Estimating interviewer variance under a measurement error model for continuous survey data

Peter Lundquist[1] and Jan H. Wretman[2]

Abstract

A simple measurement error model is introduced for continuous data collected by interviewers. The model makes a clear distinction between three different sources of randomness, namely, sample selection, interviewer assignment, and interviewing. The concept of interviewer variance is defined in the context of this measurement error model, and the problem of estimating the interviewer variance is considered, assuming simple random sampling. A simulation study indicates that estimates of the interviewer variance are unstable, especially when the interviewer variance is small (in which case the effect of interviewer variability on the main survey results may still be severe).

Key words: Response variance; survey nonsampling error; interviewer effects; interpenetration.

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[1] Statistics Sweden, S-10451 Stockholm and Department of Statistics, Stockholm University, S-10691 Stockholm.

[2] Department of Statistics, Stockholm University, S-10691 Stockholm.

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1. Introduction

Sample survey data is usually more or less affected by measurement errors. This paper deals with measurement errors in surveys of individuals or households, and it is assumed that data is collected by interviewers, for example, via telephone. Our interest will be focused on the particular part of the measurement error that is due to the behavior of the interviewers. The problem we consider is how to estimate the *interviewer variance*, a quantity which in some meaning measure the amount of variability ascribable to the interviewers.

To be able to discuss the statistical aspects of measurement errors, we need a statistical model describing how measurement errors arise. Two main types of measurement error models for sample survey data are found in the literature. One is the analysis-of-variance (ANOVA) type of model used by Kish (1962) and further developed by Hartley and Rao (1978) and others. The other type of model is the Census Bureau Model, introduced by Hansen, Hurwitz and Bershad (1961) and extended by Fellegi (1964, 1974) and others. In the present paper we will use an ANOVA type of model, and our terminology will closely follow Wolter (1985), and Särndal, Swensson and Wretman (1992).

This paper is of an introductory character. We make the simplifying assumptions that the sampling design is simple random sampling without replacement, and that there is no nonresponse. In other reports to follow, we will make less restrictive assumptions.

The main purpose of this paper is to introduce an estimator of the interviewer variance. In Section 2, a simple measurement error model is specified, and the concept of interviewer variance is introduced. Basic assumptions about the interviewer assignment are also made. In Section 3, we look at the problem of estimating a population mean under the assumed measurement error model. In Section 4, we discuss how to estimate the variance of an estimator of the population mean. In Section 5, we suggest an estimator of the interviewer variance. In Section 6, the results of a simulation study are presented. The paper concludes with a discussion in Section 7.

2. Sampling design, interviewer assignment, and measurement error model

We will describe how a sample of elements is selected from the population, how the sampled elements are assigned to interviewers, and how measurement errors arise. We start by introducing some notation. We consider a finite population U with N elements labeled $k = 1, 2, \ldots, N$; let $U = \{1, \ldots, k, \ldots, N\}$. Let μ_k be the unknown true value for element k with respect to the actual study variable. The purpose of the survey is to estimate the true population mean

$$\bar{\mu} = \frac{1}{N} \sum\nolimits_U \mu_k$$

(For short, $\sum_A$ will be used for $\sum_{k \in A}$, where $A \subseteq U$ is any subset of U.)

Let s be a sample (that is, a subset of U) consisting of n elements drawn from U by *simple random sampling without replacement*. Ideally, we would like to observe the true value μ_k for each element $k \in s$, but what we will really observe is a value y_k affected by measurement error, that is,

$$y_k = \mu_k + d_k = \text{true value} + \text{measurement error}$$

The problem now is to estimate $\bar{\mu}$ using observed data y_k for $k \in s$. After we have described the interviewer assignment, we will specify a model for the measurement errors d_k, $k \in s$.

We assume that there is a set of I interviewers available for the survey. Sampled elements are assigned to these interviewers in the following way: Each interviewer is given a randomly chosen subset of elements from the sample s, under the restriction that the subsets should be nonoverlapping and of equal size.

The following notation will be used. Let the interviewers be labeled $i = 1, 2, \ldots, I$. Let the sample s be partitioned at random into I nonoverlapping groups of equal size $m = n/I$. (We assume that m is an integer.) These groups are denoted $s_1, \ldots, s_i, \ldots,$

s_I. Now, the rule is that interviewer i is to make all the interviews in group s_i, $i = 1, 2, \ldots, I$.

The groups $s_1, s_2, \ldots, s_I$ could be called a set of "interpenetrating subsamples" from the population U, since each s_i is a simple random sample from the same population U. (The subsamples are not independent, however, because of the restriction that they should be non-overlapping.) As for the concept of interpenetration, reference is given to Mahalanobis (1946) and Bailar (1983). Following the terminology of Wolter (1985) groups $s_1, s_2, \ldots, s_I$ could be called a set of "dependent random groups".

We now introduce the measurement error model, denoted M. The model is specified conditionally on a given sample s and given interviewer assignments $s_1, s_2, \ldots, s_I$. Following Biemer and Trewin (1997) we assume that the measurement error is the sum of two components, an "interviewer error" due to the interviewer, and a "response error" which depends on the respondent (and possibly other remaining sources of error). Thus, the measurement error model says that when element $k \in s_i$ is interviewed by interviewer i, the observed value y_k can be written as

$$y_k = \mu_k + b_i + \varepsilon_k$$

where

- μ_k is the "true value", assumed to be an unknown constant associated with respondent k.

- b_i is the *interviewer error*, or *interviewer effect*, ascribed to interviewer i. It is assumed to be a random variable with expected value B_b and variance σ_b^2, the same for all interviewers i. By definition, the interviewer effect b_i is the same for all interviews made by the same interviewer i, in the same survey. The variance σ_b^2 will be called the *interviewer variance.*

- ε_k is the *response error*, ascribed to respondent k. The response error ε_k is assumed to be a random variable with expected value B_ε and variance σ_ε^2, assumed to be the same for all $k \in s$.

- All the $I+n$ random variables $b_1, \ldots, b_I$, and ε_k, $k \in s$, in the measurement model M are assumed to be independent of each other.

In the present set-up, the survey is thus viewed as a three-stage process, where randomness is involved in each stage:

Stage 1: A sample s is drawn from the population U.
Stage 2: The sample s is partitioned into subsamples $s_1, s_2, \ldots, s_I$.
Stage 3: Observed values y_k are obtained for $k \in s_i$, $i = 1, 2, \ldots, I$.

