



Beslutsträd och andra prediktiva modeller

Mathias Lanner Sas Institute

THE POWER TO KNOW®

Agenda

Introduktion till Prediktiva modeller
Beslutsträd
Pruning
Regressioner
Neurala Nätverk
Utvärdering av modeller

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Predictive Modeling Applications



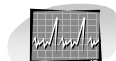
Database marketing



Financial risk management



Fraud detection



Process monitoring



Pattern detection

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Predictive Modeling Training Data

Training Data

case 1:	inputs	target
case 2:	inputs	target
case 3:	inputs	target
case 4:	inputs	target
case 5:	inputs	target

— Numeric or categorical values

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Predictive Modeling Score Data

Training Data

case 1:	inputs	target
case 2:	inputs	target
case 3:	inputs	target
case 4:	inputs	target
case 5:	inputs	target

Score Data

case 1:	inputs	?
case 2:	inputs	?
case 3:	inputs	?
case 4:	inputs	?
case 5:	inputs	?

Only input values known

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Predictions

Training Data

case 1:	inputs	target
case 2:	inputs	target
case 3:	inputs	target
case 4:	inputs	target
case 5:	inputs	target

Predictions

Score Data

case 1:	inputs	?
case 2:	inputs	?
case 3:	inputs	?
case 4:	inputs	?
case 5:	inputs	?

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Predictions

Training Data

case 1:	inputs	target
case 2:	inputs	target
case 3:	inputs	target
case 4:	inputs	target
case 5:	inputs	target

Predictions

prediction
prediction
prediction
prediction
prediction

Score Data

case 1:	inputs	?
case 2:	inputs	?
case 3:	inputs	?
case 4:	inputs	?
case 5:	inputs	?

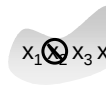
prediction
prediction
prediction
prediction
prediction

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Predictive Modeling Essentials



Predict new cases



Select useful inputs



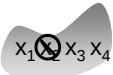
Optimize complexity

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Predictive Modeling Essentials



Predict new cases



Select useful inputs



Optimize complexity

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Three Prediction Types

Training Data

case 1: inputs	target
case 2: inputs	target
case 3: inputs	target
case 4: inputs	target
case 5: inputs	target

Predictions

prediction
prediction
prediction
prediction
prediction

- Decisions
- Rankings
- Estimates

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Decision Predictions

Training Data

case 1: inputs	target
case 2: inputs	target
case 3: inputs	target
case 4: inputs	target
case 5: inputs	target

Decisions

primary
secondary
tertiary
primary
secondary

Trained model uses input measurements to make best decision for each case.

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Ranking Predictions

Training Data

case 1: inputs	target
case 2: inputs	target
case 3: inputs	target
case 4: inputs	target
case 5: inputs	target

Rankings

720
520
620
580
470

Trained model uses input measurements to optimally rank each case.

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Estimate Predictions

Training Data		Estimates	Trained model uses input measurements to optimally estimate target value.
case 1: inputs	target	0.65	
case 2: inputs	target	0.33	
case 3: inputs	target	0.54	
case 4: inputs	target	0.47	
case 5: inputs	target	0.28	

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Model Essentials – Predict Review



Predict new cases

Decide, rank, estimate



Select useful inputs



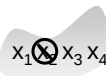
Optimize complexity

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Model Essentials – Select Review



Predict new cases



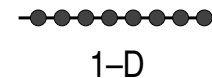
Select useful inputs



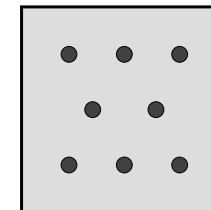
Optimize complexity

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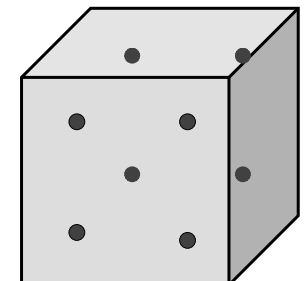
Curse of Dimensionality



1-D



2-D

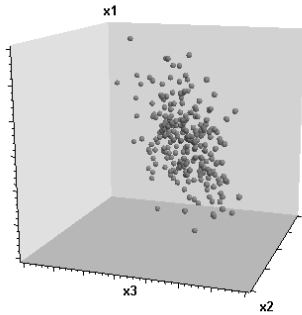


3-D

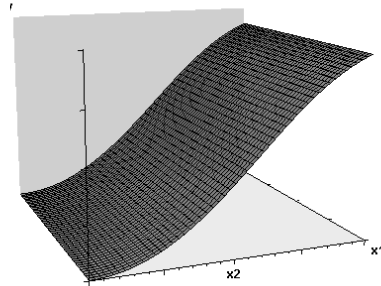
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Input Selection

Redundancy



Irrelevancy



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Model Essentials – Select Review



