PRICE INDEX THEORY

Course lectures within Economic Statistics at Stockholm University

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Fixed basket price index 1

$$I = \frac{\sum_{i=1}^{n} q_i \, p_{1,i}}{\sum_{i=1}^{n} q_i \, p_{0,i}} = \frac{q_1 \, p_{1,1} + q_2 \, p_{1,2} + \ldots + q_n \, p_{1,n}}{q_1 \, p_{0,1} + q_2 \, p_{0,2} + \ldots + q_n \, p_{0,n}}$$

- **Variables**
  q = quantity (volume)
  p = price
- **Objects and times**
  i = Product (good/service from given seller)
  0 = Base period (alias: reference period)
  1 = Current period

Fixed basket price index 2

$$I = \frac{\sum_{i=1}^{n} q_i \, p_{1,i}}{\sum_{i=1}^{n} q_i \, p_{0,i}} = \frac{q_1 \, p_{1,1} + q_2 \, p_{1,2} + \ldots + q_n \, p_{1,n}}{q_1 \, p_{0,1} + q_2 \, p_{0,2} + \ldots + q_n \, p_{0,n}}$$

Exempel

$$I = \frac{50 \times 98 + 100 \times 49 + 20 \times 195}{50 \times 88 + 100 \times 48 + 20 \times 195} \times 100 = 104.6$$

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Price and volume indices

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Price index

$$I = \frac{\sum_i q_{0,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}}$$

Laspeyres

$$I = \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{1,i}\, p_{0,i}}$$

Paasche

Volume index

$$I = \frac{\sum_i q_{1,i}\, p_{0,i}}{\sum_i q_{0,i}\, p_{0,i}}$$

Laspeyres

$$I = \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{1,i}}$$

Paasche

Factors of a value index

$$\frac{\sum_i q_{1,i}\, p_{0,i}}{\sum_i q_{0,i}\, p_{0,i}} \cdot \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{1,i}\, p_{0,i}} = \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} = \frac{\text{Total value(1)}}{\text{Total value(0)}}$$

Volume index x Price index = Value index

Laspeyres Paasche

$$\frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{1,i}} \cdot \frac{\sum_i q_{0,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} = \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} = \frac{\text{Total value(1)}}{\text{Total value(0)}}$$

Paasche Laspeyres

Practical uses

- *Deflating* is to compute

$$\text{Volume index} = \frac{\text{Value index}}{\text{Price index}}$$

↳ ***Eliminates price change***

- *Implicit price index* is computed as

$$\text{Price index} = \frac{\text{Value index}}{\text{Volume index}}$$

'Laspeyres type' (Lowe index)

$$I_{2006,\mathrm{Dec}}^{2007,\mathrm{April}} = \frac{\sum_i q_{2005;i}\, p_{2007,\mathrm{April};i}}{\sum_i q_{2005;i}\, p_{2006,\mathrm{Dec};i}}$$

- A useful generalisation of Laspeyres index
- Example: Annual link in HICP (Harmonised index of consumer prices)
- Price base period = Dec 2006
- Weight base period = entire year 2005

Laspeyres in another form

$$I = \frac{\sum_i q_{0,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} = \sum_i \frac{q_{0,i}\, p_{0,i}}{\sum_k q_{0,k}\, p_{0,k}} \cdot \frac{p_{1,i}}{p_{0,i}} = \sum_i w_i \cdot \frac{p_{1,i}}{p_{0,i}}$$

with weights $w_i = \dfrac{q_{0,i}\, p_{0,i}}{\sum_k q_{0,k}\, p_{0,k}}$, satisfying $\sum_i w_i = 1$

Problems with fixed baskets

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- Laspeyres > Paasche price index
  - *True almost always – due to altered consumption pattern*
- Fixed basket gets out of date – at new prices, new choices give better value for money
  - *Products with larger price rises are "substituted away" by buyers*

  Ex.: Petrol price up → car use down

Chaining in Swedish CPI

$$I_{1980}^{2007,\text{Jan}} = I_{1980}^{1980,\text{Dec}} \times I_{1980,\text{Dec}}^{1981,\text{Dec}} \times I_{1981,\text{Dec}}^{1982,\text{Dec}} \times \ldots$$

$$\ldots \times I_{2002,\text{Dec}}^{2003,\text{Dec}} \times I_{2003,\text{Dec}}^{2004} \times I_{2004}^{2005} \times I_{2005}^{2007,\text{Jan}}$$