The randomness in the first stage comes from the sampling design, denoted p, which in the actual case means simple random sampling without replacement of n elements. The randomness in the second stage comes from the random division of the sample into subsets assigned to the interviewers. The randomness in the third stage comes from the measurement error model M just described. Sometimes it will be found convenient to consider the first two stages jointly as constituting one coherent stage, which will then be denoted p^*.

In what follows, estimators will usually be judged by their bias and variance with respect to the joint distribution induced by the three stages above, which will be called the p^*M-distribution. It will sometimes be found convenient to express expected values and variances using conditional probabilities in the following way:

$$E_{p^*M}(\ .\ ) = E_{p^*}[E_M(\ .\ \mid s; s_1, \ldots, s_I)]$$

and

$$Var_{p^*M}(\ .\ ) = E_{p^*}[Var_M(\ .\ \mid s; s_1, \ldots, s_I)] + Var_{p^*}[E_M(\ .\ \mid s; s_1, \ldots, s_I)]$$

where $E_{p^*M}(\ .\ )$ denotes expectation with respect to the stochastic mechanisms in stage 1, 2, and 3 simultaneously, $E_{p^*}(\ .\ )$ denotes expectation with respect to stage 1 and 2 only, and $E_M(\ .\ \mid s; s_1, \ldots, s_I)$ denotes conditional expectation with respect to stage 3, given the outcome of stage 1 and 2. Analogous principles of notation hold for the variances. Note, this set-up implies that the order of E_{p^*} and E_M are not interchangeable. In the rest of this paper we will, for the sake of simplicity, write $E_M(\ .\ )$ instead of the longer and more exact expression $E_M(\ .\ \mid s; s_1, \ldots, s_I)$. Thus, in what follows,

$$E_M(\ .\ ) = E_M(\ .\ \mid s; s_1, \ldots, s_I)$$

The assumptions of the measurement error model can now be expressed formally.

Model assumptions:

For a given sample s and given subsamples $s_1, s_2, \ldots, s_I$,

$$y_k = \mu_k + b_i + \varepsilon_k \qquad \text{for } k \in s_i,\ i = 1, \ldots, I$$

$$E_M(b_i) = B_b \quad \text{and} \quad Var_M(b_i) = \sigma_b^2 \qquad \text{for } i = 1, \ldots, I$$

$$E_M(\varepsilon_k) = B_\varepsilon \quad \text{and} \quad Var_M(\varepsilon_k) = \sigma_\varepsilon^2 \qquad \text{for } k \in s_i,\ i = 1, \ldots, I$$

$b_1, b_2, \ldots, b_I, \varepsilon_k$ $(k \in s)$ are independent random variables

The following result follows immediately from the model assumptions.

Result 2.1: Under Model M, it holds that

$$E_M(y_k) = \mu_k + B_b + B_\varepsilon \qquad \text{for } k \in s_i,\ i = 1, \ldots, I$$

$$Var_M(y_k) = \sigma_b^2 + \sigma_\varepsilon^2 \qquad \text{for } k \in s_i,\ i = 1, \ldots, I$$

$$Cov_M(y_k, y_l) = \begin{cases} \sigma_b^2 & \text{for } k \neq l,\ k \in s_i,\ l \in s_i,\ i = 1, \ldots, I \\ 0 & \text{for } k \neq l,\ k \in s_i,\ l \in s_j,\ i = 1, \ldots I,\ j = 1, \ldots, I,\ i \neq j \end{cases}$$

Thus, conditionally on the sample s and on the interviewer assignments $s_1, s_2, \ldots, s_I$, observed values for different elements obtained by different interviewers are uncorrelated, while values for different elements obtained by the same interviewer are correlated. The block diagonal covariance structure is illustrated in the Appendix.

It follows from the structure of the covariances that two respondents interviewed by the same interviewer have a constant *model correlation*, denoted ρ_M, of the y-values, namely,

$$\rho_M = \rho_M(y_k, y_l) = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_\varepsilon^2}$$

Note that this is a conditional correlation, for a given sample s and given subsamples s_1, s_2, …, s_I. For two respondents interviewed by different interviewers, the model correlation is zero. Another measure of how two observations made by the same interviewer are both influenced by the same interviewer effect, is the *intra-interviewer correlation*, ρ_W, which is defined as

$$\rho_W = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_\varepsilon^2 + S_\mu^2}$$

The intra-interviewer correlation may be interpreted as the correlation of the measurements made on two elements, which are drawn at random from the population and then interviewed by the same interviewer.

3. Estimating the population mean

We will now discuss the problem of estimating the population mean, when measurement errors are present. The sampling design, interviewer allocation, and measurement error model are assumed to be as specified in Section 2.

The population characteristic to be estimated is the true mean

$$\bar{\mu} = \frac{1}{N}\sum_{U} \mu_k$$

If, hypothetically, we had sample data, μ_k for $k \in s$, *without* measurement errors, the true population mean would, under simple random sampling and in the absence of auxiliary information, usually be estimated by the sample mean

$$\bar{\mu}_s = \frac{1}{n}\sum_{s} \mu_k$$

The estimator that we are going to consider is the sample mean based on data *with* measurement errors, namely,

$$\bar{y}_s = \frac{1}{n}\sum_{s} y_k \qquad (3.1)$$

We will now find expressions for the expected value and the variance of the estimator (3.1). The kind of expectation and variance that we are interested in is with respect to all the three stages, introduced in Section 2, jointly.

The main result on expectation is the following:

Result 3.1: Under the sampling design, interviewer assignment, and model assumptions of Section 2,

$$E_{p^*M}(\bar{y}_s) = \bar{\mu} + B_b + B_\varepsilon \qquad (3.2)$$

Thus, the estimator is biased

$$Bias_{p^*M}(\overline{y}_s) = E_{p^*M}(\overline{y}_s) - \overline{\mu} = B_b + B_\varepsilon \tag{3.3}$$

Result 3.1 can be obtained as follows, using the tool of conditional probabilities:

$$\begin{aligned}
E_{p^*M}(\overline{y}_s) &= E_{p^*}[E_M(\frac{1}{n}\sum\nolimits_s y_k)] \\
&= E_{p^*}[E_M(\frac{1}{n}\sum_{i=1}^{I}\sum\nolimits_{s_i} y_k)] \\
&= E_{p^*}[\frac{1}{n}\sum_{i=1}^{I}\sum\nolimits_{s_i}(\mu_k + B_b + B_\varepsilon)] \\
&= E_{p^*}(\overline{\mu}_s + B_b + B_\varepsilon) \\
&= \overline{\mu} + B_b + B_\varepsilon
\end{aligned}$$

because, under simple random sampling, $\overline{\mu}_s$ would be unbiased for $\overline{\mu}$. (Note that $\overline{\mu}_s$ is not affected by the interviewer assignment due to the simple random sampling design.)

