Lec 4: Experiment with Blocking Factors

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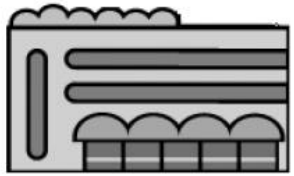
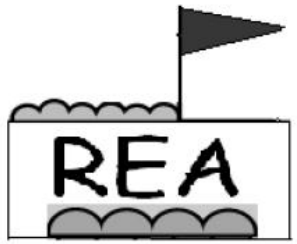
- Three end-aisle displays
- Which is the best?

1

2



3



nuisance factor

A design factor that probably has an effect on the response, but we are not interested in that effect.

- unknown & uncontrolled $\Longleftarrow$ Randomization
- known but uncontrolled $\Longleftarrow$ Analysis of covariance
- known & controlled $\Longleftarrow$ Blocking

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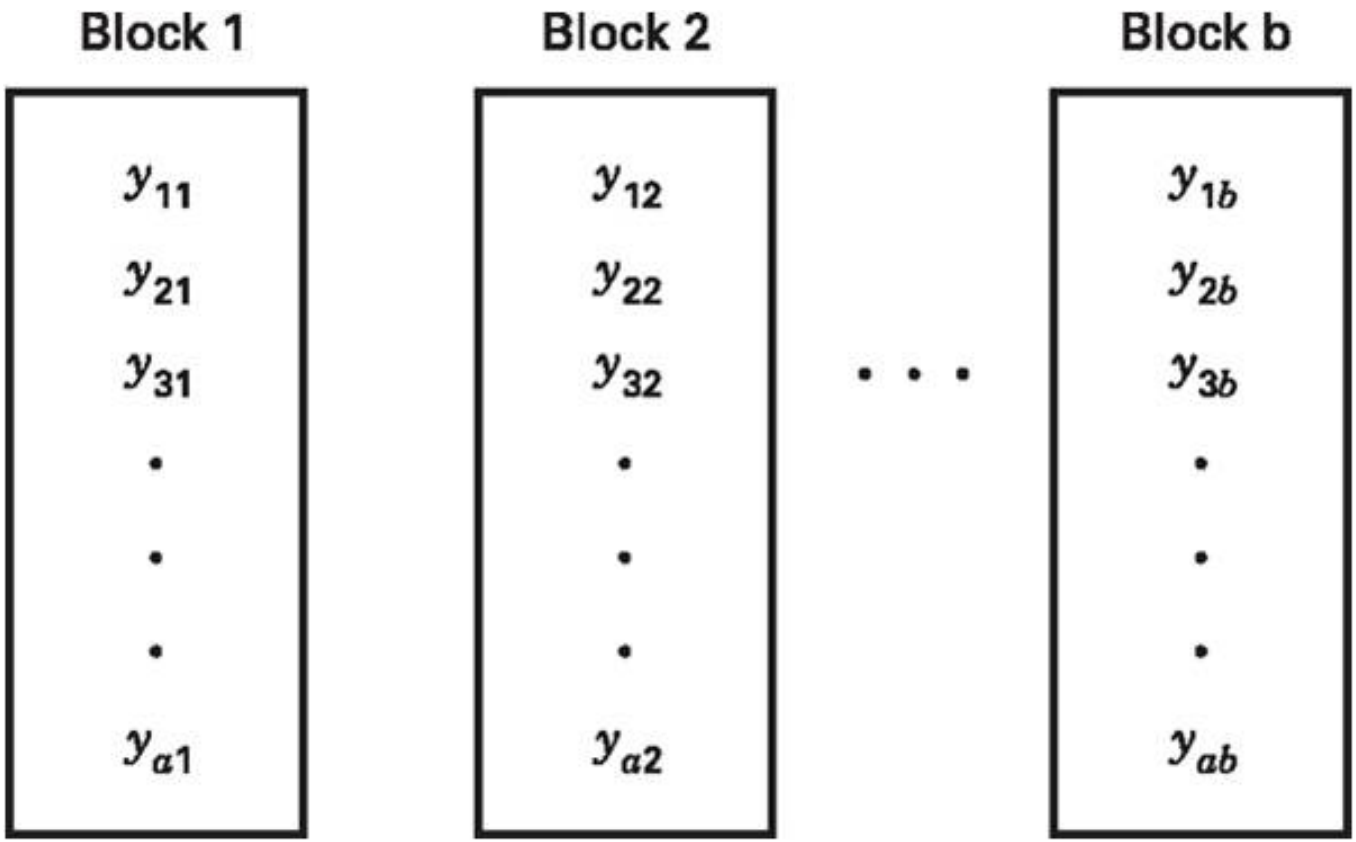
- known & controlled $\Longleftarrow$ Blocking