Predict new cases

**Decide, rank,
estimate**



Select useful inputs

**Eradicate
redundancies
irrelevancies**



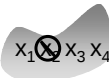
Optimize complexity

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Model Essentials – Optimize



Predict new cases



Select useful inputs



Optimize complexity

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Fool's Gold

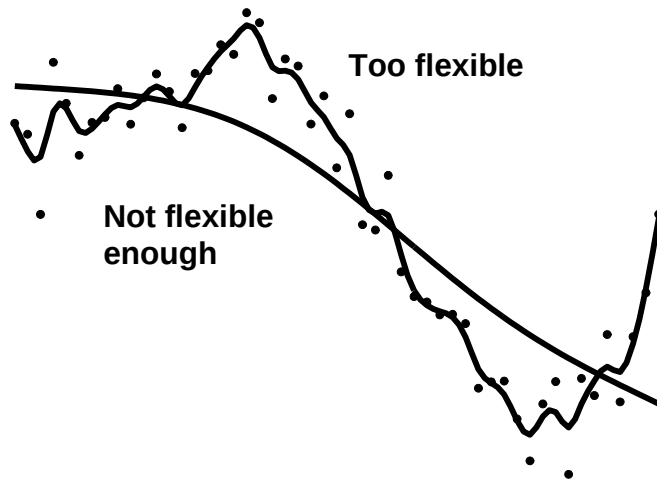
**My model fits the
training data perfectly..**

I've struck it rich!



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Model Complexity



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Data Splitting



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Training Data Role

Training Data

case 1: inputs	target
case 2: inputs	target
case 3: inputs	target
case 4: inputs	target
case 5: inputs	target



Training data gives sequence of predictive models with increasing complexity.

Validation Data

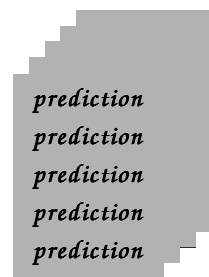
case 1: inputs	target
case 2: inputs	target
case 3: inputs	target
case 4: inputs	target
case 5: inputs	target

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Validation Data Role

Training Data

case 1: inputs	target
case 2: inputs	target
case 3: inputs	target
case 4: inputs	target
case 5: inputs	target



Validation Data

case 1: inputs	target
case 2: inputs	target
case 3: inputs	target
case 4: inputs	target
case 5: inputs	target

Validation data helps select best model from sequence

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Validation Data Role

Training Data

case 1: inputs target
case 2: inputs target
case 3: inputs target
case 4: inputs target
case 5: inputs target

prediction
prediction
prediction
prediction
prediction

Validation Data

case 1: inputs target
case 2: inputs target
case 3: inputs target
case 4: inputs target
case 5: inputs target

prediction
prediction
prediction
prediction
prediction

Validation data helps select best model from Sequence.

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Model Essentials – Optimize



Predict new cases

Decide, rank, estimate



Select useful inputs

Eradicate redundancies irrelevancies



Optimize complexity

Tune models with validation data

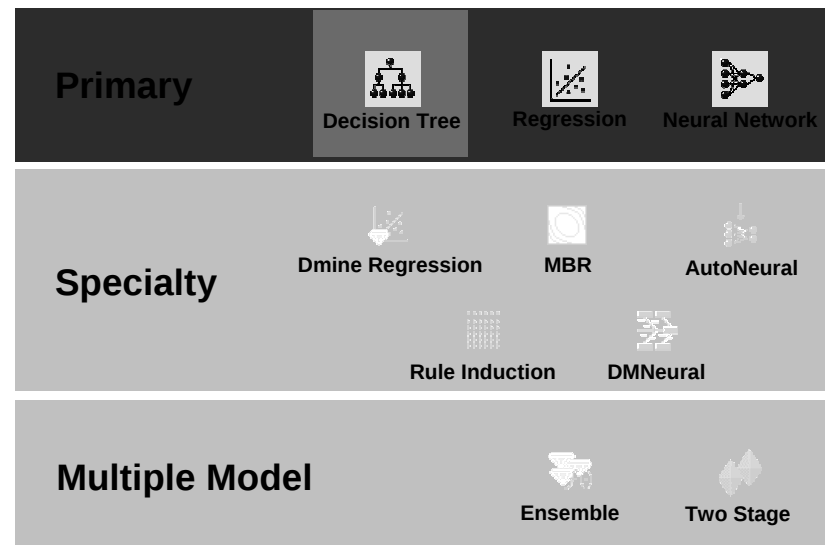
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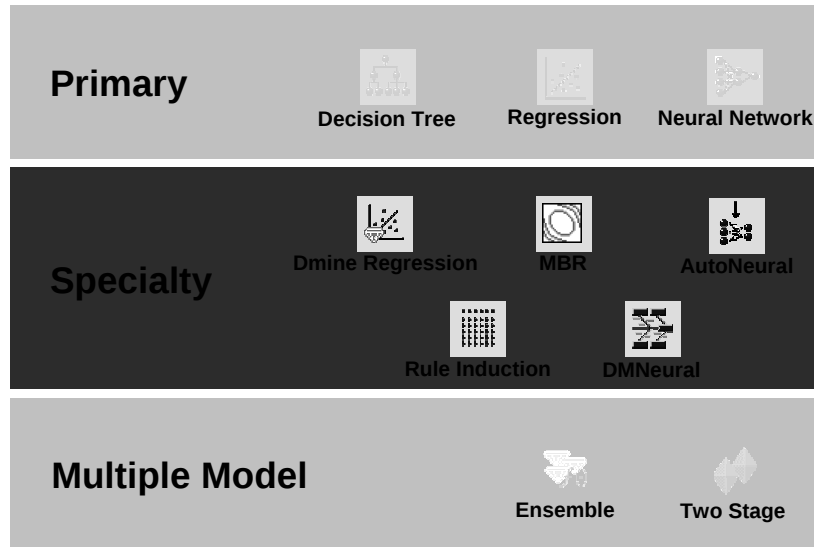
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Predictive Modeling Tools



