationary and heteroscedastic.

3.2 The Baxter-King filter

The above filters are all approximations of an ideal high-pass filter, which would remove *only* the lowest frequencies from the data. The ideal band-pass filter removes *both* low and high frequencies, passing the intervening frequencies. Baxter and King (1999) proposed a moving average type approximation of the business cycle band defined by Burns and Mitchell (1946). That is, the BK filter is designed to pass through time series components with frequencies between 6 and

32 quarters, while dampening higher and lower frequencies. This is done using a symmetric finite odd-order $k = 2p + 1$ moving average. The cyclical component using the BK-filter takes the form

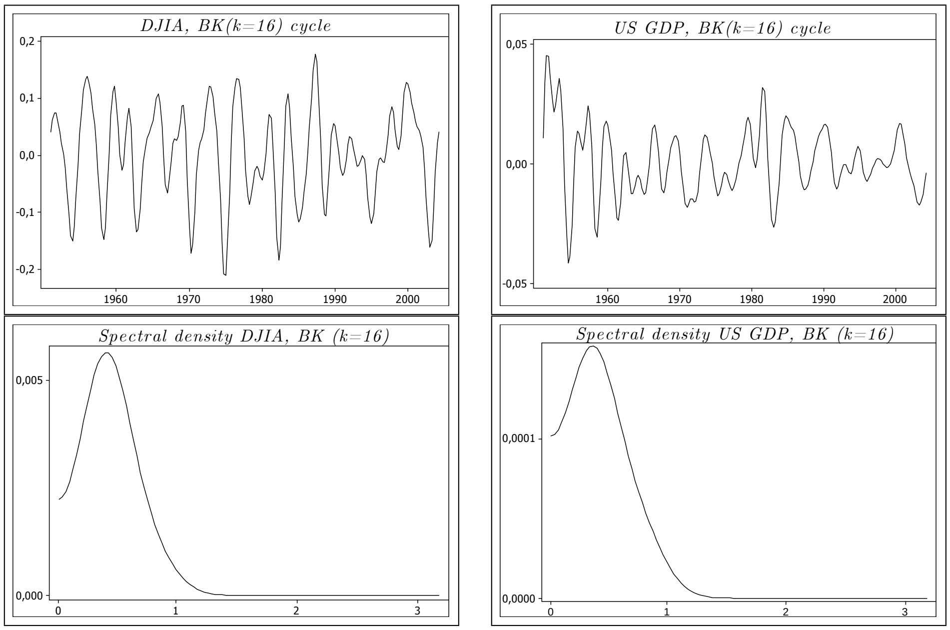
$$y_t^{c,BK} = \sum_{i=-p}^{p} h_i B^i y_t, \qquad (3.2)$$

where $y_t^{c,BK}$ is the BK filtered series and h_i are the filter weights obtained by solving the optimization problem

$$\min_{h_i} Q = \int_{-\pi}^{\pi} |\delta(w)|^2 \, dw, \qquad (3.3)$$

where $\delta(w) = \beta(w) - \alpha(w)$ is the error arising from approximating the Fourier transform of the ideal filter, $\beta(w)$, by an approximation of the same, $\alpha(w)$. For the solutions of (3.3), see Baxter and King (1999). Since the BK-filter is symmetric it does not induce phase shifts. Also the filter is designed to produce stationary output. The BK filter has the ability to remove up to two unit roots. Ibid suggested a value of $p = 12$ for the frequency band 6 to 32 quarters, and argued that the filtering is basically equivalent for larger values of p. Iacobucci and Noullez (2005) showed that the size of p beyond $p = 12$ matters, and suggested the value $p \geq 12$ irrespective of the sample size and the band to be extracted. This might cause response leakage due to the truncation. By adding a constant to the ideal filter coefficients in (3.3), a discontinuity at the endpoints results, which makes the leakage at high frequencies worse. Note that, like all moving average smoothers, p observations will be lost at the beginning and at the end of the filtered series. Figure 3.7 shows the cycle and spectral density of the BK ($k = 16$) filtered series (using bandpass 6 to 32 quarters).

Figure 3.7: The cycle and spectral densities of the cyclical components calculated from the BK filter using $k = 16$.



The shape of the spectral densities using $k = 12$, 24 and 36 are very similar. As the window length becomes larger the peak is shifted slightly to the right. The spectral densities of the cyclical component from the BK filter is very similar in shape to the HP filter.

The standard frequency band of 6 to 32 quarters used to extract business cycles seems to work well when applied to individual series. Section 5 will reveal that this choice of band greatly influences the shape of the coherency and phase functions, especially at frequencies shorter than 6 quarters. We argue that high frequency comovements are important encouraging us to extend the BK frequency band to between 2 to 32 quarters. This does not change the frequency domain properties of the individual filtered series much, see Table 3.1. The two BK filter alternatives are hereafter denoted $\text{BK}^{(6,32)}(k)$ and $\text{BK}^{(2,32)}(k)$. The spectral densities of the BK filter using frequency band 1 to 32, showed peaks at zero frequency for $k = 24$ and $k = 36$, and will therefore not be included in this study.

