Lec 2: Sample size calculation & Linear Models

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Sample size determination

- FAQ in designed experiments
- Answer depends on lots of things: type of experiment, desired sensitivity
- Generally, increasing the number of replications increases the sensitivity or it makes it easier to detect small differences in means

Sample Size Determination for Fixed Effects Case

Notation

- Fixed effect
- Random effect

Type I error The hypothesis is correct, but the test reject it.

Type II error The hypothesis is wrong, but the test does not reject it.

- $$\alpha = Pr(\text{Type I error})$$
- $$\beta = Pr(\text{Type II error})$$

$$\text{Significant level} = \alpha$$

$$Power = 1 - \beta$$

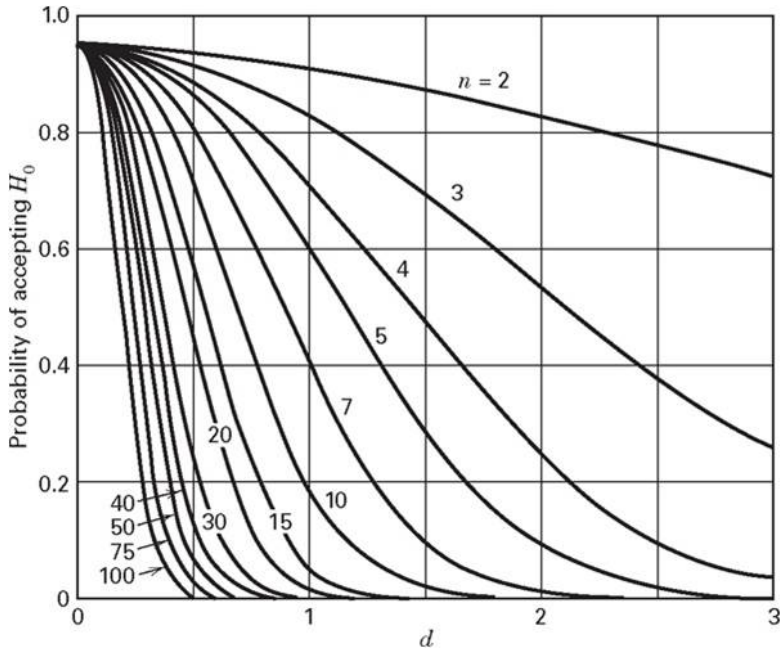
Non-central t distribution

If $Y_1 \sim N(0,1)$ and $Y_2 \sim \chi^2(v)$ then

$$T = \frac{Y_1 + \lambda}{\sqrt{Y_2/v}}$$

is non-central t-distributed with v degrees of freedom and non-centrality parameter λ

OC curve



$$d = \frac{|\mu_1 - \mu_2|}{2\sigma} = \frac{|\delta|}{2\sigma}$$

In the graph, the n is specially defined as $2n^* - 1$, n^* is the replicates in each treatment, which is different from our usual notation!

Sample size calculation: two samples

- Consider an experiment with two treatments (i.e. one factor with two levels)
- We want to reject the null hypothesis with probability 90% if the true difference is 1.
- How many experimental units are needed if the standard deviation is 0.5?

R output

```
> power.t.test(sd=0.5,power=0.9, sig.level=0.05,delta=1)

     Two-sample t test power calculation

              n = 6.386756
          delta = 1
             sd = 0.5
      sig.level = 0.05
          power = 0.9
    alternative = two.sided

 NOTE: n is number in *each* group
```

Sample size calculation: ANOVA

If we have more levels, we need to use OC curve against a different parameter

$$\Phi = \frac{n\sum_{i=1}^{a}\tau_i^2}{a\sigma^2}$$

The corresponding OC curve is given in A V

- Consider an experiment with three treatments (i.e. one factor with three levels)
- We want to reject the null hypothesis with probability 90 % given three true expected values: 5.320, 6.238, 8.318
- How many experimental units are needed if the standard deviation is 0.5?

R output

```
> groupmeans <- c(5.12,6.238,8.318)
> power.anova.test(groups=3,
  between.var=var(groupmeans),within.var=0.25,power=0.9)

     Balanced one-way analysis
     of variance power calculation

         groups = 3
              n = 2.009089
    between.var = 2.633921
     within.var = 0.25
      sig.level = 0.05
          power = 0.9

NOTE: n is number in each group
```

SAS

```
proc power;
   onewayanova
      groupmeans = 35.6 | 33.7 | 30.2 | 29 28 | 25.9
      stddev = 3.75
      alpha = 0.025
      ntotal = .
      power = 0.9
      contrast = (4 -1 -1 -1 -1) (0  1  1 -1 -1)
                 (0  1 -1  0  0) (0  0  0  1 -1);
run;
```

Notations

orthogonality

Two vector u and v are orthogonal if $u'v = 0$

Projection

- Let A be a square matrix.
- A is idempotent if $A^2 = A$.
- If A is idempotent, then A is a projection matrix.