Link of Previous type ($I_{2002,\text{Dec}}^{2003,\text{Dec}}$)

Special link ($I_{2003,\text{Dec}}^{2004}$)

Link of New type (I_{2004}^{2005})

Final link ($I_{2005}^{2007,\text{Jan}}$)

Price indices (in Sweden) 1

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- *CPI – Consumer Price Index*
  KPI – Konsumentprisindex
- *HICP – Harmonised Index for Consumer Prices*
  HIKP
- NPI – *Net Price Index*
- KPIX – *Underlying Inflation (Core Inflation)*

Price indices (in Sweden) 2

- PPI – *Producer Price Index (goods)*
- *SPPI* – *Producer Price Index for Services*
  TPI – Tjänsteprisindex
- BPI – *Building Price Index*
- *Real Estate Price Index*
- *CCI* – *Construction Cost Index for*
  E84 *Buildings*
  (building materials, labour)

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International classification standards for breakdown

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- COICOP – Classification of Individual Consumption by Purpose – *in CPI*
- NACE – Industry classification standard / Nomenclature statistique des Activités économiques dans la Communauté Européenne – *in PPI, SPPI*

Classification levels in CPI

- 00 CPI overall (all items index)
- 01 Food and non-alcoholic beverages
- 01.1 Food
- 01.1.8 Sugar, jam, chocolate etc.
- 1819 Ice cream
- 1819-80 Ice cream brand X, type Y

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Swedish CPI basket in 2010

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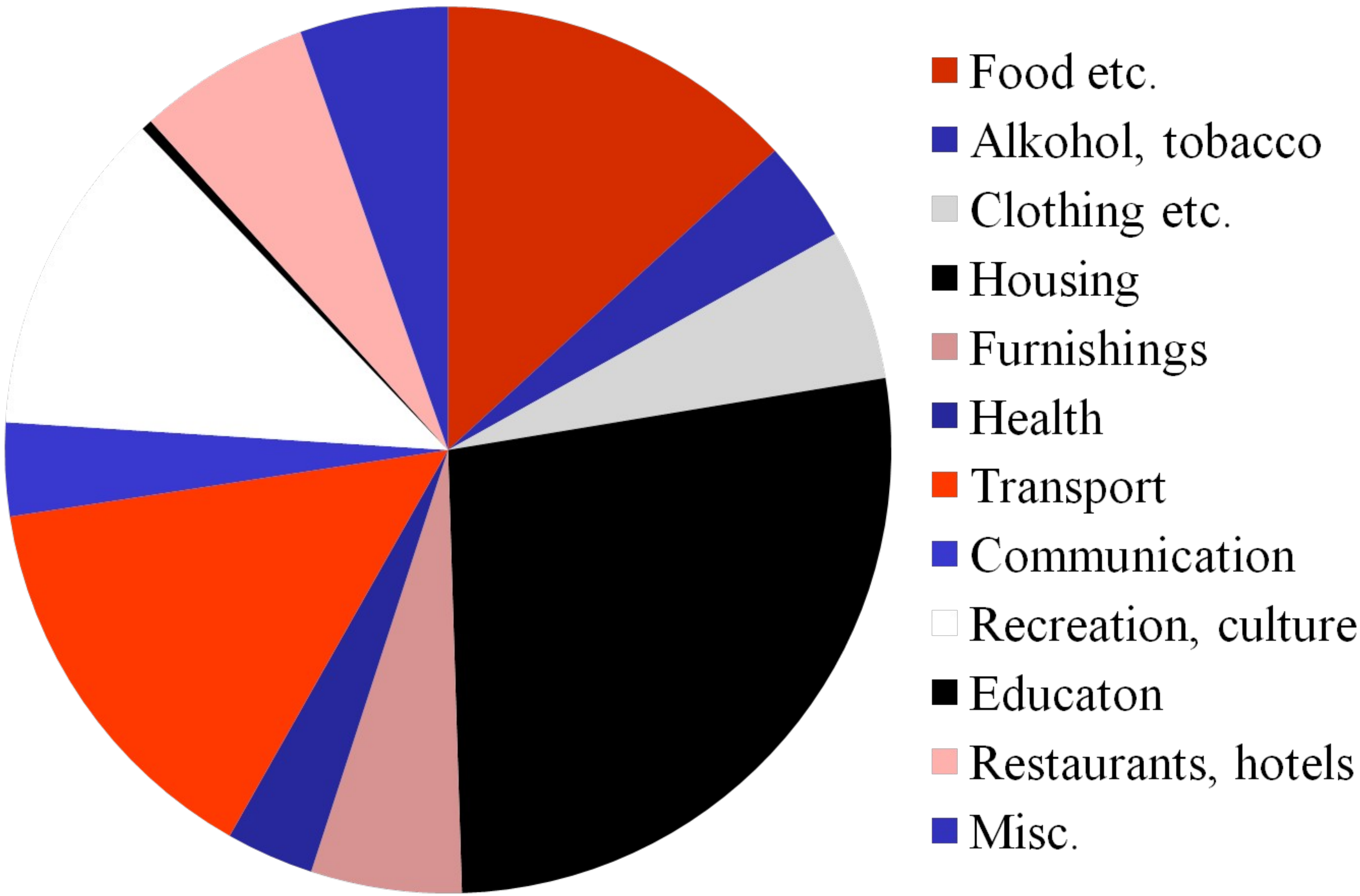