3.3 The Beveridge-Nelson filter

The model-based decomposition suggested by Beveridge and Nelson (1981) is based on Wold's representation theorem and separates a time series into a permanent (P) and transitory (T) component. A shock at time t results in a permanent change to the series if it affects the permanent component, while the effect of the shock will damp down over time if it affects the transitory component. The series, y_t is thus decomposed as follows:

$$y_t = P_t + T_t.$$

It is further assumed that y_t is an $ARIMA(p, 1, q)$ process (and thus $\Delta y_t = \Delta P_t + \Delta T_t$). The first difference, Δy_t, of an $ARIMA(p, 1, q)$ process can be expressed as an infinite order moving average process

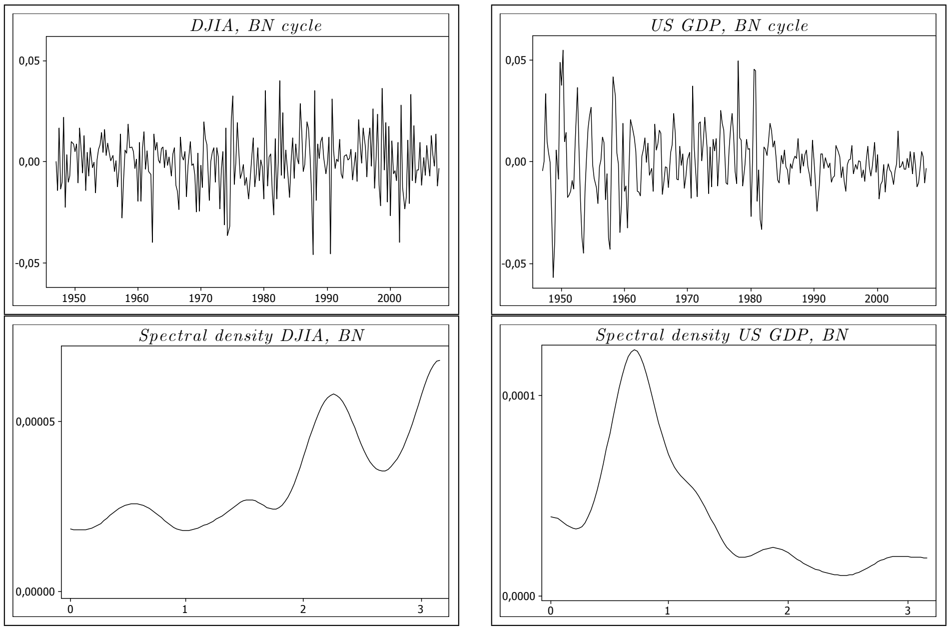
$$\begin{aligned} \Delta y_t &= c(B)a_t \\ &= c_0 a_t + \psi(B)(1 - B)a_t, \end{aligned}$$

where $\psi(B) = \psi_0 + \psi_1(B) + ...$, is an infinite order polynomial. The permanent and transitory components are identified as

$$\begin{aligned} \Delta P_t &= c_0 a_t \\ \Delta T_t &= \psi(B)(1 - B)a_t, \end{aligned}$$

Note that $T_t = \psi(B)a_t$, thus P_t is a process of integrating order one, $I(1)$, whereas T_t is $I(0)$. That is, one difference is required to make P_t stationary wheras T_t is stationary by definition. There are basically two ways to estimate the BN components (Morley, 2007). Here the approach suggested by Beveridge and Nelson (1981) is used. Figure 3.8 shows the cyclical series and the corresponding spectral densities.

Figure 3.8: Cyclical components calculated from the BN filter (upper panel) and the corresponding spectral densities (lower panel)[7]



Note the high-pass properties of the BN filter for the DJIA, for which the filter eliminates most of the low frequency variation, including the business cycles. The first hump represents the business cycle, almost overshadowed by the high frequency variations. The spectral densities of the US GDP series is quite similar in shape to the ones in Figures 3.6 and 3.7. As before, heteroscedasticity is revealed in the upper panel of Figure 3.8, as noted before, especially in the US GDP series.

3.4 A trend and heteroscedasticity removing filter

None of the above filters accounts for the highly possible event of heteroscedasticity in economic and financial series. Despite first order stationarity in the detrended series, the null hypothesis of homoscedasticity is rejected for every one of them. This is a major drawback when applying a frequency domain approach (as in this study), see e.g. Engle (1974). Because of this, ÖS(2007) proposed the following detrending and heteroscedasticity removing filter

[7] The models with the smallest AIC among the adequate ones were an ARIMA(1,1,1) and ARIMA(2,1,2), for DJIA and US GDP, respectively.