General rule

"Block what you can, randomize what you cannot."

The randomized complete block design (RCBD)



Statistical model for RCBD

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$$

$$i = 1, 2, \cdots, a$$

$$j = 1, 2, \cdots, b$$

■ TABLE 4.2

Analysis of Variance for a Randomized Complete Block Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}}$	$a-1$	$\frac{SS_{\text{Treatments}}}{a-1}$	$\frac{MS_{\text{Treatments}}}{MS_E}$
Blocks	SS_{Blocks}	$b-1$	$\frac{SS_{\text{Blocks}}}{b-1}$	
Error	SS_E	$(a-1)(b-1)$	$\frac{SS_E}{(a-1)(b-1)}$	
Total	SS_T	$N-1$		

Example 4.1

A medical device manufacturer produces vascular grafts (artificial veins).
The extrusion pressure (PSE) affect the quality of the artificial veins.
However, batch to batch variation may also be significant.

■ TABLE 4.3
Randomized Complete Block Design for the Vascular Graft Experiment

Extrusion Pressure (PSI)	Batch of Resin (Block) 1	2	3	4	5	6	Treatment Total
8500	90.3	89.2	98.2	93.9	87.4	97.9	556.9
8700	92.5	89.5	90.6	94.7	87.0	95.8	550.1
8900	85.5	90.8	89.6	86.2	88.0	93.4	533.5
9100	82.5	89.5	85.6	87.4	78.9	90.7	514.6
Block Totals	350.8	359.0	364.0	362.2	341.3	377.8	$y_{..} = 2155.1$

Example 4.1

TABLE 4.4
Analysis of Variance for the Vascular Graft Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Treatments (extrusion pressure)	178.17	3	59.39	8.11	0.0019
Blocks (batches)	192.25	5	38.45		
Error	109.89	15	7.33		
Total	480.31	23			

Example 4.1

TABLE 4.4
Analysis of Variance for the Vascular Graft Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Treatments (extrusion pressure)	178.17	3	59.39	8.11	0.0019
Blocks (batches)	192.25	5	38.45		
Error	109.89	15	7.33		
Total	480.31	23			

TABLE 4.5
Incorrect Analysis of the Vascular Graft Experiment as a Completely Randomized Design