The main result on the variance of $\overline{y}_s$ is:

Result 3.2: Under the sampling design, interviewer assignment, and model assumptions of Section 2,

$$Var_{p^*M}(\overline{y}_s) = (\frac{\sigma_b^2}{I} + \frac{\sigma_\varepsilon^2}{n}) + (1 - \frac{n}{N})\frac{S_\mu^2}{n} \tag{3.4}$$

where

$$S_\mu^2 = \frac{1}{N-1}\sum\nolimits_U (\mu_k - \overline{\mu})^2$$

The variance of the estimator $\overline{y}_s$ can thus be seen as made up of two components. The first one,

$$\frac{\sigma_b^2}{I}+\frac{\sigma_\varepsilon^2}{n}$$

is a measurement error component which depends on the variability of the measurement error components, and on the number of interviewers (and the sample size). We call this component the *measurement variance.*

The second component, which we call the *sampling variance,*

$$(1-\frac{n}{N})\frac{S_\mu^2}{n}$$

is a pure sampling component which depends on the variability of the true values in the population, and on the sample size.

Result 3.2 is also obtained using conditional probabilities. We first write

$$Var_{p^*M}(\overline{y}_s)=\underbrace{E_{p^*}[Var_M(\overline{y}_s)]}_{V_1}+\underbrace{Var_{p^*}[E_M(\overline{y}_s)]}_{V_2}=V_1+V_2$$

Looking at the component V_1, we first see that

$$Var_M(\overline{y}_s)=\frac{1}{n^2}Var_M(\sum\nolimits_s y_k)$$

$$=\frac{1}{n^2}[\sum\nolimits_s Var_M(y_k)+\sum\sum\nolimits_{k,l\in s;k\neq l}Cov_M(y_k,y_l)]$$

$$=\frac{1}{n^2}[n(\sigma_b^2+\sigma_\varepsilon^2)+n(m-1)\sigma_b^2]$$

$$=\frac{\sigma_b^2}{I}+\frac{\sigma_\varepsilon^2}{n}$$

and it follows that

$$V_1 = \frac{\sigma_b^2}{I} + \frac{\sigma_\varepsilon^2}{n}$$

The second variance component, V_2, is found in a similar way. We first find that

$$E_M(\overline{y}_s) = \frac{1}{n}\sum\nolimits_s (\mu_k + B_b + B_\varepsilon) = \overline{\mu}_s + B_b + B_\varepsilon$$

and because the sampling design is simple random sampling ($\overline{\mu}_s$ is unaffected by the interviewer assignment), it follows that

$$V_2 = Var_{p^*}(\overline{\mu}_s) = (1 - \frac{n}{N})\frac{S_\mu^2}{n}$$

Remark 1. The variance (3.4) can be rewritten in different ways. One way is to write it as

$$Var_{p^*M}(\overline{y}_s) = [1 + (m-1)\rho_M]\frac{\sigma_b^2 + \sigma_\varepsilon^2}{n} + (1 - \frac{n}{N})\frac{S_\mu^2}{n}$$

where $\rho_M = \sigma_b^2 / (\sigma_b^2 + \sigma_\varepsilon^2)$ is the model correlation between two measurements made by the same interviewer, defined in Section 2. If the sampling fraction is negligible, that is, if we let $n/N = 0$, we can write the variance (3.4) as

$$Var_{p^*M}(\overline{y}_s) = [1 + (m-1)\rho_w]\frac{\sigma_b^2 + \sigma_\varepsilon^2 + S_\mu^2}{n}$$

where $\rho_w = \sigma_b^2 / (\sigma_b^2 + \sigma_\varepsilon^2 + S_\mu^2)$ is defined in Section 2 as the intra-interviewer correlation.

4. Estimating the variance of $\overline{y}_s$

Suppose that we want to estimate the variance of the sample mean, $\overline{y}_s$, which was given by equation (3.4) as

$$Var_{p*M}(\overline{y}_s) = \frac{\sigma_b^2}{I} + \frac{\sigma_\varepsilon^2}{n} + (1 - \frac{n}{N})\frac{S_\mu^2}{n}$$

We will consider two different estimators of this variance. One is the traditional estimator, given in textbooks, for the case of simple random sampling without replacement and no measurement errors. This variance estimator is

$$\hat{V} = (1 - \frac{n}{N})\frac{S_{ys}^2}{n} \tag{4.1}$$

where

$$S_{ys}^2 = \frac{1}{n-1}\sum\nolimits_s (y_k - \overline{y}_s)^2$$

The other variance estimator to be considered is based on the means of the subsamples assigned to the different interviewers:

$$\hat{V}_B = \frac{1}{I(I-1)}\sum_{i=1}^{I}(\overline{y}_{s_i} - \overline{y}_s)^2 \tag{4.2}$$

It will be found that both of these two estimators are biased. The bias that we are talking of here is with respect to sampling design, interviewer allocation design, and measurement error model jointly. From the bias point of view, however, we will prefer the second estimator, (4,2).

The main result on the bias of the variance estimator (4.1) is the following.

Result 4.1: Under the sampling design, interviewer assignment, and model assumptions of Section 2,

$$E_{p^*M}(\hat{V}) = (1-\frac{n}{N})[\frac{I-1}{I(n-1)}\sigma_b^2 + \frac{\sigma_\varepsilon^2}{n} + \frac{S_\mu^2}{n}]$$

$$= Var_{p^*M}(\overline{y}_s) - [\frac{m-1}{n-1} + \frac{m(I-1)}{N(n-1)}]\sigma_b^2 - \frac{\sigma_\varepsilon^2}{N} \qquad (4.3)$$

Although this variance estimator would be unbiased in a situation without measurement errors, it is now seen to have a negative bias when measurement errors are present. That is, it will tend to underestimate the true variance. Even when the population size, N, is large, this tendency could be disturbing.