$$\widetilde{z}_t = s_y \left[\frac{\left(z_t^{(i)}\right)^d}{HP^{(\gamma)}\left(\sqrt{\sum_{\tau=t-\nu}^{t+\nu}\left(z_\tau^{(i)}\right)^{2d}/2\nu}\right)} \right] + \overline{y}, \qquad (3.4)$$

where $t = \max[k-\eta, l-\nu], \max[k-\eta+1, l-\nu+1], ..., k$ and l (both odd) is the window length in the numerator and denominator, respectively. $\eta = (k-1)/2$, $\nu = (l-1)/2$, $\widetilde{z}_t$ is the filtered series and $i = a, b$ from the detrending operations

$$(a) \qquad z_t^{(a)} = \Delta y_t - \sum_{\tau=t-\eta}^{t+\eta} \Delta y_\tau /k \ , \qquad t = \eta+1, \eta+2, ..., n-\eta \qquad (3.5a)$$

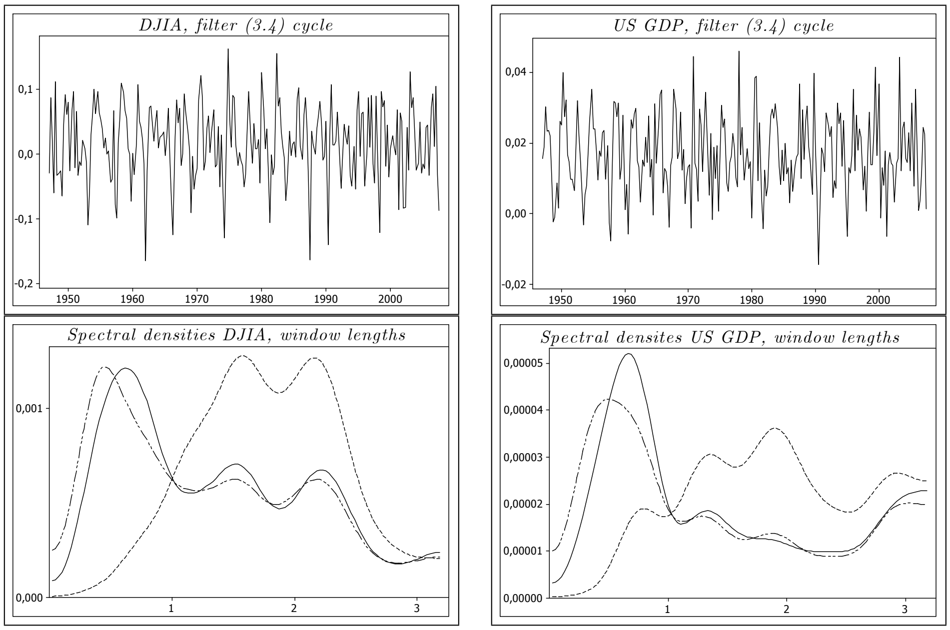
and with y_τ delayed one period:

$$(b) \qquad z_t^{(b)} = \Delta y_t - \sum_{\tau=t-\eta}^{t+\eta} \Delta y_{\tau-1} /k \ , \qquad t = \eta+2, \eta+3, ..., n-\eta+1 \qquad (3.5b)$$

where $\Delta y_t = y_t - y_{t-1}$, y_t is the logaritmic series at time t, k (odd) is the window length, and $\eta = (k-1)/2$ and even. This transformation is generalized by raising z_t to the power d, $\left(z_t^{(i)}\right)^d$. Using different values on η in (3.5), different degrees of integration are achieved. There are two extremes. For $\eta = (n-1)/2$, the term $\sum_{\tau=t-\eta}^{t+\eta} \Delta y_\tau /k$ equals $\overline{\Delta y}$ assuming that the original series is $I(1)$ centered at zero. The other extreme appears when k equals one, that is $\eta = 0$. Operation (3.5b) is used only in the latter case and is equivalent to the second difference operation, $\Delta^2 y_t$. The choice of η depends on the series studied. If it is close to $I(1)$ then you should just choose η close to $(n-1)/2$, and if the series is close to $I(2)$ then choose $\eta = 0$ in (b) or a small value on η in (a).

Filter (3.4) was designed to remove heteroscedasticity in time series data in a simple, yet efficient way. As proposed by Hodrick and Prescott (1997), $\lambda = 1600$ is suitable for quarterly data, ÖS (2007) further suggest the use of $k = 15$ in (3.4). The upper panel in Figure 3.9 shows the filtered series using this filter. The spectral density functions of the two time series filtered by (3.4) (using $\lambda = 1\ 600$ and different window lengths) are presented in the lower panel.

Figure 3.9: The detrended and heteroscedasticity corrected filtered series (upper panel). The lower panel shows the spectral densities using filter (3.4) with $\lambda =$ 1 600 and $k = 5$ (dashed line), $k = 15$ (solid line) and $k = 25$ (dashed/dotted line)



Both filtered series are found to be stationary according to ADF and PP tests, and (contrary to all other filters discussed in this section) homoscedastic according to the ARCH-LM test. The window length proposed in ÖS(2007), $k = 15$, results in a spectral peak at 10.4 quarters both for DJIA and US GDP. Increasing the window length to $k = 25$ shift the peaks close to 14 quarters. The operations in (3.5) contain first differencing implying a small phase shift, but everything else is symmetrical.