Producer and Import Price Indices (PPI)

- PPI – Producer Price Index
- ITPI – Price Index for Domestic Supply
- EXPI – Export Price Index
- IMPI – Import Price Index
- HMPI – Producer Price Index of Home Sales

PPI			
ITPI			
EXPI			
IMPI			
HMPI			

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Actual prices: CPI

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CPI follows:

- **Price on price tag (shown to consumer)**
- **After any sales deduction**
- **After deduction of general discounts**
- **But before deduction of individual discounts, loyalty rebates etc.**
  - ***Not quite ideal, e.g. for cars***
- **Inluding VAT and other indirect taxes**
- **After deduction of subventions**

Actual prices: PPI, SPPI

PPI, SPPI follow:

- **Invoiced price – transaction (ideally)**
- **After deduction of any discounts**
- **Excluding taxes, VAT**
- **List price rather not, maybe as ”proxy”**
- **Ex. *chargeout rate* (charged hour rate) for consultant services in SPPI – not ideal but practically feasible solution**

Indices – aims – targets

- CPI – Main aim is compensation
  Target is Cost Of Living Index
- HICP– Main aim is monetary politics
  Target is Laspeyres type (?)
- SPPI – Main aim is deflating
  Ideal target is Paasche
  - *Deflating with Paasche price index yields volyme index series i base period prices*
  - *But take Laspeyres i practice*

Levels of aggregation in the Swedish CPI

Full-year base, Walsh index ↑	Overall index
	Coicop classes
	350 Product groups
December base, Jevons index ↑	Elementary aggregates

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Elementary aggregates 0

- Weithting data are available on higher levels of aggregation
- Overall index is practically computed by weighting together of subindices
- *Elementary aggregates* are on lowest level of aggregation – weights usally not available

  ↳ ***Index formulas ”without q” needed***

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Elementary aggregates 1

$$I = \frac{\frac{1}{n}\sum_{i=1}^{n} p_{1,i}}{\frac{1}{n}\sum_{i=1}^{n} p_{0,i}} = \frac{\sum_{i=1}^{n} p_{1,i}}{\sum_{i=1}^{n} p_{0,i}}$$

- Ratio of mean prices [Dutot]

$$I = \frac{1}{n}\sum_{i=1}^{n} \frac{p_{1,i}}{p_{0,i}}$$

- Mean of price relatives [Carli] *Beware – bias!*

Elementary aggregates 2

$$I = \frac{\prod_{i=1}^{n} p_{1,i}^{1/n}}{\prod_{i=1}^{n} p_{0,i}^{1/n}} = \left(\prod_{i=1}^{n} \frac{p_{1,i}}{p_{0,i}} \right)^{1/n} = \sqrt[n]{\prod_{i=1}^{n} \frac{p_{1,i}}{p_{0,i}}}$$

- **Geometric mean [Jevons]**
  - ***Handles disparate price levels adequately***
  - ***Partially accounts for substitution***

Elementary aggregates 3

$$I = \left(\prod_{i=1}^{n} \left(\frac{p_{1,i}}{p_{0,i}} \right)^{V_i} \right)^{1/\sum_{i=1}^{n} V_i} = \exp \left(\frac{\sum_{i=1}^{n} V_i \ln \left(\frac{p_{1,i}}{p_{0,i}} \right)}{\sum_{i=1}^{n} V_i} \right)$$