The main result on the bias of the second variance estimator, (4.2), is:

Result 4.2: Under the sampling design, interviewer assignment, and model assumptions of Section 2,

$$E_{p^*M}(\hat{V}_B) = \frac{\sigma_b^2}{I} + \frac{\sigma_\varepsilon^2}{n} + \frac{S_\mu^2}{n} = Var_{p^*M}(\overline{y}_s) + \frac{S_\mu^2}{N} \qquad (4.4)$$

This second variance estimator has a positive bias (which would remain even in a situation without measurement errors). The bias is small, however, in most situations when N is large. Since we prefer a variance estimator with a small positive bias to one with a possibly large negative bias, the conclusion is that we prefer the variance estimator $\hat{V}_B$ to $\hat{V}$.

We will now prove Results 4.1 and 4.2. In doing so, we first prove a slightly more general result, Result 4.3 below, from which the results above will easily follow, and which will also be useful later on. We first introduce some new notation. For a fixed set of subsamples, $s_1, s_2, \ldots, s_I$, the *total sum of squares* (SST_y) of the observed y_k-values can then be thought of as a sum of squares *beween* subsamples (SSB_y), and a sum of squares *within* subsamples (SSW_y), in the same way as in analysis of variance:

$$\underbrace{\sum\nolimits_s (y_k - \overline{y}_s)^2}_{SST_y} = \underbrace{m\sum_{i=1}^{I} (\overline{y}_{s_i} - \overline{y}_s)^2}_{SSB_y} + \underbrace{\sum_{i=1}^{I} \sum\nolimits_{s_i} (y_k - \overline{y}_{s_i})^2}_{SSW_y}$$

We also define the analogous sums of squares based on the μ_k-values:

$$\underbrace{\sum\nolimits_s (\mu_k - \overline{\mu}_s)^2}_{SST_\mu} = \underbrace{m\sum_{i=1}^{I} (\overline{\mu}_{s_i} - \overline{\mu}_s)^2}_{SSB_\mu} + \underbrace{\sum_{i=1}^{I} \sum\nolimits_{s_i} (\mu_k - \overline{\mu}_{s_i})^2}_{SSW_\mu}$$

Finally, we define

$$MSB_y = \frac{SSB_y}{I-1} \qquad \text{and} \qquad MSW_y = \frac{SSW_y}{I(m-1)}$$

Result 4.3: Under the sampling design, interviewer assignment, and model assumptions of Section 2,

$$E_{p*M}\left(\frac{SST_y}{n-1}\right) = \frac{m(I-1)}{n-1}\sigma_b^2 + \sigma_\varepsilon^2 + S_\mu^2 \qquad (4.5)$$

$$E_{p*M}(MSB_y) = m\sigma_b^2 + \sigma_\varepsilon^2 + S_\mu^2 \qquad (4.6)$$

$$E_{p*M}(MSW_y) = \sigma_\varepsilon^2 + S_\mu^2 \qquad (4.7)$$

In proving this result, we will make use of the following auxiliary result (which will not be proved here).

Auxiliary Result: For jointly distributed random variables $\xi_1, \xi_2, \ldots, \xi_n$, with expected values $\mu_1, \mu_2, \ldots, \mu_n$, variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$, and covariances σ_{ij} (for $i \neq j$), it holds that

$$E\left[\sum_{i=1}^{n}(\xi_i - \bar{\xi})^2\right] = \frac{n-1}{n}\sum_{i=1}^{n}\sigma_i^2 - \frac{1}{n}\sum\sum_{i \neq j}\sigma_{ij} + \sum_{i=1}^{n}(\mu_i - \bar{\mu})^2$$

where $\bar{\xi} = (1/n)\sum_{i=1}^{n}\xi_i$ and $\bar{\mu} = (1/n)\sum_{i=1}^{n}\mu_i$.

Let us first prove (4.5). Inserting y_k for ξ_i in the Auxiliary Result and using the model assumptions from Section 2, we have

$$\begin{aligned} E_M(SST_y) &= E_M\left[\sum_s (y_k - \bar{y}_s)^2\right] \\ &= \frac{n-1}{n} n(\sigma_b^2 + \sigma_\varepsilon^2) - \frac{1}{n} I \;\; m(m-1)\sigma_b^2 + \sum_s (\mu_k + B_b + B_\varepsilon - \bar{\mu}_s - B_b - B_\varepsilon)^2 \\ &= m(I-1)\sigma_b^2 + (n-1)\sigma_\varepsilon^2 + SST_\mu \end{aligned}$$

which makes

$$E_M\left(\frac{SST_y}{n-1}\right) = \frac{m(I-1)}{n-1}\sigma_b^2 + \sigma_\varepsilon^2 + \frac{SST_\mu}{n-1}$$

and finally, since we have simple random sampling,

$$E_{p^*M}\left(\frac{SST_y}{n-1}\right) = E_{p^*}\left[E_M\left(\frac{SST_y}{n-1}\right)\right] = \frac{m(I-1)}{n-1}\sigma_b^2 + \sigma_\varepsilon^2 + S_\mu^2$$

Next we want to prove (4.7). Using the Auxiliary Result separately within each subsample we have

$$E_M(SSW_y) = E_M\left[\sum_{i=1}^{I}\sum\nolimits_{s_i}(y_k - \overline{y}_{s_i})^2\right] = \sum_{i=1}^{I} E_M\left[\sum\nolimits_{s_i}(y_k - \overline{y}_{s_i})^2\right]$$

$$= \sum_{i=1}^{I}\left[\frac{m-1}{m} m(\sigma_b^2 + \sigma_\varepsilon^2) - \frac{1}{m} m(m-1)\sigma_b^2 + \sum\nolimits_{s_i}(\mu_k + B_b + B_\varepsilon - \overline{\mu}_{s_i} - B_b - B_\varepsilon)^2\right]$$

$$= \sum_{i=1}^{I}\left[(m-1)\sigma_\varepsilon^2 + \sum\nolimits_{s_i}(\mu_k - \overline{\mu}_{s_i})^2\right]$$

$$= I(m-1)\sigma_\varepsilon^2 + SSW_\mu$$

Thus,

$$E_{p^*M}(MSW_y) = E_{p^*}\left[E_M\left(\frac{SSW_y}{I(m-1)}\right)\right] = \sigma_\varepsilon^2 + E_{p^*}\left[\frac{SSW_\mu}{I(m-1)}\right]$$

Since each subsample s_i is a simple random sample from the population of N elements,

$$E_{p^*}\left[\frac{SSW_\mu}{I(m-1)}\right] = \frac{1}{I}\sum_{i=1}^{I}\underbrace{E_{p^*}\left[\frac{1}{m-1}\sum\nolimits_{s_i}(\mu_k - \overline{\mu}_{s_i})^2\right]}_{=S_\mu^2} = S_\mu^2$$

and equation (4.7) is derived.