Table 3.1 summarizes the main spectral properties of the above filters. The peak frequencies are measured in quarters.

Table 3.1: Spectral density peaks

	DJIA		US GDP	
	Peak freq. (quarters)	value	Peak freq. (quarters)	value
FD	12.8	0.0009	10.6	0.00005
MA($k = 3$)	7.9*	0.0002	9.0*	0.00001
MA($k = 9$)	9.8	0.0017	9.8	0.00011
MA($k = 15$)	13.5	0.0046	11.1	0.00024
HP($\lambda = 100$)	12.0	0.0025	12.0	0.00013
HP($\lambda = 1\ 600$)	16.5	0.0052	16.5	0.00023
HP($\lambda = 50\ 000$)	–	–	–	–
$\text{BK}^{(6,32)}(12)$, $\text{BK}^{(2,32)}(12)$	16.9 15.7	0.0054 0.0054	18.3 18.3	0.0002 0.0002
$\text{BK}^{(6,32)}(16)$, $\text{BK}^{(2,32)}(16)$	16.5 16.3	0.0056 0.0055	17.8 17.7	0.0002 0.0002
$\text{BK}^{(6,32)}(24)$, $\text{BK}^{(2,32)}(24)$	14.9 15.1	0.0048 0.0049	16.2 15.1	0.0001 0.0001
$\text{BK}^{(6,32)}(36)$, $\text{BK}^{(2,32)}(36)$	14.5 14.3	0.0045 0.0044	15.8 15.6	0.0001 0.0001
BN	11.6*	0.0000	9.0	0.00012
(3.4),($k = 5$,$\lambda = 1\ 600$)	4.2*	0.0012	7.6*	0.00002
(3.4),($k = 15$,$\lambda = 1\ 600$)	10.4	0.0011	10.4	0.00005
(3.4),($k = 25$,$\lambda = 1\ 600$)	14.1	0.0011	14.1	0.00004
Mean	**13.2**		**13.6**	

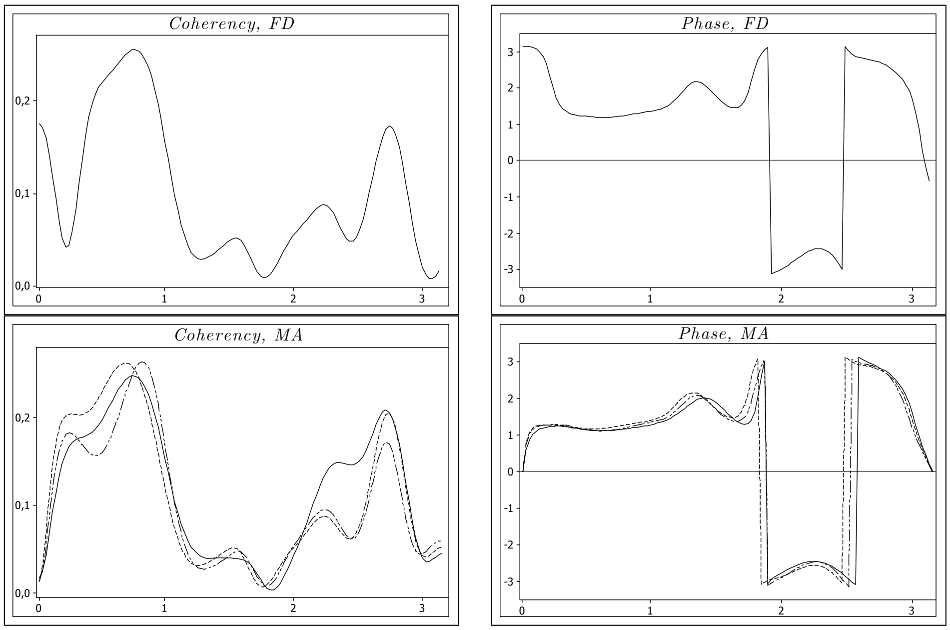
*The high frequency peaks are assumed to be spurious, c.f. Figures 3.4, 3.8 and 3.9.

As summarized in Table 3.1, the length of the business cycle depends on the choice of detrending filter. The spectral peaks for MA($k = 9$), HP($\lambda = 100$), HP($\lambda = 1\ 600$) and filter (3.4) (using $\lambda = 1\ 600$, and $k = 15, 25$), are located at the same frequency. That is a favourable feature improving the estimates of cross-spectral densities. The BK filter seems rather robust to different window lengths, but with peak frequencies between 14 and 18 quarters. This accords well with the HP($\lambda = 1\ 600$) filter. Accounting for heteroscedasticity using filter (3.4) and the suggested window length $k = 15$, slightly shortens the cycle.

4. Comovements between the two series

The choice of detrending filter not only affects the shape of the spectral densities, but also the cross-spectral functions. The cross-spectral differences of the detrending procedures is the issue of this section. Figure 4.1 shows the coherency and phase spectrum between the cyclical components of DJIA and US GDP using the FD filter and the MA filter. In creating all subsequent phase spectrums, the DJIA series have been put before US GDP.