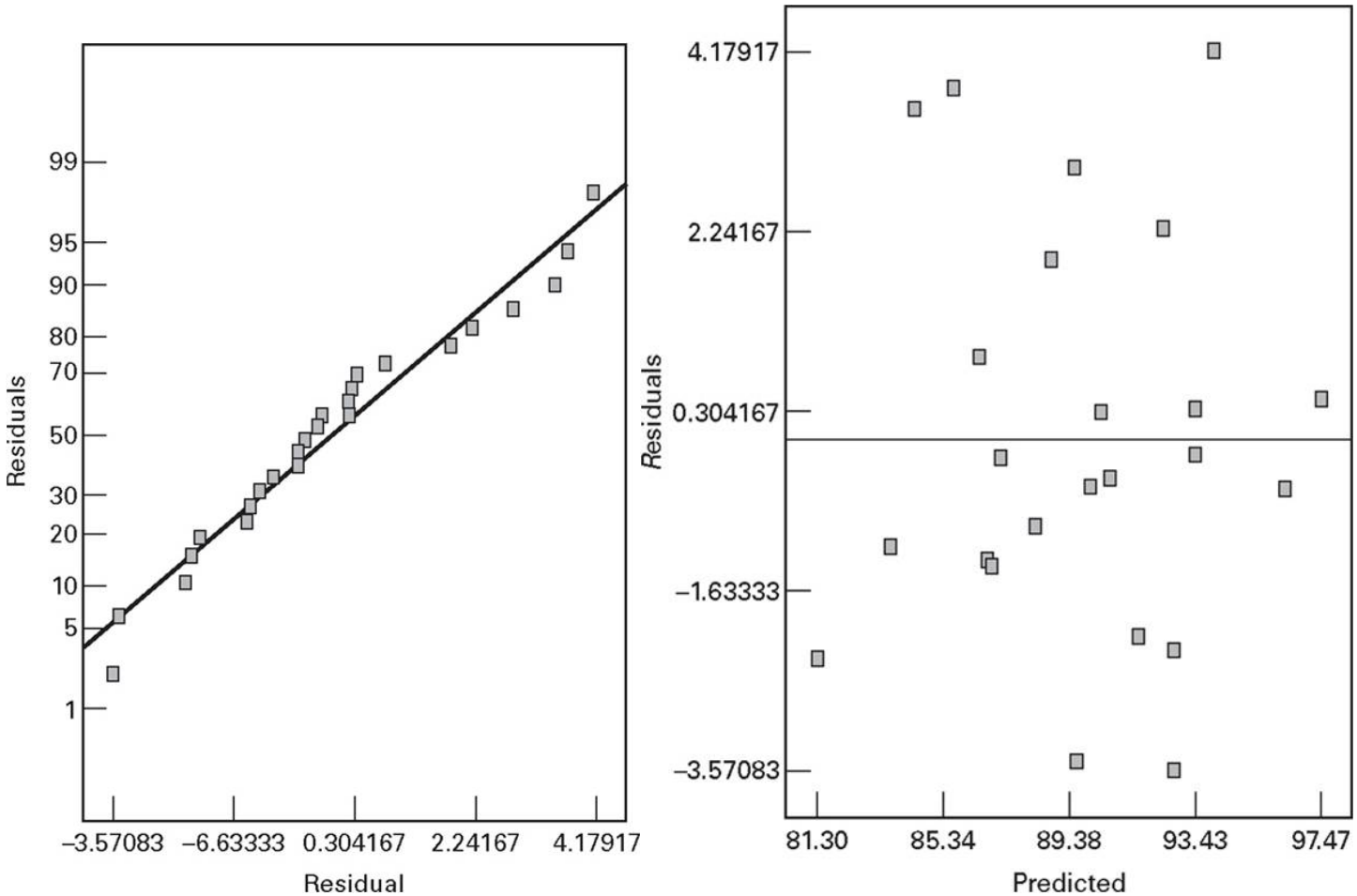
Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Extrusion pressure	178.17	3	59.39	3.95	0.0235
Error	302.14	20	15.11		
Total	480.31	23			