- **Weighted geometric mean**
  - *Weighted by value (turnover)* V_i

Jevons index combined with low-level weights

$$I^{y,m}_{y-1,\mathrm{Dec};d} = \left(\prod_{k=1}^{n_d} p_{y,m;k} / p_{y-1,\mathrm{Dec};k} \right)^{1/n_d}$$

$$I^{y,m}_{y-1,\mathrm{Dec};g} = \prod_{d \in D(g)} \left(I^{y,m}_{y-1,\mathrm{Dec};d} \right)^{w_d}$$

$$= \exp\left(\sum_{d \in D(g)} w_d \log I^{y,m}_{y-1,\mathrm{Dec};d} \right), \quad \sum_{d \in D(g)} w_d = 1$$

Features of the Jevons index

- ☺ Not disturbed by spread in price level
- ☺ Accounts for consumer substitution to some extent – suitable for Cost-Of-Living Index (coli)
- 😐 Index sensitive to EA level choice
- ☹ Breaks down for zero prices
  - ↳ *Special fix required*

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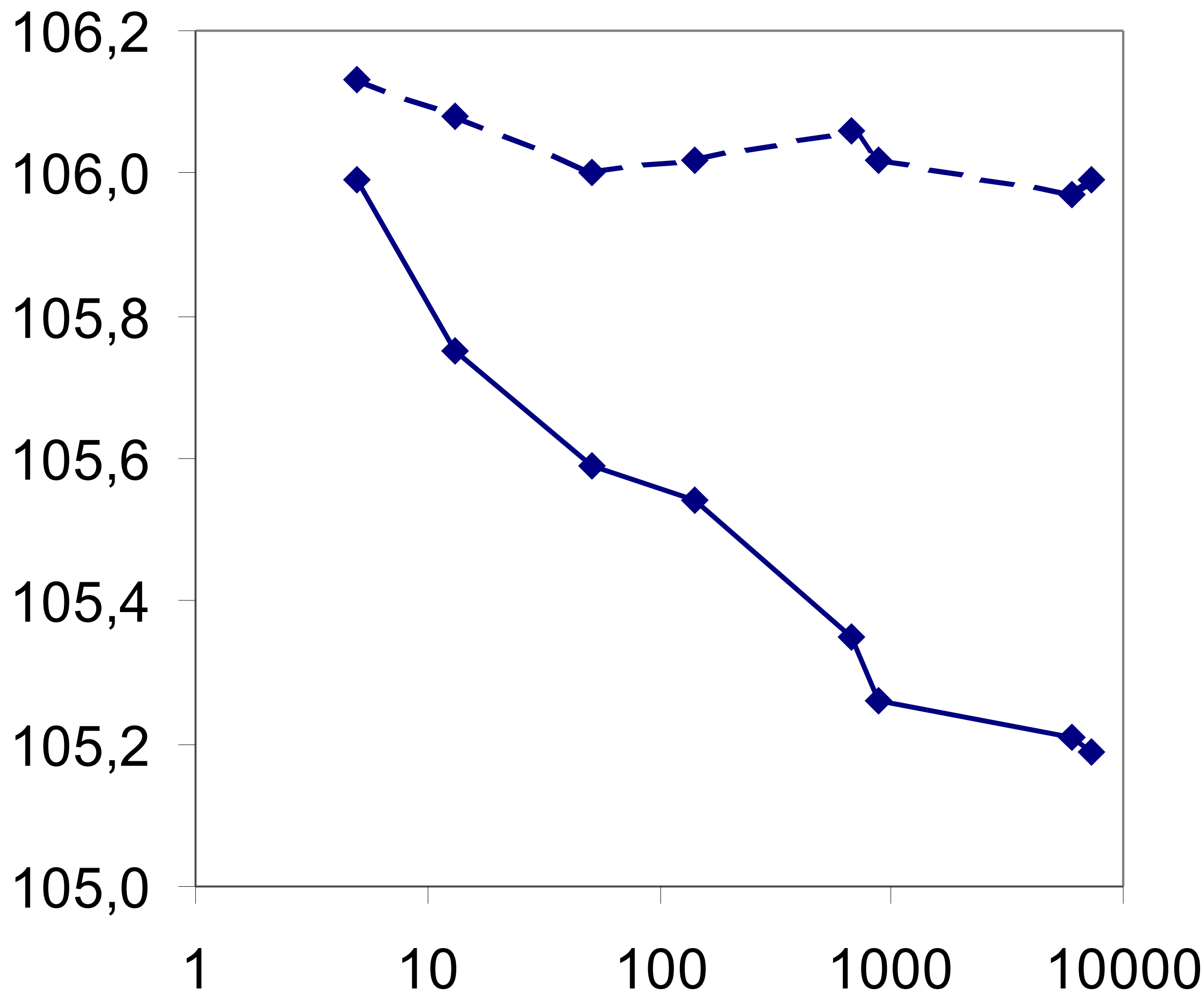
Index by EA size