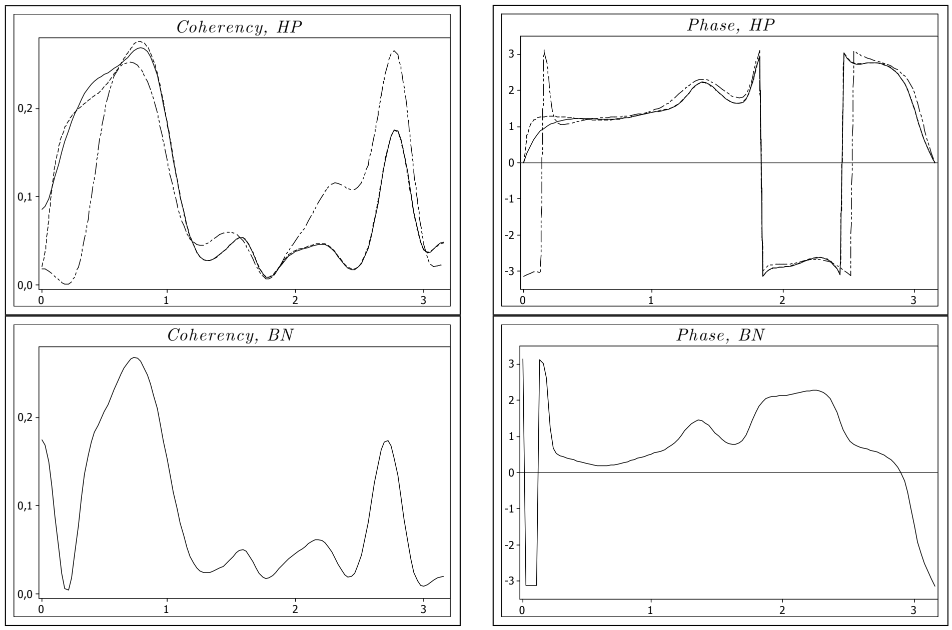
Figure 4.1: Cross-spectral densities of the cyclical components of DJIA and US GDP using the FD filter (upper panel) and the MA(k = 3) (dashed line), MA(k = 9) (solid line) and MA(k = 15) (dashed/dotted line) (lower panel).



The coherency function using the FD filter in Figure 4.1 (upper panel) peaks ($K_{1,2}^2(w = 0.72) = 0.26$) at 8.7 quarters, which means that the relationship between the two series is closest at a frequency of just over two years. The spectra of the two individual series, $f_1(w)$ and $f_1(w)$, peak at $10 - 12$ quarters, essentially the frequency to focus on in the coherency plots. At these frequencies, the approximate coherency is $K_{1,2}^2(w) = 0.23$. Extending the window length in the MA filter shifts the coherency peaks to the right.

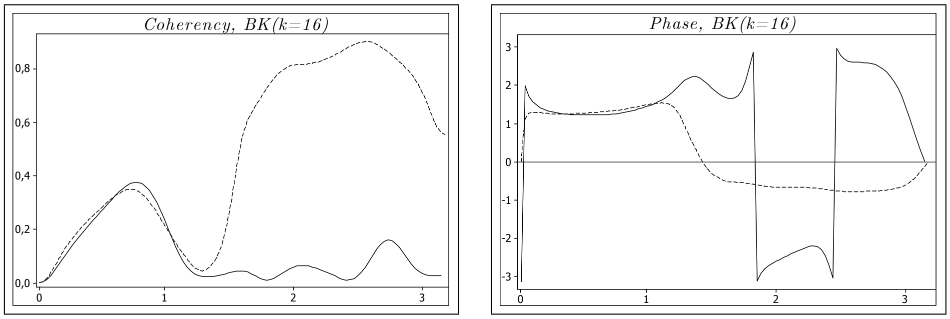
The frequency to focus on in the phase spectrum is the peak coherency frequency. In most cases in this study this corresponds to a relatively linear (positively) part of the phase spectrum, see e.g. Figure 4.1. The slope is estimated using linear regression on the frequency of interest and four observations on each side. Most filtered series also indicates rather high coherency at high frequencies (around $2 - 2.5$ quarters). At this frequency, the trend in the phase spectrum is typically negative and is again estimated using a linear regression on nine observations surrounding the (high frequency) peak coherency frequency. Put in practice, the phase spectrum for the FD filtered series indicates that DJIA leads US GDP on the business cycle frequencies by on the average of 0.45 quarters. It also shows signs of feedback at high frequencies, where US GDP leads DJIA by on average 1.34 quarters. Figure 4.2 shows the same measures using the HP and BN filters.