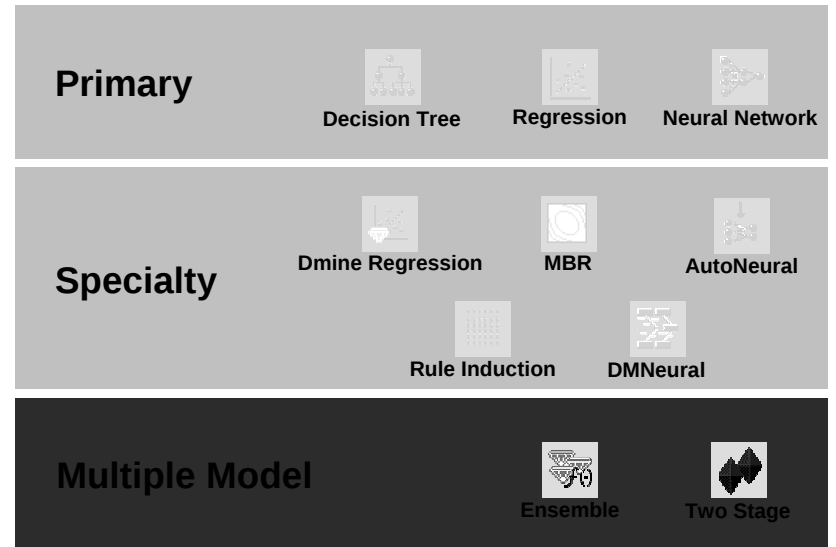
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Predictive Modeling Tools



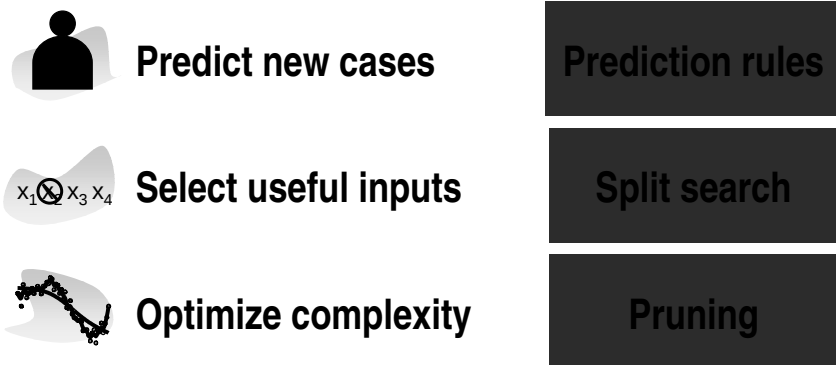
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Predictive Modeling Tools



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Model Essentials – Decision Trees

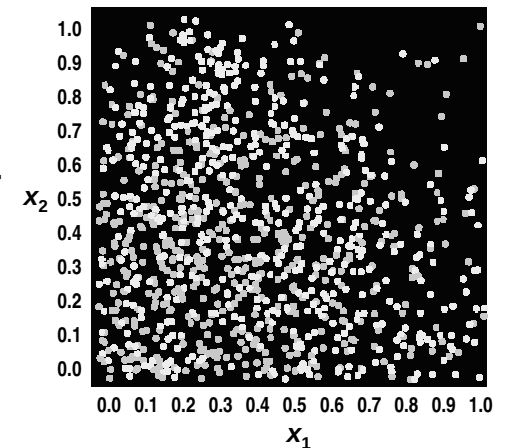


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Simple Prediction Illustration

Analysis goal:

Predict the color of a dot based on its location in a scatter plot.