Equation (4,6), finally, is easily obtained from the relation

$$SSB_y = SST_y - SSW_y$$

Results 4.1 and 4.2 are now easily obtained from Result 4.3, using the fact that

$$\hat{V} = (1 - \frac{n}{N})\frac{SST_y}{n(n-1)}$$

and

$$\hat{V}_B = \frac{MSB_y}{n}$$

5. Estimating the interviewer variance and the interviewer correlation

Suppose that we want to estimate the interviewer variance σ_b^2. The estimator that we suggest is

$$\hat{\sigma}_b^2 = I(\hat{V}_B - \frac{\hat{V}_W}{n}) \tag{5.1}$$

where

$$\hat{V}_W = \frac{1}{I(m-1)} \sum_{i=1}^{I} \sum\nolimits_{s_i} (y_k - \overline{y}_{s_i})^2 = MSW_y \tag{5.2}$$

Using Result 4.3 and the fact that

$$\hat{\sigma}_b^2 = \frac{MSB_y - MSW_y}{m}$$

it is easy to see that the variance estimator (5.1) is an unbiased estimator of the interviewer variance (that is, unbiased with respect to the $p{*}M$ distribution).

Result 5.1: Under the sampling design, interviewer assignment, and model assumptions of Section 2,

$$E_{p*M}(\hat{\sigma}_b^2) = \sigma_b^2 \tag{5.3}$$

We conclude this section by giving an estimator of the intra-interviewer correlation

$$\rho_W = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_\varepsilon^2 + S_\mu^2} = \frac{\sigma_b^2}{\sigma_{tot}^2}$$

introduced in Section 2. We have already seen that a p^*M-unbiased estimator of σ_b^2 was given by (5.1). It also follows from Result 4.3 that a p^*M-unbiased estimator of σ_{tot}^2 is given by

$$\hat{\sigma}_{tot}^2 = I \;\; \hat{V}_B + \frac{m-1}{m}\hat{V}_W = \frac{MSB_y + (m-1)MSW_y}{m} \tag{5.4}$$

The intra-interviewer correlation can then be estimated by taking the ratio

$$\hat{\rho}_W = \frac{\hat{\sigma}_b^2}{\hat{\sigma}_{tot}^2} = \frac{n\hat{V}_B - \hat{V}_W}{n\hat{V}_B + (m-1)\hat{V}_W} = \frac{MSB_y - MSW_y}{MSB_y + (m-1)MSW_y} \tag{5.5}$$

This estimator will produce the same estimates as the estimator suggested by Kish (1962), also presented in Groves (1989, p.318).

6. Simulation study

It was seen in the preceding section that the interviewer variance estimator $\hat{\sigma}_b^2$, given by (5.1), is an unbiased estimator of the interviewer variance σ_b^2. The variance of this interviewer variance estimator will now be studied by means of Monte Carlo simulation. The behavior of the estimated intra-interviewer correlation will also be studied.

These results, especially how the variance of the estimators depend on the sample size n and the number of interviewers I, will be of interest when an interviewer variance study is being planned and we have to choose n and I so that the interviewer variance, or the intra-interviewer correlation, can be estimated with reasonable accuracy.

The simulation study is done in the following way.

- An artificial finite population of size $N =$ 100,000 is used. "True values", μ_k ($k =$ 1, …, 100,000), are created by generating 100,000 independent random numbers from a standard normal distribution, N(0, 1). For this finite population we found

$$\bar{\mu} = 0.00175458 \quad \text{and} \quad S_\mu^2 = 0.99674963$$

- From the finite population 5,000 samples of the same size are drawn by simple random sampling without replacement. Each sample is replaced before the next sample is drawn, so that all the samples are drawn from the same population. Two different sample sizes are used, $n =$ 2,000 and $n =$ 10,000.

- For each sample, the sampled elements are assigned at random to I fictitious interviewers, in conformity with the assumptions made in Section 2, so that every interviewer gets the same number, $m = n/I$, of respondents. Nine different values of I are used for $n =$ 10,000 and eight values for $n =$ 2,000.

- For each sample, with given I, and given interviewer assignments, interviewer effects, b_i ($i =$ 1,…, I) are obtained by generating I independent random numbers from an N(0, σ_b^2) distribution. Different values of σ_b^2 were chosen as described below.

- For each sample, response errors, ε_k ($k \in s$), are obtained by generating n independent random numbers from an N(0, σ_ε^2) distribution. Different values of σ_ε^2 were chosen as described below.

- The following three different combinations of values were used for σ_b^2 and σ_ε^2:

	$\sigma_b^2 / \sigma_\varepsilon^2$	σ_b^2	σ_ε^2
Combination 1	0.1	0.0255571	0.2555771
Combination 2	1.0	0.0207656	0.0207656
Combination 3	10.0	0.0203834	0.0020383

  These combinations of values were chosen in order to illustrate various relations between the two variances involved, while at the same time, together with the actual value of $S_\mu^2 = 0.99674963$, giving a constant intra-interviewer correlation of $\rho_w = 0{,}02$ in all the three cases. The value $\rho_w = 0{,}02$ could be considered realistic for a centralized group of telephone interviewers; see Biemer and Trewin (1997, p. 611).

- Finally, for each sample s we obtain realized values y_k ($= \mu_k + b_i + \varepsilon_k$) for all $k \in s$. Using these values, we then calculate, for each sample, $\overline{y}_s$ (3.1), $\hat{V}_B$ (4.2), $\hat{V}_W$ (5.2), $\hat{\sigma}_b^2$ (5.1), and $\hat{\rho}_w$ (5.5).

The numerical results from the simulation study are used to obtain approximations to the expected value and the variance of the estimators under consideration. Calculations will be made in accordance with the following pattern. Let t_s denote some sample quantity calculated from observed data in a sample s, and let t_{sj} be the realized value of t_s for the jth simulation sample ($j = 1, \ldots, 5{,}000$). (For example, if t_s is the sample mean $\overline{y}_s$, then $t_{sj} = \overline{y}_{sj}$ is the observed sample mean in the jth simulation sample.) We then calculate the simulation mean

$$\overline{t}_s = \frac{1}{5{,}000} \sum_{j=1}^{5{,}000} t_{sj}$$

which is the simulation estimate of the expected value $E_{p^*M}(t_s)$. We also calculate the simulation variance

$$S_{t_s}^2 = \frac{1}{5{,}000-1}\sum_{j=1}^{5{,}000}(t_{sj} - \overline{t}_s)^2$$

which is the simulation estimate of the variance $Var_{p^*M}(t_s)$.

