Stockholms universitet

Optimisation Algorithms in Statistics II – Spring 2021
Assignment 2

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Perform the solutions individually and send your report **until April 26** to me. Try to keep this deadline. However, if you have problems with it, there will be a final deadline on August 31 for all assignments.
Please send me one **pdf-file** with your report (alternatively, Word is ok, too), and additionally, please send me your code in one separate **plain-text file** (an R-markdown, .rmd, is possible but not required).

Problem 2.1

Consider a unidimensional minimisation problem where the minimum is attained at 0. Consider further a particle in the PSO algorithm with $x^{(1)} = 0$ and $x^{(2)} = -2$ ($v^{(2)} = -2$). For this particle, we have $p_{\text{best}}^{(t)} = g_{\text{best}}^{(t)} = 0$ for all t (the starting value happened to identify already the minimum; the stagnation assumption is fulfilled here). We consider a standard PSO with parameters w, $c_1 = c_2 =: c$ which generates the sequence $x^{(t)}, t = 1, 2, \dots$ for this particle.

a. Compute the sequence $E[x^{(t)}]$ for iteration number $t = 1, 2, \dots, 40$ and plot $E[x^{(t)}]$ versus t for different combinations of w and c. Use the pairs $(0.721, 1.193)$ (default in `R`-package `pso`), $(0.9, 1.193), (0.721, 2.2), (0.2, 3)$ for (w, c) and at least one further pair of your choice.

b. Simulate the sequence $x^{(t)}, t = 1, 2, \dots, 40$, around 1000 times. Compute the Monte Carlo estimate for $\text{Var}(x^{(t)})$ for each $t = 3, \dots, 40$ (which is simply the variance of the say 1000 simulated values for $x^{(t)}$). Plot the estimated variance versus iteration number. Do this for the (w, c)-pairs which you have used in a.

c. Based on your results from a. and the empirical results from b.: Can you confirm the theoretical results about order-1 and order-2 stability?

Problem 2.2

An iterative algorithm is given with the iteration rule

$$x^{(t+1)} = x^{(t)} + R_{t+1}(x^* - x^{(t-1)}) + S_{t+1}(x^* - x^{(t-2)}),$$

where R_i are independent Unif$[0, r]$-distributed random variables ($r \geq 0$), S_i are independent Unif$[0, s]$-distributed random variables ($s \geq 0$), and $x^{(1)}, x^{(2)}, x^{(3)}$ are given starting values and x^* is a fixed value.

a. Consider for simplicity unidimensional values for $x^{(t)}$ and x^*. Determine (with help of `R`) the pairs (r, s) $(0 \leq r \leq 2, 0 \leq s \leq 2)$ which ensure an order-1 stable sequence $(x^{(t)})$ and show them in a plot.

b. What does the results from a. say for the multidimensional case? It could be either when

$$\mathbf{x}^{(t+1)} = \mathbf{x}^{(t)} + R_{t+1}(\mathbf{x}^* - \mathbf{x}^{(t-1)}) + S_{t+1}(\mathbf{x}^* - \mathbf{x}^{(t-2)}),$$

or when

$$\mathbf{x}^{(t+1)} = \mathbf{x}^{(t)} + \mathbf{R}_{t+1} \otimes (\mathbf{x}^* - \mathbf{x}^{(t-1)}) + \mathbf{S}_{t+1} \otimes (\mathbf{x}^* - \mathbf{x}^{(t-2)}),$$

where $\otimes$ denotes the component-wise product and the components of the random vectors $\mathbf{R}_i, \mathbf{S}_i$ are all independent, uniformly distributed, Unif$[0, r]$, Unif$0, s]$, respectively. Discuss it briefly; no derivations or programming are expected for this Part b.