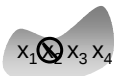
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Model Essentials – Decision Trees



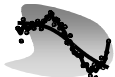
Predict new cases

Prediction rules



Select useful inputs

Split search

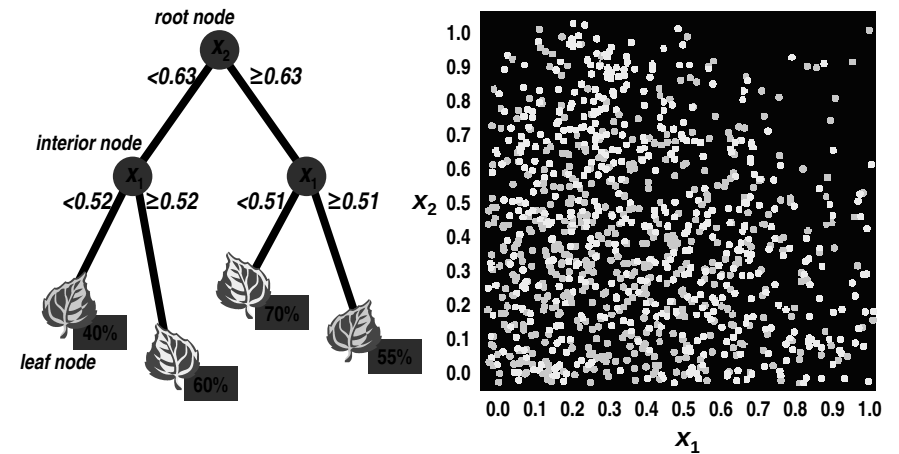


Optimize complexity

Pruning

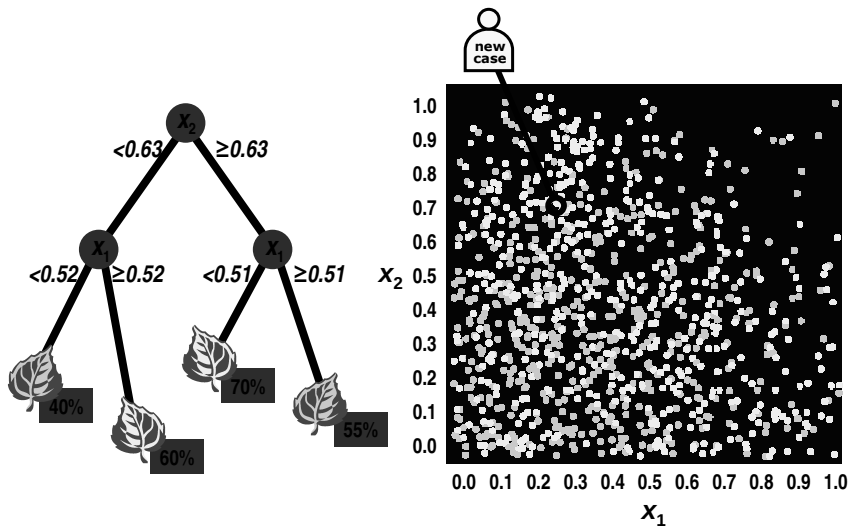
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Decision Tree Prediction Rules



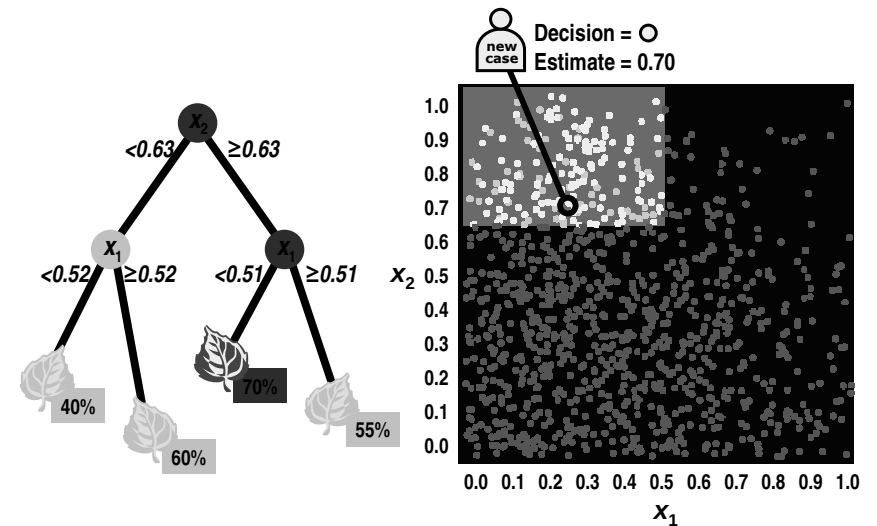
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Decision Tree Prediction Rules



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Decision Tree Prediction Rules



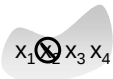
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Model Essentials – Decision Trees



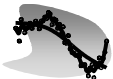
Predict new cases

Prediction rules



Select useful inputs

Split search



Optimize complexity

Pruning

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Demo Beslutsträd

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Model Essentials – Decision Trees