Let us first look at the simulation results for the sample mean and the sample variance of the true values,

$$\overline{\mu}_s = \frac{1}{n}\sum\nolimits_s \mu_k \qquad \text{and} \qquad S_{\mu s}^2 = \frac{1}{n-1}\sum\nolimits_s (\mu_k - \overline{\mu}_s)^2$$

We already know from elementary sampling theory that (because the sampling design is simple random sampling without replacement):

$$E_p(\overline{\mu}_s) = \overline{\mu} = 0.175458 \times 10^{-2}$$

$$Var_p(\overline{\mu}_s) = (1-\frac{n}{N})\frac{S_\mu^2}{n} = \begin{cases} 4.88407319 \times 10^{-4} & \text{when } n = 2{,}000 \\ 0.89707467 \times 10^{-4} & \text{when } n = 10{,}000 \end{cases}$$

$$E_p(S_{\mu s}^2) = S_\mu^2 = 0.99674963$$

Since the true values are fixed constants, the measurement error model is not considered here.

The results of the simulation study are given in Table 6.1 below. The simulation estimates are seen to be rather close to the exact values of the quantities that they are supposed to approximate.

Table 6.1. Results from the simulation study on sample mean and sample variance of the true values. 5,000 repeated samples.

		Sample size	
		n=2,000	n=10,000
$E_p(\overline{\mu}_s)$	Exact value	0.175×10^{-2}	0.175×10^{-2}
	Simulation estimate	0.142×10^{-2}	0.158×10^{-2}
$Var_p(\overline{\mu}_s)$	Exact value	4.88×10^{-4}	0.897×10^{-4}
	Simulation estimate	4.95×10^{-4}	0.861×10^{-4}
$E_p(S_{\mu s}^2)$	Exact value	99.7×10^{-2}	99.7×10^{-2}
	Simulation estimate	99.7×10^{-2}	99.7×10^{-2}
$Var_p(S_{\mu s}^2)$	Exact value	Not computed	Not computed
	Simulation estimate	9.82×10^{-4}	1.74×10^{-4}

Simulation mean and simulation variance for each of $\overline{y}_s$, $\hat{V}_B$, $\hat{V}_W$, $\hat{\sigma}_b^2$, and $\hat{\rho}_w$ are given in Tables 6.2 – 6.7 below, for sample size n = 2,000 and 10,000; for various values of I (and $m = n/I$); and for $\sigma_b^2 / \sigma_\varepsilon^2 = 0.1$, 1, and 10. Some comments on these simulation results follow here.

- It seems that the relation between the interviewer variance σ_b^2 and the response error variance σ_ε^2 is not of great importance in this simulation example. If the interviewer variance is a tenth of, equal to, or ten times the elementary error variance does not affect the results very much.

- We note that the simulation variance of $\overline{y}_s$ increases when the number of interviewers, I, decreases (that is, when the subsample size, m, increases). This is a fact that has been commented on in the literature (see for example Biemer and Trewin 1997), and it is also what we expect when we look at equation (3.4) or Remark 1.

- We also note that in most cases the simulation mean of the variance estimator $\hat{V}_B$ is slightly larger than the simulation variance of $\overline{y}_s$. This is in accordance with equation (4.4) which says that $\hat{V}_B$ (considered as an estimator of $Var_{p*M}(\overline{y}_s)$) has a positive bias equal to S_μ^2 / N which in the actual simulation example is approximately equal to 1/100,000 = 0.00001.

- (From equations 4.7) and (5.2) we have that $E_{p^*M}(\hat{V}_W) = \sigma_\varepsilon^2 + S_\mu^2$. The simulations verify this result. The simulation mean of $\hat{V}_W$ is about the same for n = 2,000 and n = 10,000, for the different values on I, and for the different values on $\sigma_b^2 / \sigma_\varepsilon^2$.

- The simulation variance of $\hat{V}_W$ is smaller for n = 10,000 than for n = 2,000. For given sample size n, it decreases slightly when the number of interviewers, I, increases (that is, when the subsample size, m, decreases).

- For the interviewer variance estimator, $\hat{\sigma}_b^2$, the simulations indicate that the choice of I and m (for given sample size n) will affect the variance of $\hat{\sigma}_b^2$. For both large and small values on I, the simulation variance of $\hat{\sigma}_b^2$ increases, and it thus seems that the simulation variance has its minimum for some I value in between.

- The findings for $\hat{\rho}_w$ in the simulation study are similar to what we found for $\hat{\sigma}_b^2$. Thus, it seems that the sample size n as well as the combination of I and m are of importance for $\hat{\rho}_w$ to have a small variance.

Table 6.2. Simulation means, E_{sim}, and variances, V_{sim}, of $\bar{y}_s, \hat{V}_B, \hat{V}_W, \hat{\sigma}_b^2$, and $\hat{\rho}_w$ from 5,000 repeated simple random samples. Sample size n = 10,000, $\sigma_b^2 / \sigma_\varepsilon^2 = 0.1$, and $\rho_w = 0.02$. ($\sigma_b^2 = 0.025557683$)

$I =$	500	250	200	125	100	80	50	40	20
$m =$	20	40	50	80	100	125	200	250	500
$\bar{y}_s$									
$E_{sim} \times 10^2$	0.13	0.13	0.15	0.19	0.13	0.15	0.22	0.16	0.11
$V_{sim} \times 10^4$	1.65	2.14	2.36	3.07	3.58	4.28	6.21	7.27	14.0
$\hat{V}_B$									
$E_{sim} \times 10^4$	1.77	2.27	2.53	3.30	3.80	4.42	6.37	7.68	14.0
$V_{sim} \times 10^8$	0.01	0.04	0.07	0.17	0.29	0.49	1.62	3.02	20.0
$\hat{V}_W$									
E_{sim}	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
$V_{sim} \times 10^4$	3.06	2.95	2.94	2.92	2.91	2.91	2.90	2.89	2.89
$\hat{\sigma}_b^2$									
$E_{sim} \times 10^2$	2.57	2.55	2.56	2.56	2.55	2.53	2.56	2.57	2.54
$V_{sim} \times 10^4$	0.32	0.27	0.26	0.27	0.29	0.32	0.41	0.48	0.80
$\hat{\rho}_w$									
$E_{sim} \times 10^2$	2.01	1.99	2.01	2.00	2.00	1.98	2.00	2.01	1.99
$V_{sim} \times 10^4$	0.19	0.16	0.16	0.16	0.17	0.19	0.24	0.28	0.47