Coicop 01 – December 2001



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Theoretical effects (by Dalén)

- Math. expectation of Jevons index falls below true mean μ by the amount:

$$\frac{\sigma^2}{2\mu} - \frac{\sigma^2}{2\mu} \cdot \frac{1}{n}$$

Effect of sample size

Effect of universe variance σ^2
= Assumed substitution gain of consumers

Sources of errors in CPI

- Sampling error in price observations
- Sampling error in weights
- Uncertainty in Quality Adjustment (QA)
- Measurement error in price observations
- Some undercoverage
- Proxies for hard-to-measure prices
- Errors by mistakes

Quality Assurance of work

- Management commitment to quality
- Staff competence
- Knowledge of markets
- Documentation of procedures
- Work instructions
- Safe procedures
- Price data validation and editing
- Output validation
- Debriefing

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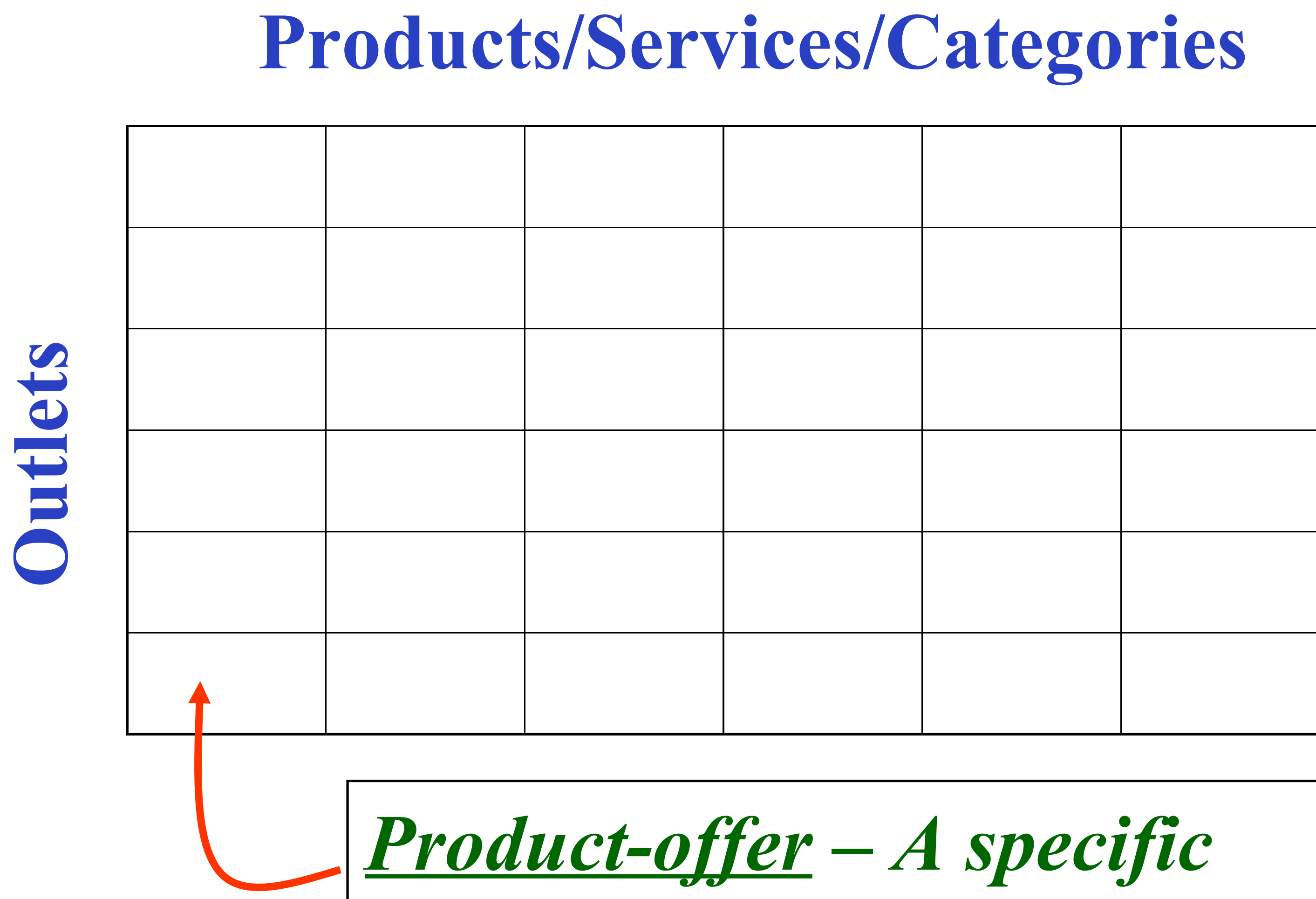
Sampling error