Predict new cases

Prediction rules



Select useful inputs

Split search

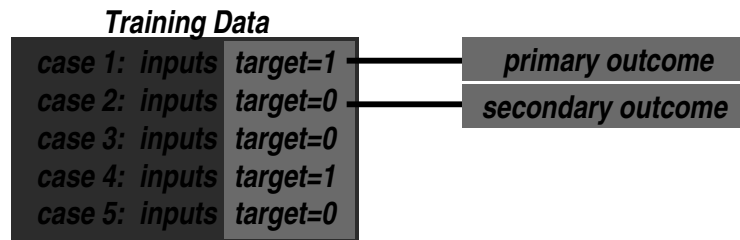


Optimize complexity

Pruning

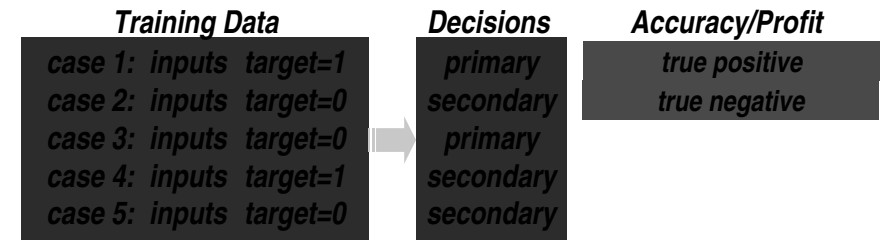
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Binary Targets



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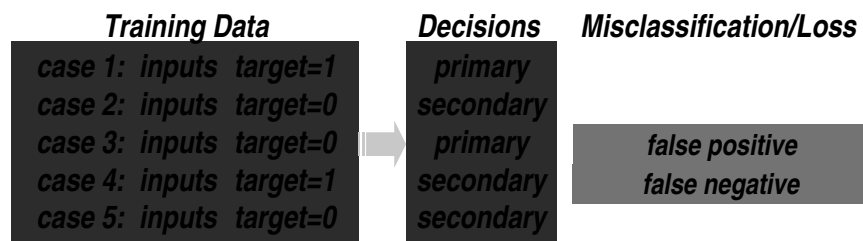
Decision Assessment



Focus on correct decisions

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Decision Assessment (for Pessimists)



Focus on incorrect decisions

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Ranking Assessment



Focus on correct ordering

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Ranking Assessment (for Pessimists)

Training Data	Rankings	Discordance
case 1: inputs target=1	720	rank(target=1) < rank(target=0)
case 2: inputs target=0	520	
case 3: inputs target=0	620	
case 4: inputs target=1	580	
case 5: inputs target=0	470	

Focus on incorrect ordering

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Estimate Assessment (only Pessimistic!)

Training Data	Estimates	Squared Error
case 1: inputs target=1	0.65	$(\text{target}-\text{estimate})^2$
case 2: inputs target=0	0.33	
case 3: inputs target=0	0.54	
case 4: inputs target=1	0.47	
case 5: inputs target=0	0.28	

Focus on incorrect estimation

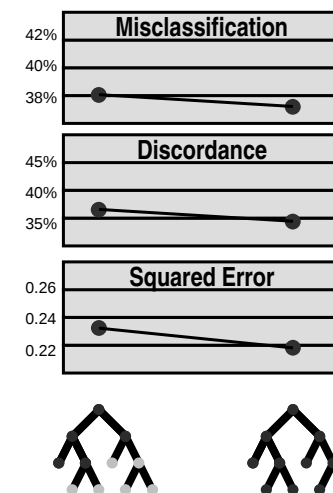
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Predictive Modeling Assessments

$\bigcirc \Rightarrow D_1$ $\bigcirc \Rightarrow D_0$	Decisions	Accuracy Misclassification
$\bigcirc \Rightarrow R$ $\bigcirc \Rightarrow r$	Rankings	Concordance Discordance
$\bigcirc \Rightarrow p_1$	Estimates	Squared Error

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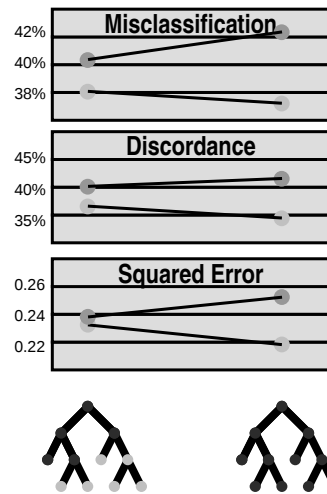
Optimistic Assessment, Pessimistic Stats



Increasing leaf count *improves* assessment measures on training data.

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Unbiased Assessment



Increasing leaf count *might worsen* assessment measures on validation data.