Table 6.3. Simulation means, E_{sim}, and variances, V_{sim}, of $\overline{y}_s, \hat{V}_B, \hat{V}_W, \hat{\sigma}_b^2$, and $\hat{\rho}_w$ from 5,000 repeated simple random samples. Sample size $n = 10{,}000$, $\sigma_b^2 / \sigma_\varepsilon^2 = 1$, and $\rho_w = 0.02$. ($\sigma_b^2 = 0.020765617$)

$I =$	500	250	200	125	100	80	50	40	20
$m =$	20	40	50	80	100	125	200	250	500
$\overline{y}_s$									
$E_{sim} \times 10^2$	0.13	0.13	0.15	0.18	0.14	0.15	0.22	0.17	0.11
$V_{sim} \times 10^4$	1.32	1.73	1.90	2.46	2.90	3.45	5.01	5.87	11.3
$\hat{V}_B$									
$E_{sim} \times 10^4$	1.43	1.85	2.06	2.68	3.09	3.59	5.18	6.24	11.4
$V_{sim} \times 10^8$	0.01	0.03	0.04	0.12	0.17	0.32	1.06	2.00	13.2
$\hat{V}_W$									
E_{sim}	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
$V_{sim} \times 10^4$	1.94	1.87	1.86	1.85	1.84	1.84	1.83	1.83	1.82
$\hat{\sigma}_b^2$									
$E_{sim} \times 10^2$	2.08	2.07	2.08	2.08	2.07	2.06	2.08	2.09	2.07
$V_{sim} \times 10^4$	0.21	0.18	0.18	0.18	0.20	0.21	0.27	0.32	0.53
$\hat{\rho}_w$									
$E_{sim} \times 10^2$	2.01	1.99	2.01	2.00	1.99	1.98	2.00	2.01	1.99
$V_{sim} \times 10^4$	0.19	0.16	0.16	0.16	0.18	0.19	0.24	0.29	0.47

Table 6.4. Simulation means, E_{sim}, and variances, V_{sim}, of $\overline{y}_s, \hat{V}_B, \hat{V}_W, \hat{\sigma}_b^2$, and $\hat{\rho}_w$ from 5,000 repeated simple random samples. Sample size $n = 10{,}000$, $\sigma_b^2 / \sigma_\varepsilon^2 = 10$ and $\rho_w = 0.02$. ($\sigma_b^2 = 0.020383428$)

$I =$	500	250	200	125	100	80	50	40	20
$m =$	20	40	50	80	100	125	200	250	500
$\overline{y}_s$									
$E_{sim} \times 10^2$	0.14	0.13	0.15	0.18	0.14	0.15	0.22	0.17	0.12
$V_{sim} \times 10^4$	1.29	1.70	1.87	2.41	2.85	3.39	4.92	5.76	11.0
$\hat{V}_B$									
$E_{sim} \times 10^4$	1.41	1.81	2.02	2.63	3.03	3.52	5.08	6.13	11.1
$V_{sim} \times 10^8$	0.01	0.03	0.04	0.11	0.19	0.31	1.02	1.93	12.7
$\hat{V}_W$									
E_{sim}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$V_{sim} \times 10^4$	1.86	1.80	1.78	1.77	1.76	1.77	1.76	1.75	1.75
$\hat{\sigma}_b^2$									
$E_{sim} \times 10^2$	2.05	2.03	2.04	2.04	2.03	2.02	2.04	2.05	2.03
$V_{sim} \times 10^4$	0.20	0.17	0.17	0.17	0.19	0.20	0.26	0.31	0.51
$\hat{\rho}_w$									
$E_{sim} \times 10^2$	2.01	1.99	2.01	2.00	1.99	1.98	2.00	2.01	1.99
$V_{sim} \times 10^4$	0.19	0.16	0.16	0.16	0.18	0.19	0.24	0.29	0.47

Table 6.5. Simulation means, E_{sim}, and variances, V_{sim}, of $\overline{y}_s, \hat{V}_B, \hat{V}_W, \hat{\sigma}_b^2$, and $\hat{\rho}_w$ from 5,000 repeated simple random samples. Sample size n = 2,000, $\sigma_b^2 / \sigma_\varepsilon^2$ = 0.1, and ρ_w = 0.02. (σ_b^2 = 0.025557683)

I =	125	100	80	50	40	25	20	16
m =	16	20	25	40	50	80	100	125
$\overline{y}_s$								
$E_{sim} \times 10^2$	0.13	0.13	0.09	0.12	0.16	0.19	0.20	0.11
$V_{sim} \times 10^4$	8.31	8.59	9.41	11.3	12.6	16.2	18.4	22.6
$\hat{V}_B$								
$E_{sim} \times 10^4$	8.30	8.81	9.46	11.4	12.7	16.6	19.2	22.2
$V_{sim} \times 10^8$	1.10	1.59	2.27	5.16	8.11	24.1	38.3	67.9
$\hat{V}_W$								
E_{sim}	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
$V_{sim} \times 10^4$	16.7	16.4	16.3	16.0	16.0	15.8	15.8	15.8
$\hat{\sigma}_b^2$								
$E_{sim} \times 10^2$	2.54	2.55	2.56	2.56	2.56	2.59	2.58	2.56
$V_{sim} \times 10^4$	1.79	1.62	1.47	1.30	1.30	1.51	1.53	1.74
$\hat{\rho}_w$								
$E_{sim} \times 10^2$	1.99	1.99	2.00	2.00	2.00	2.02	2.01	1.99
$V_{sim} \times 10^4$	1.08	0.97	0.88	0.77	0.77	0.89	0.90	1.02

Table 6.6. Simulation means, E_{sim}, and variances, V_{sim}, of $\overline{y}_s, \hat{V}_B, \hat{V}_W, \hat{\sigma}_b^2$, and $\hat{\rho}_w$ from 5,000 repeated simple random samples. Sample size n = 2,000, $\sigma_b^2 / \sigma_\varepsilon^2$ = 1, and ρ_w = 0.02. (σ_b^2 = 0.020765617)