Figure 4.2: Cross-spectral densities of the cyclical components of DJIA and US GDP using the HP filter with $\lambda = 100$ (dashed line), $\lambda = 1\ 600$ (solid line) and $\lambda = 50\ 000$ (dashed/dotted line) (upper panel) and using the BN filter (lower panel)



In Figure 4.2 (upper panel), the coherency function using the HP(100) and HP(1600) peaks at 8.1 quarters. Increasing λ to 50 000 slightly shifts the peak to the left. Using HP(1 600), the spectra of the two individual series, $f_1(w)$ and $f_2(w)$, peak at around 16.5 quarters, for which the coherency is $K^2_{1,2}(w = 0.38) = 0.23$. The coherency function using the BN filter is similar, but note that its phase spectrum does not have the typical discontinuities around $w = 1.8$ and $w = 2.5$. Also, the HP(50 000) and the BN filters both have a low frequency discontinuities in the phase spectrum. The phase function is defined as the arctan of the ratio between the quadrature and the co-spectrum resulting as discontinuities at frequency multiples of $\frac{\pi}{2}$, see e.g. Jenkins and Watts (1968) for details. In Figure 4.3 the coherency and phase spectrum for the BK$^{(2,32)}(k = 16)$ and BK$^{(6,32)}(k = 16)$ filtered series are presented. The densities using other values on k show very similar patterns.

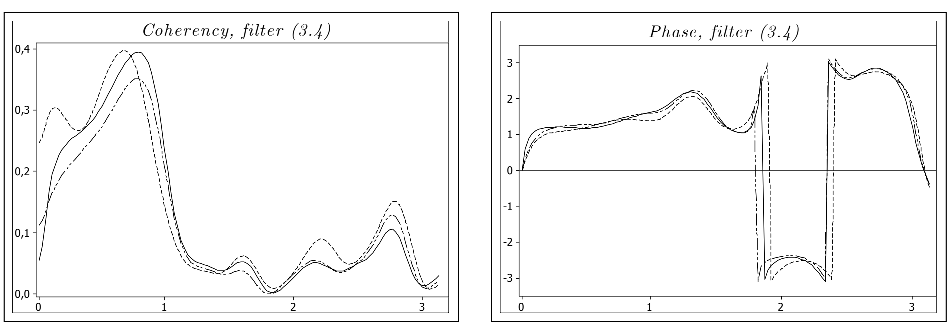
Figure 4.3: Cross-spectral densities of DJIA and US GDP using the $BK^{(2,32)}$(k=16) (solid line) and *$BK^{(6,32)}$(k=16) (dashed line) filter*



The choice of frequency band to extract in the BK filter has a large effect on the coherency and phase. The BK standard frequency band, $6-32$ quarters, by definition completely miss the information that can be extracted at high frequencies. This results in a peculiar shape of the coherency function which peaks far from the business cycle. As with the BN filter, the phase spectrum does not have the typical discontinuity at high frequencies. On the contrary, the cross-spectral densities of the $\mathrm{BK}^{(2,32)}(k=16)$ filtered series are similar to the filtered series discussed above and below.

The cross-spectral densities between the two series detrended by filter (3.4) with $\lambda = 1\ 600$ and $k = 5$, 15 and 25 are shown in Figure 4.4.

Figure 4.4: Cross-spectral densities of DJIA and US GDP using filter (3.4) with $\lambda = 1\ 600$ and $k = 5$ (dashed line), $k = 15$ (solid line) and $k = 25$ (dashed/dotted line)



The coherency function for the (3.4) filtered series (using $\lambda = 1\ 600$ and $k = 15$) peaks on the average at 8.2 quarters at which $K^2_{1,2}(0.76) = 0.40$. At this frequency, $\phi_{1,2}(0.76) = 1.20$ quarters, with feedback $\phi_{1,2}(2.77) = -1.48$. At the peak spectral frequencies of 10.4 quarters (see Table 3.1), $K^2_{1,2}(0.60) = 0.35$. The results are summarized in Table 4.1.

Table 4.1: Cross-spectral density peaks

	Coherency			Phase	
	Peak freq.(q)	Value	Value at peak spectral freq.	At peak coh. freq.(q)	At high freq.(q)
FD	8.7	0.26	0.23	0.45	-1.34
MA($k=3$)	9.4	0.26	0.25	0.49	-1.59
MA($k=9$)	8.7	0.25	0.24	0.33	-1.84
MA($k=15$)	7.8	0.26	0.17	0.73	-1.51
HP($\lambda=100$)	8.1	0.28	0.23	0.65	-1.45
HP($\lambda=1\ 600$)	8.1	0.27	0.23	0.51	-1.40
HP($\lambda=50\ 000$)	9.0	0.25	–	0.26	-1.39
BK$^{(6,32)}$(12)	8.5*	0.29	0.21	0.35	-0.23
BK$^{(6,32)}$(16)	8.9*	0.35	0.23	0.41	-0.10
BK$^{(6,32)}$(24)	8.1*	0.39	0.24	0.24	0.14
BK$^{(6,32)}$(36)	7.6*	0.35	0.17	0.09	0.13
BK$^{(2,32)}$(12)	8.5	0.28	0.22	0.16	2.54
BK$^{(2,32)}$(16)	8.2	0.38	0.21	0.51	-1.02
BK$^{(2,32)}$(24)	8.5	0.36	0.26	-0.18	-0.44
BK$^{(2,32)}$(36)	7.8	0.35	0.17	-0.23	-1.45
BN	8.7	0.27	0.24	0.67	-1.49
(3.4),($k=5$,$\lambda=1\ 600$)	9.3	0.40	0.10	0.73	-1.35
(3.4),($k=15$,$\lambda=1\ 600$)	8.2	0.40	0.35	1.20	-1.48
(3.4),($k=25$,$\lambda=1\ 600$)	8.1	0.35	0.26	0.99	-1.02
Mean	**8.4**	**0.32**	**0.22**	**0.44**	**-0.86**