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Demo på Pruning

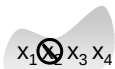
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Model Essentials – Regressions



Predict new cases

Prediction
formula



Select useful inputs

Sequential
selection



Optimize complexity

Optimal sequence
model

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Model Essentials – Regressions



Predict new cases

Prediction
formula



Select useful inputs

Sequential
selection



Optimize complexity

Optimal sequence
model

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Linear Regression Prediction Formula

$$\hat{y} = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

intercept estimate
parameter estimate
input measurement
prediction estimate

Choose intercept and parameter estimates to *minimize*.

squared error function

$$\sum (y_i - \hat{y}_i)^2$$

training data

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Logistic Regression Prediction Formula

$$\log\left(\frac{\hat{p}}{1 - \hat{p}}\right) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

logit scores

Choose intercept and parameter estimates to *maximize*.

log-likelihood function

$$\sum \log(\hat{p}_i) + \sum \log(1 - \hat{p}_i)$$

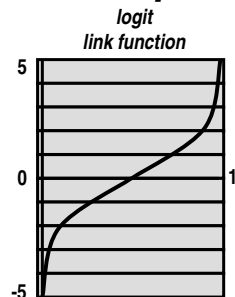
primary outcome training cases
secondary outcome training cases

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Logit Link Function

$$\log\left(\frac{\hat{p}}{1 - \hat{p}}\right) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

logit scores



Δx_i consequence
 $1 \Rightarrow \text{odds} \times \exp(w_i)$ odds ratio
 $\frac{0.69}{w_i} \Rightarrow \text{odds} \times 2$
 doubling amount
Model interpretation

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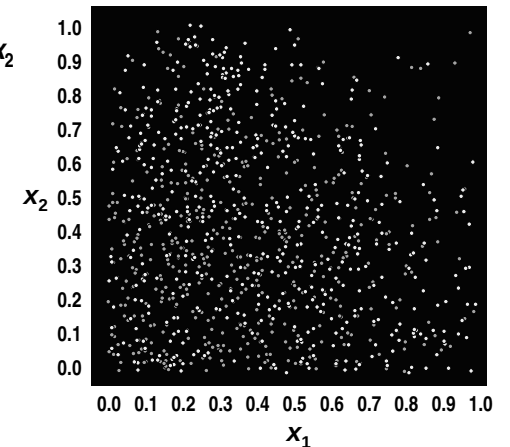
Simple Prediction Illustration – Regressions

logit equation

$$\text{logit}(\hat{p}) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

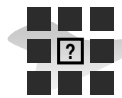
$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

logistic equation



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Beyond the Prediction Formula



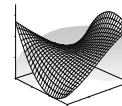
Missing values



Extreme or unusual values



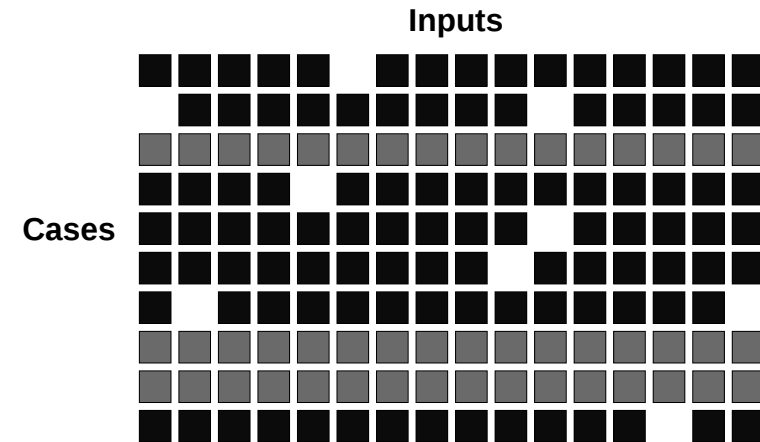
Non-numeric inputs



Nonlinearity and Non-additivity

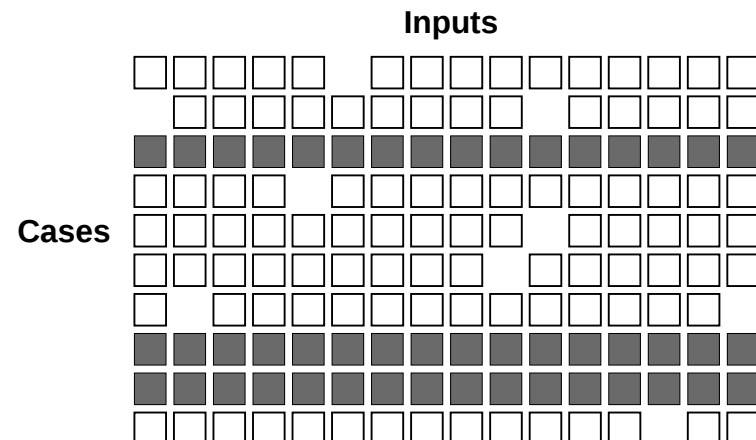
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Missing Values and Regression Modeling



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Missing Values and Regression Modeling



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Missing Values and the Prediction Formula