RCBD vs CRD

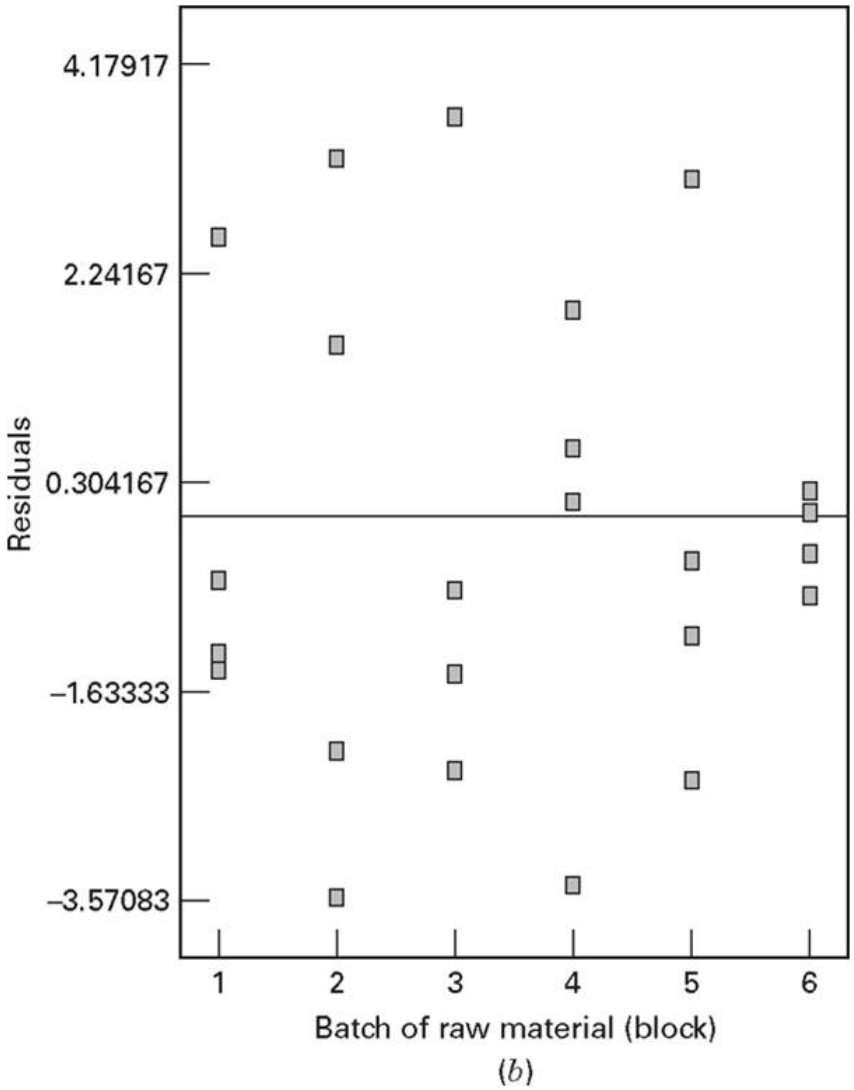
Disadvantage of RCBD

- Error df is smaller than that for the CRD (problem with a small number of treatments).
- If there are missing data, a RCBD experiment may be less efficient than a CRD.
- Difficult to make inferences about blocking variable.

Model Adequacy Checking



Model Adequacy Checking



(b)

Example

A Study of different formulation of a rocket propellants used in aircrew escape systems on the burning rate.

- 5 different formulations.
- batch of raw material
- Operators

Latin Squares

■ TABLE 4.8

Latin Square Design for the Rocket Propellant Problem

Batches of Raw Material	Operators				
	1	2	3	4	5
1	$A = 24$	$B = 20$	$C = 19$	$D = 24$	$E = 24$
2	$B = 17$	$C = 24$	$D = 30$	$E = 27$	$A = 36$
3	$C = 18$	$D = 38$	$E = 26$	$A = 27$	$B = 21$
4	$D = 26$	$E = 31$	$A = 26$	$B = 23$	$C = 22$
5	$E = 22$	$A = 30$	$B = 20$	$C = 29$	$D = 31$

■ TABLE 4.12

Standard Latin Squares and Number of Latin Squares of Various Sizes[a]