$$\text{Standard\_error}(I) \approx \frac{\sigma\left(\frac{p_{1,i}}{p_{0,i}}\right)}{\sqrt{n}} \quad [\times (\mathbf{deft})]$$

$$\approx \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_{1,i}}{p_{0,i}} - \frac{1}{N}\sum_{i=1}^{N}\frac{p_{1,i}}{p_{0,i}}\right)^{2}}}{\sqrt{n}} \quad \times (\mathbf{deft})$$

Two sampling dimensions



Product-offer ***– A specific product in a specific outlet (shop)***

Sampling principles

Sampling of outlets (shops etc.):

- Sampling with pps from business register (used in Swedish practice)
- Cluster sampling of regions

Sampling of products:

- Sampling with pps from product register (if available)
- Judgmental sampling of product specifications
- Judgmental sampling of models in shops

Aggregation examples (SPPI)

Architects:

- Prices for 3 categories (differ between firms)

  ↳ *2 steps:* 1) Mean price for firm

  2) Index = ratio of mean prices

Technical consultants:

- Prices for 5 work areas – weights available

  ↳ *2 steps:* 1) Sub-index for work area

  = ratio of mean prices

  2) Index = weithting of sub-indices

Survey design weights

- **Laspeyres index:**

$$I = \frac{\sum_i q_{0,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} = \sum_i w_i \cdot \frac{p_{1,i}}{p_{0,i}}$$

- **Estimation with design weights:**

$$I = \sum_i \frac{w_i}{\pi_i} \cdot \frac{p_{1,i}}{p_{0,i}}$$

where π_i = sampling probability

For pps sampling:

$$\pi_i = n\, w_i$$

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More problems of baskets

<u>***Problem:***</u>

- **Product models vanish, new ones appear**

<u>***Remedies:***</u>

- ***Annual re-sampling* of products for price observation**
- ***Replacement* of products in sample**
- ***Quality Adjustment* at replacement**
  - ***Various methods***

Replacement is restricted by product specifications

1) Tight product specifications
   Ex. ”Biscuits brand X, 300 g”
   + Strong theory, simple practice
   – May miss price changes

2) Loose product specifications
   Ex. ”Rye loaf 300-750 g, in slices”
   + Adapts to real world
   – Weak theory, hard practice

A basic dilemma

- Index has to follow basket – sample
  - *Representative sample*
  - *Laspeyres principle: Basket is fixed*
- But also, index should reflect the current market

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Structure changes

<u>*Example*</u>

- A firm in SPPI sample joins another by merger

<u>*Solution*</u>
<u>*– guided by Laspeyres principle*</u>

- Continue with prices from the new firm
- If both firms were in the sample, take the new firm's prices for both

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Re-sampling frequency

- Pros of frequent re-sampling
  - *Sample reflects current market*
  - *Adaptive to dynamic markets*
  - *Statistically scientifically correct*
- Pros of infrequent re-sampling
  - *Respondents get experience: easier for them + better response quality*
  - *(Controversial linking avoided)*

Cost Of Living Index (COLI)

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- Pertains to unchanged standard of living
- ***Ideal solution*:**
  Konüs index compares two baskets
- **Both baskets yield the same *utility* – at minimal cost**
  - ***Substitutions alter the basket***
- ***Practical solution*:**
  A fixed basket of a ”compromise” kind
  - ***Yields index that approximates coli!***