I =	125	100	80	50	40	25	20	16
m =	16	20	25	40	50	80	100	125
$\overline{y}_s$								
$E_{sim} \times 10^2$	0.14	0.14	0.10	0.13	0.17	0.19	0.20	0.12
$V_{sim} \times 10^4$	6.75	7.02	7.70	9.23	10.3	13.3	14.9	18.4
$\hat{V}_B$								
$E_{sim} \times 10^4$	6.75	7.16	7.68	9.25	10.3	13.5	15.5	18.1
$V_{sim} \times 10^8$	0.73	1.04	1.50	3.37	5.37	15.7	25.1	44.8
$\hat{V}_W$								
E_{sim}	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
$V_{sim} \times 10^4$	11.0	10.8	10.7	10.5	10.5	10.4	10.4	10.3
$\hat{\sigma}_b^2$								
$E_{sim} \times 10^2$	2.07	2.07	2.07	2.08	2.07	2.10	2.09	2.08
$V_{sim} \times 10^4$	1.20	1.07	0.98	0.85	0.86	0.99	1.00	1.15
$\hat{\rho}_w$								
$E_{sim} \times 10^2$	2.00	1.99	1.99	2.00	1.99	2.02	2.01	1.99
$V_{sim} \times 10^4$	1.10	0.97	0.89	0.76	0.78	0.88	0.89	1.01

Table 6.7. Simulation means, E_{sim}, and variances, V_{sim}, of $\overline{y}_s, \hat{V}_B, \hat{V}_W, \hat{\sigma}_b^2$, and $\hat{\rho}_w$ from 5,000 repeated simple random samples. Sample size n = 2,000, $\sigma_b^2 / \sigma_\varepsilon^2 = 10$, and $\rho_w = 0.02$. ($\sigma_b^2 = 0.020383428$)

	I =	125	100	80	50	40	25	20	16
	m =	16	20	25	40	50	80	100	125
$\overline{y}_s$									
$E_{sim} \times 10^2$		0.14	0.14	0.11	0.13	0.17	0.19	0.20	0.12
$V_{sim} \times 10^4$		6.63	6.91	7.58	9.07	10.1	13.1	14.7	18.0
$\hat{V}_B$									
$E_{sim} \times 10^4$		6.63	7.03	7.53	9.08	10.1	13.3	15.3	17.7
$V_{sim} \times 10^8$		0.71	1.00	1.45	3.24	5.19	15.1	24.1	43.1
$\hat{V}_W$									
E_{sim}		0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
$V_{sim} \times 10^4$		10.5	10.4	10.3	10.1	10.1	9.95	9.95	9.92
$\hat{\sigma}_b^2$									
$E_{sim} \times 10^2$		2.04	2.04	2.03	2.04	2.04	2.07	2.05	2.04
$V_{sim} \times 10^4$		1.16	1.03	0.95	0.82	0.83	0.95	0.97	1.11
$\hat{\rho}_w$									
$E_{sim} \times 10^2$		2.00	2.00	1.99	2.00	1.99	2.02	2.01	1.99
$V_{sim} \times 10^4$		1.10	0.97	0.89	0.76	0.78	0.87	0.89	1.01

7. Discussion

We have in this paper tried to introduce a theoretical framework for interviewer variance studies for continuous data. Our goal is to provide a framework for a survey organization that wants to conduct a study on interviewer variance and intra-interviewer correlation. The present paper is of an introductory nature, and more specific problems will be addressed in forthcoming papers.

The theory in this paper is not new; already Kish (1962) used an ANOVA model. During the years several models of similar kind have been employed in this field. As witnessed by Lessler and Kalsbeek (1992), it is not always easy to understand the difference between different measurement error models found in the literature, because the meaning of basic terms seems to vary somewhat among different authors. That is why we have spent some effort in this paper defining the basic concepts that we are using. Our terminology and notation is to a large extent the same as in Särndal et al. (1992).

The interviewer variance (and the intra-interviewer correlation) is often numerically small. But even if it is small, it may have a severe effect on the precision of survey estimates. One of the problems in designing an interviewer variance study is that one often has to detect and estimate a quantity that is near zero. A large number of observations are then needed which may be practically inconvenient or impossible. This fact was already noted by Groves (1989, page 380) who wrote: "It is clear that the instability of estimates from most single surveys is an impediment to understanding interviewer variability."

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Appendix Covariance matrix with elements $Cov_M(y_k,\ y_l)$

	s_1				s_2					s_I			
s_1	$\sigma_b^2+\sigma_\varepsilon^2$	σ_b^2	$\cdots$	σ_b^2	0	0	$\cdots$	0	$. \ . \ .$	0	0	$\cdots$	0
	σ_b^2	$\sigma_b^2+\sigma_\varepsilon^2$	$\cdots$	σ_b^2	0	0	$\cdots$	0	$. \ . \ .$	0	0	$\cdots$	0
	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$			$\vdots$		$\vdots$
	σ_b^2	σ_b^2	$\cdots$	$\sigma_b^2+\sigma_\varepsilon^2$	0	0	$\cdots$	0	$. \ . \ .$	0	0	$\cdots$	0
s_2	0	0	$\cdots$	0	$\sigma_b^2+\sigma_\varepsilon^2$	σ_b^2	$\cdots$	σ_b^2	$. \ . \ .$	0	0	$\cdots$	0
	0	0	$\cdots$	0	σ_b^2	$\sigma_b^2+\sigma_\varepsilon^2$	$\cdots$	σ_b^2	$. \ . \ .$	0	0	$\cdots$	0
	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$
	0	0	$\cdots$	0	σ_b^2	σ_b^2	$\cdots$	$\sigma_b^2+\sigma_\varepsilon^2$	$. \ . \ .$	0	0	$\cdots$	0
	.	.		.	.	.		.	.	.	.		.
	.	.		.	.	.		.	.	.	.		.
	.	.		.	.	.		.	.	.	.		.
s_I	0	0	$\cdots$	0	0	0	$\cdots$	0	$. \ . \ .$	$\sigma_b^2+\sigma_\varepsilon^2$	σ_b^2	$\cdots$	σ_b^2
	0	0	$\cdots$	0	0	0	$\cdots$	0	$. \ . \ .$	σ_b^2	$\sigma_b^2+\sigma_\varepsilon^2$	$\cdots$	σ_b^2
	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\ddots$	$\vdots$
	0	0	$\cdots$	0	0	0	$\cdots$	0	$. \ . \ .$	σ_b^2	σ_b^2	$\cdots$	$\sigma_b^2+\sigma_\varepsilon^2$

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