*The high frequency peaks are assumed to be spurious, c.f. Figure 4.3.

The column to the far right shows the phase shift at the rightmost coherency peak frequency, see Figures 4.1-4.4. The coherency seems quite robust to different filters. Its peak frequencies varies from 7.6 to 9.4 quarters, with an average of 8.4 quarters. The choice of BK$^{(6,32)}(k)$ or BK$^{(2,32)}(k)$ does not seem to matter much, but this is true only for the Burns and Mitchell (6 to 32 quarters) business cycle frequencies. Due to the extended high frequency band, the BK$^{(2,32)}(k)$ has the ability to also describe variations at shorter frequencies, see Figure 4.3. It is therefore in comovement studies advisable to use this filter (if the series are homoscedastic). The phase at peak coherency frequency is less robust with values varying from -0.23 to 1.20 quarters (average 0.44). All filters (with the exeptions of the BK$^{(2,32)}$(24) and BK$^{(2,32)}$(36) filters) reports that DJIA leads USGDP at peak coherency frequency. Both BK filters show scattered phase. Accounting for heteroscedasticity using filter (3.4) shows coherency peaks approximately on average frequency, with larger than average coherency values. Also, the homoscedastic series induce the longest lead shifts.

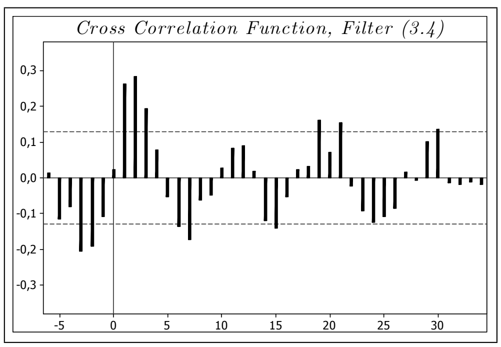
It is possible from cross-spectral analysis to detect lead or lag of the series under a common cyclical period. The phase densities show both the lead times and reveal the direction of the comovements. As a double check of the results, the time domain Granger causality test was performed on the (3.4) filtered and stationary series. The results are presented in table 4.2.

Table 4.2: Granger-causality tests, p-values

\Lag	1	2	3	4	5	6	7	8
DJIA doesn´t Granger-cause USGDP	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
USGDP doesn´t Granger-cause DJIA	0.08	0.02	0.00	0.12	0.02	0.14	0.36	0.45

Table 4.2 shows that the null hypotheses of Granger-noncausality is rejected for all lags from DJIA to US GDP, using the 0.05 significance level. At lags 2, 3 and 5, there seems to exist a feedback - US GDP leads DJIA. This further confirms the phase densities in Figures 4.1-4.3 and is also supported by the cross correlation function between the (3.4) filtered series:

Figure 4.5: The cross correlations between DJIA and US GDP



where the dashed lines denotes ± 2 standard errors for the estimates. Table 4.3 presents the cross correlations (at lag 1) between DJIA and US GDP for the entire period, and for three subperiods (standard errors in paranthesis and significant correlations in bold figures). The subperiods were chosen in accordance with US GDP volatility, where 1947-1960, 1961-1983 and 1984-2007 are periods denoted as high, medium and low volatility, respectively. This is also in compliance with Stock and Watson (2003) who reported that the US GDP variance declined over 50 per cent from 1960-1983 to 1984-2002 when averaged over four quarters. The decline in volatility was even larger in e.g. Italy and Japan, and also widespread across sectors within the US. As indicated by ÖS (2007, 2008), the volatility in the US GDP was even larger before 1960. Note that the decreasing volatility does not apply to DJIA, cf. Figure 3.1.