Prediction Formula:

$$\text{logit}(p) = -2.1 + 0.072x_1 - 0.89x_2 - 1.24x_3$$

New Case:

$$(x_1, x_2, x_3) = (2, ?, -1)$$

Predicted Value:

$$\text{logit}(p) = -2.1 + 0.144x_1 - 0.8 ? + 1.24$$

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Missing Value Causes

N/A Not applicable

? No match

⊘ Non-disclosure

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Missing Value Remedies

N/A Not applicable

? No match

⊘ Non-disclosure

Synthetic distribution



Estimation

$$x_i = f(x_1, \dots, x_p)$$

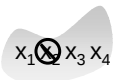
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Model Essentials – Regressions



Predict new cases

Prediction
formula



Select useful inputs

Sequential
selection



Optimize complexity

Optimal sequence
model

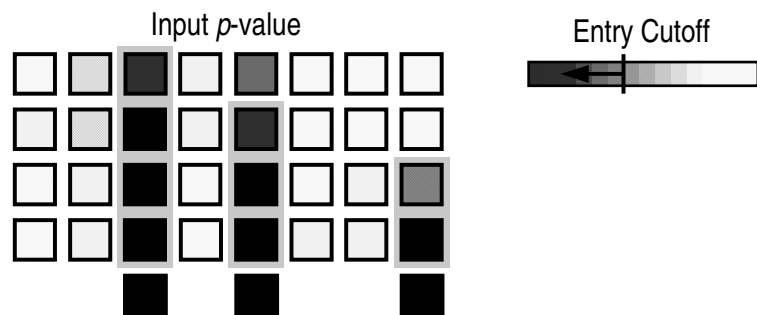
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Sequential Selection – Forward



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Sequential Selection – Forward



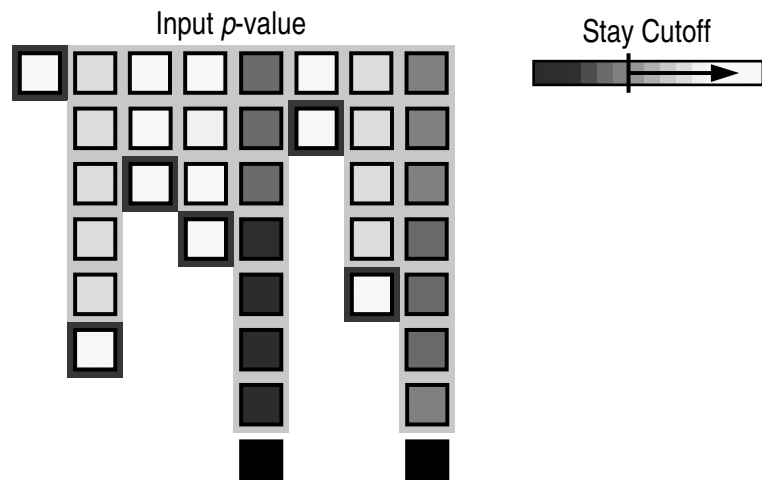
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Sequential Selection – Backward



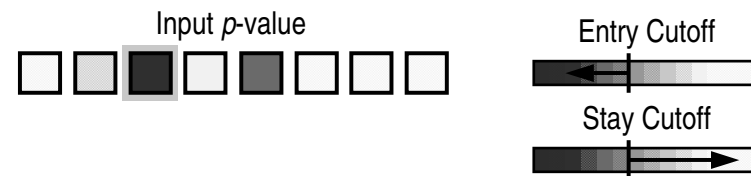
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Sequential Selection – Backward



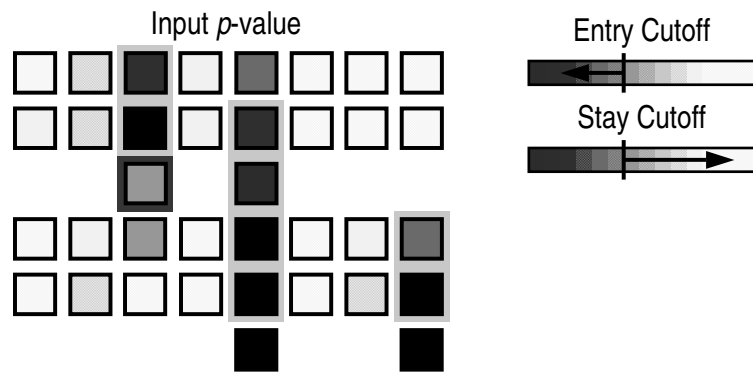
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Sequential Selection – Stepwise



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Sequential Selection – Stepwise



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Model Essentials – Regressions