Size	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	$p \times p$
Examples of standard squares	$A\ B\ C$ $B\ C\ A$ $C\ A\ B$	$A\ B\ C\ D$ $B\ C\ D\ A$ $C\ D\ A\ B$ $D\ A\ B\ C$	$A\ B\ C\ D\ E$ $B\ A\ E\ C\ D$ $C\ D\ A\ E\ B$ $D\ E\ B\ A\ C$ $E\ C\ D\ B\ A$	$A\ B\ C\ D\ E\ F$ $B\ C\ F\ A\ D\ E$ $C\ F\ B\ E\ A\ D$ $D\ E\ A\ B\ F\ C$ $E\ A\ D\ F\ C\ B$ $F\ D\ E\ C\ B\ A$	$A\ B\ C\ D\ E\ F\ G$ $B\ C\ D\ E\ F\ G\ A$ $C\ D\ E\ F\ G\ A\ B$ $D\ E\ F\ G\ A\ B\ C$ $E\ F\ G\ A\ B\ C\ D$ $F\ G\ A\ B\ C\ D\ E$ $G\ A\ B\ C\ D\ E\ F$	$ABC \ldots P$ $BCD \ldots A$ $CDE \ldots B$ ⋮ $PAB \ldots (P-1)$
Number of standard squares	1	4	56	9408	16,942,080	—
Total number of Latin squares	12	576	161,280	818,851,200	61,479,419,904,000	$p!(p-1)! \times$ (number of standard squares)

ANOVA for Latin Squares

■ TABLE 4.9

Analysis of Variance for the Latin Square Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}} = \frac{1}{p}\sum_{j=1}^{p} y_{\cdot j\cdot}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Treatments}}}{p-1}$	$F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$
Rows	$SS_{\text{Rows}} = \frac{1}{p}\sum_{i=1}^{p} y_{i\cdot\cdot}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Rows}}}{p-1}$	
Columns	$SS_{\text{Columns}} = \frac{1}{p}\sum_{k=1}^{p} y_{\cdot\cdot k}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Columns}}}{p-1}$	
Error	SS_E (by subtraction)	$(p-2)(p-1)$	$\frac{SS_E}{(p-2)(p-1)}$	
Total	$SS_T = \sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{\cdots}^2}{N}$	p^2-1		

Replication of Latin Squares: case1

Operators

Case 1, P. 143

Batches				
	AC	CA	DB	BD
	BD	DB	CA	AC
	CB	AD	BC	DA
	DA	BC	AD	CB

Four batches and four operators

Replication of Latin Squares: case1

■ TABLE 4.13

Analysis of Variance for a Replicated Latin Square, Case 1

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$\frac{1}{np}\sum_{j=1}^{p} y_{\cdot j \cdot}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Treatments}}}{p-1}$	$\frac{MS_{\text{Treatments}}}{MS_E}$
Rows	$\frac{1}{np}\sum_{i=1}^{p} y_{i \cdot \cdot}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Rows}}}{p-1}$	
Columns	$\frac{1}{np}\sum_{k=1}^{p} y_{\cdot\cdot k}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Columns}}}{p-1}$	
Replicates	$\frac{1}{p^2}\sum_{l=1}^{n} y_{\cdot\cdot\cdot l}^2 - \frac{y_{\cdots}^2}{N}$	$n-1$	$\frac{SS_{\text{Replicates}}}{n-1}$	
Error	Subtraction	$(p-1)[n(p+1)-3]$	$\frac{SS_E}{(p-1)[n(p+1)-3]}$	
Total	$\sum\sum\sum\sum y_{ijkl}^2 - \frac{y_{\cdots}^2}{N}$	np^2-1		

Replication of Latin Squares: case2

Operators

Batches				
	A	C	D	B
	B	D	C	A
	C	A	B	D
	D	B	A	C
	A	D	B	C
	B	C	A	D
	C	B	D	A
	D	A	C	B

Case 2, p. 143

Eight batches and four operators

Replication of Latin Squares: case2

Case 2, p. 143

Batches	Operators							
	A	C	D	B	A	D	B	C
	B	D	C	A	B	C	A	D
	C	A	B	D	C	B	D	A
	D	B	A	C	D	A	C	B