Table 4.3: First order cross correlations between DJIA and US GDP

	Full sample	1947-1960	1961-1983	1984-2007
FD	**0.173** (0.064)	**0.534** (0.135)	0.167 (0.104)	0.125 (0.102)
MA($k = 3$)	**0.151** (0.064)	**0.448** (0.134)	0.061 (0.104)	0.060 (0.102)
MA($k = 9$)	**0.335** (0.064)	**0.559** (0.134)	**0.339** (0.104)	0.139 (0.102)
MA($k = 15$)	**0.336** (0.064)	**0.574** (0.134)	**0.294** (0.104)	0.152 (0.102)
HP($\lambda = 100$)	**0.338** (0.064)	**0.642** (0.134)	**0.294** (0.104)	0.074 (0.102)
HP($\lambda = 1\ 600$)	**0.329** (0.064)	**0.573** (0.134)	**0.250** (0.104)	0.196 (0.102)
HP($\lambda = 50\ 000$)	0.092 (0.064)	**0.309** (0.134)	−0.190 (0.104)	0.112 (0.102)
$\mathrm{BK}^{(6,32)}(12)$, $\mathrm{BK}^{(2,32)}(12)$	**0.285** (0.067) **0.275** (0.067)	**0.514** (0.151) **0.537** (0.151)	**0.225** (0.104) **0.242** (0.104)	0.083 (0.109) 0.048 (0.109)
$\mathrm{BK}^{(6,32)}(16)$, $\mathrm{BK}^{(2,32)}(16)$	**0.309** (0.069) **0.288** (0.069)	**0.621** (0.158) **0.597** (0.158)	**0.231** (0.104) **0.208** (0.104)	0.039 (0.112) 0.066 (0.112)
$\mathrm{BK}^{(6,32)}(24)$, $\mathrm{BK}^{(2,32)}(24)$	**0.223** (0.071) **0.218** (0.071)	**0.606** (0.177) **0.578** (0.177)	0.193 (0.104) 0.150 (0.104)	**−0.321** (0.118) **−0.265** (0.118)
$\mathrm{BK}^{(6,32)}(36)$, $\mathrm{BK}^{(2,32)}(36)$	0.131 (0.076) 0.133 (0.076)	0.335 (0.224) **0.750** (0.224)	**0.368** (0.104) 0.018 (0.104)	**0.484** (0.129) **−0.427** (0.129)
BN	**0.170** (0.064)	0.192 (0.134)	**0.318** (0.104)	0.080 (0.104)
(3.4),($k = 5$,$\lambda = 1\ 600$)	**0.168** (0.064)	0.139 (0.134)	0.044 (0.104)	0.066 (0.102)
(3.4),($k = 15$,$\lambda = 1\ 600$)	**0.263** (0.064)	**0.275** (0.134)	**0.222** (0.104)	0.090 (0.102)
(3.4),($k = 25$,$\lambda = 1\ 600$)	**0.236** (0.064)	0.210 (0.134)	0.166 (0.104)	0.077 (0.102)

Table 4.3 further corroborates that the detrending method used has effect on estimated cross correlations. This is expected due to the one-to-one relationship betweeen cross covariances and spectral densities, see Section 3. In addition, Table 4.3 shows that the cross correlation decresases with US GDP volatility, but less so using filter (3.4). The choice of $\mathrm{BK}^{(6,32)}(k)$ or $\mathrm{BK}^{(2,32)}(k)$ has little effect on correlations exept when $k = 36$. Odd looking negative correlation is also present in the mid volatilty part using the HP(50 000) filter. The BN filter shows for the same period its the maximum cross correlation. This could be due to the cross correlation between opposite cyclical phases in the two series (see the feedback discussion above).

5. Conclusions

This paper reveals frequency domain relationships between the Dow-Jones industrial average stock prices and US GDP growth. Both series are heteroscedastic, making standard detrending procedures, such as Hodrick-Prescott or Baxter-King, inadequate. Neglecting heteroscedasticity distorts frequency domain results and render inefficient estimation of the spectral densities. Surprisingly, many frequency domain studies do not take notice of this and mechanically use standard detrending filters. Prior to the comovement study, the univariate and comovement frequency domain results from these filters are compared to the results from the heteroscedasticity removing filter suggested by ÖS (2007). Thus, the effect of the often neglected heteroscedasticity is measured.

Accounting for the heteroscedasticity somewhat shortens the business cycles.

No matter which filter is used, significant comovements exist between the DJIA and US GDP series. The coherency seems quite robust to the different filters. Accounting for heteroscedasticity slightly shifts the coherency peak to the left and with larger than average coherency values. The phase shift is less robust, especially for the BK filtered series. Most filters report that DJIA leads US GDP at peak coherency frequency ($7.6 - 9.4$ quarters), but also reveal a feedback from US GDP to DJIA at around $2 - 2.5$ quarters. The filtered series using the suggested heteroscedasticity removing filter induce the longest lead shifts (1.2 quarters) at peak coherency frequency, and also above average feedback lag (1.48 quarters). Using the Baxter-King filter with frequency band 6 to 32 quarters (as first suggested by Burns and Mitchell, 1946) by definition completely misses this information. The same apply to the Beveridge-Nelson filter. It is therefore advisable to extend the frequency bands to 2 to 32 quarters in comovement studies like this (under the condition that the series are homoscedastic). The frequency domain results were also confirmed in the time domain using cross correlations and Granger-causality tests. When applied on subperiods in accordance with US GDP volatility, most filtered series showed scattered first order cross correlations, but less so in the homoscedastic series.

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