Predict new cases

Prediction
formula



Select useful inputs

Sequential
selection

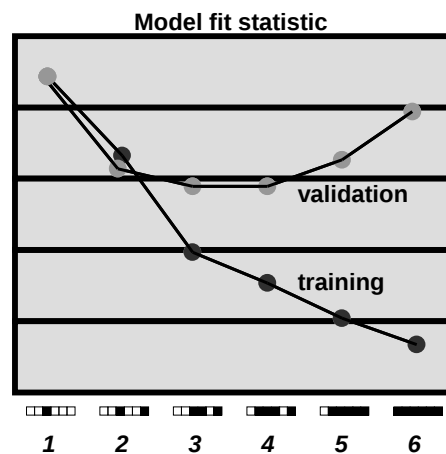


Optimize complexity

Optimal sequence
model

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Model Fit versus Complexity

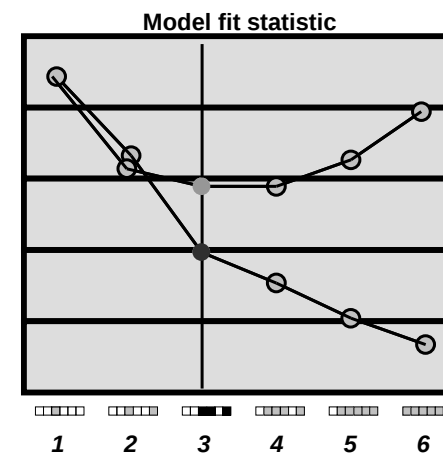


Evaluate each
sequence step.

Choose simplest
optimal model.

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Select Model with Optimal Validation Fit



Evaluate each
sequence step.

Choose simplest
optimal model.

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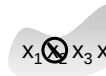
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Model Essentials – Neural Networks



Predict new cases

Prediction
formula



Select useful inputs

None



Optimize complexity

Stopped training

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Model Essentials – Neural Networks



Predict new cases

Prediction
formula



Select useful inputs

None



Optimize complexity

Stopped training

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Neural Network Prediction Formula

$$\begin{aligned}
 \text{prediction estimate } \hat{y} &= \underbrace{\hat{w}_{00}}_{\text{bias estimate}} + \underbrace{\hat{w}_{01}}_{\text{weight estimate}} H_1 + \underbrace{\hat{w}_{02}}_{\text{weight estimate}} H_2 + \underbrace{\hat{w}_{03}}_{\text{weight estimate}} H_3 \\
 H_1 &= \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2) \\
 H_2 &= \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2) \\
 H_3 &= \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)
 \end{aligned}$$

hidden unit

activation function

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Neural Network Binary Prediction Formula

$$\underset{\substack{\text{logit} \\ \text{link function}}}{\log\left(\frac{\hat{p}}{1-\hat{p}}\right)} = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

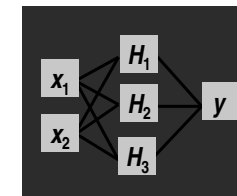
$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$

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Neural Network Diagram

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$



input layer hidden layer target layer

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$

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Prediction Illustration – Neural Networks

logit equation

$$\text{logit}(\hat{p}) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$

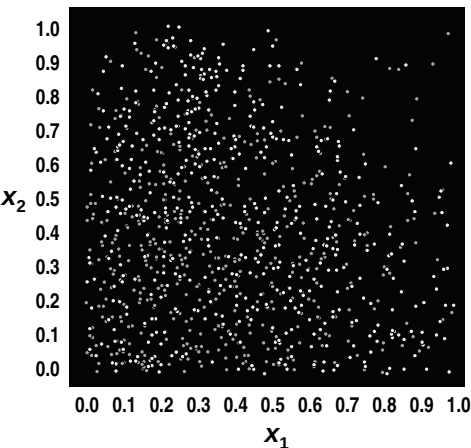
$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

logistic equation



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Agenda

Introduktion till Prediktiva modeller
Beslutsträd
Pruning
Regressioner
Neurala Nätverk
Utvärdering av modeller

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Assessment Types

The Model Comparison tool provides

**KS^C
ASE** Summary statistics

 Statistical graphics

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Summary Statistics Summary

Prediction Type

Statistic



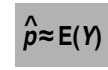
Decisions

Accuracy / Misclassification
Profit / Loss
KS-statistic



Rankings

ROC Index (concordance)
Gini coefficient



Estimates

Average squared error
SBC / Likelihood

82

Summary Statistics Summary

Prediction Type

Statistic



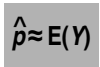
Decisions

Accuracy / Misclassification
Profit / Loss
KS-statistic



Rankings

ROC Index (concordance)
Gini coefficient



Estimates

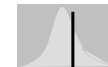
Average squared error
SBC / Likelihood

83

Summary Statistics Summary

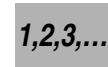
Prediction Type

Statistic



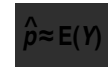
Decisions

Accuracy / Misclassification
Profit / Loss
KS-statistic



Rankings

ROC Index (concordance)
Gini coefficient



Estimates

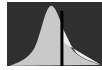
Average squared error
SBC / Likelihood

84

Statistical Graphics Summary

Prediction Type

Statistic



Decisions

1,2,3,...

Rankings

$\hat{p} \approx E(\gamma)$

Estimates

Sensitivity charts
Response rate charts