Four batches and eight operators

Replication of Latin Squares: case2

■ TABLE 4.14

Analysis of Variance for a Replicated Latin Square, Case 2

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$\frac{1}{np}\sum_{j=1}^{p} y_{\cdot j \cdot \cdot}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Treatments}}}{p-1}$	$\frac{MS_{\text{Treatments}}}{MS_E}$
Rows	$\frac{1}{p}\sum_{l=1}^{n}\sum_{i=1}^{p} y_{i \cdot \cdot l}^2 - \sum_{l=1}^{n}\frac{y_{\cdots l}^2}{p^2}$	$n(p-1)$	$\frac{SS_{\text{Rows}}}{n(p-1)}$	
Columns	$\frac{1}{np}\sum_{k=1}^{p} y_{\cdot \cdot k \cdot}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Columns}}}{p-1}$	
Replicates	$\frac{1}{p^2}\sum_{l=1}^{n} y_{\cdots l}^2 - \frac{y_{\cdots}^2}{N}$	$n-1$	$\frac{SS_{\text{Replicates}}}{n-1}$	
Error	Subtraction	$(p-1)(np-1)$	$\frac{SS_E}{(p-1)(np-1)}$	
Total	$\sum_i\sum_j\sum_k\sum_l y_{ijkl}^2 - \frac{y_{\cdots}^2}{N}$	np^2-1		

Replication of Latin Squares: case3

Case 3, p. 143

Batches	Operators							
	A	C	D	B				
	B	D	C	A				
	C	A	B	D				
	D	B	A	C				
					A	D	B	C
					B	C	A	D
					C	B	D	A
					D	A	C	B

Eight batches and eight operators

Replication of Latin Squares: case3

■ TABLE 4.15

Analysis of Variance for a Replicated Latin Square, Case 3

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$\frac{1}{np}\sum_{j=1}^{p} y_{\cdot j \cdot}^2 - \frac{y_{\cdots}^2}{N}$	$p-1$	$\frac{SS_{\text{Treatments}}}{p-1}$	$\frac{MS_{\text{Treatments}}}{MS_E}$
Rows	$\frac{1}{p}\sum_{i=1}^{n}\sum_{l=1}^{p} y_{i\cdot l}^2 - \sum_{i=1}^{n}\frac{y_{i\cdot\cdot}^2}{p^2}$	$n(p-1)$	$\frac{SS_{\text{Rows}}}{n(p-1)}$	
Columns	$\frac{1}{p}\sum_{l=1}^{n}\sum_{k=1}^{p} y_{\cdot kl}^2 - \sum_{l=1}^{n}\frac{y_{\cdot\cdot l}^2}{p^2}$	$n(p-1)$	$\frac{SS_{\text{Columns}}}{n(p-1)}$	
Replicates	$\frac{1}{p^2}\sum_{l=1}^{n} y_{\cdot\cdot l}^2 - \frac{y_{\cdots}^2}{N}$	$n-1$	$\frac{SS_{\text{Replicates}}}{n-1}$	
Error	Subtraction	$(p-1)[n(p-1)-1]$	$\frac{SS_E}{(p-1)[n(p-1)-1]}$	
Total	$\sum_i\sum_j\sum_k\sum_l y_{ijkl}^2 - \frac{y_{\cdots}^2}{N}$	np^2-1		

Crossover Design

Latin Squares

	I		II		III		IV		V		VI		VII		VIII		IX		X	
Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Period 1	*A*	*B*	*B*	*A*	*B*	*A*	*A*	*B*	*A*	*B*	*B*	*A*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*
Period 2	*B*	*A*	*A*	*B*	*A*	*B*	*B*	*A*	*B*	*A*	*A*	*B*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*

■ TABLE 4.16

Analysis of Variance for the Crossover Design in Figure 4.7

Source of Variation	Degrees of Freedom
Subjects (columns)	19
Periods (rows)	1
Fluids (letters)	1
Error	18
Total	39

Graeco-Latin Square Design

■ TABLE 4.17

4 × 4 Graeco-Latin Square Design

	Column			
Row	**1**	**2**	**3**	**4**
1	$A\alpha$	$B\beta$	$C\gamma$	$D\delta$
2	$B\delta$	$A\gamma$	$D\beta$	$C\alpha$
3	$C\beta$	$D\alpha$	$A\delta$	$B\gamma$
4	$D\gamma$	$C\delta$	$B\alpha$	$A\beta$