Target and accuracy of CPI

- *Target* of CPI is coli
- *Practical computation* is based on a suitable fixed basket
- *Statistical accuracy:* How closely the computation hits the target

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Theory of COLI

- Simplified assumption: 1 consumer

- In each period the consumer maximises her/his utility within a budget constraint

  ➡ *Theoretical utility function*

  $U(q_1, \ldots, q_G) = \max!$

- Index should reflect the development of cost for retaining a constant utility in the most cost-efficient way

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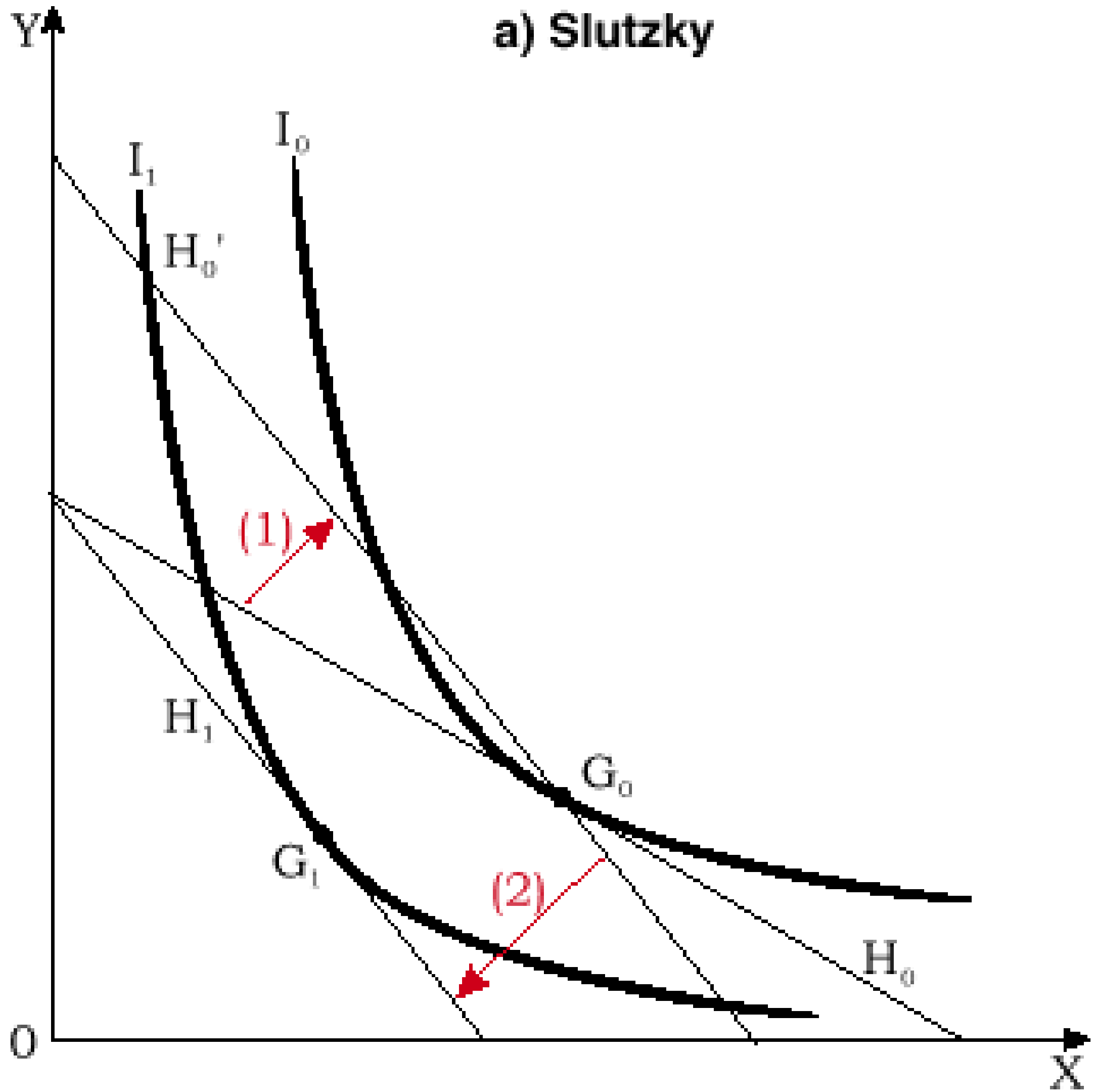
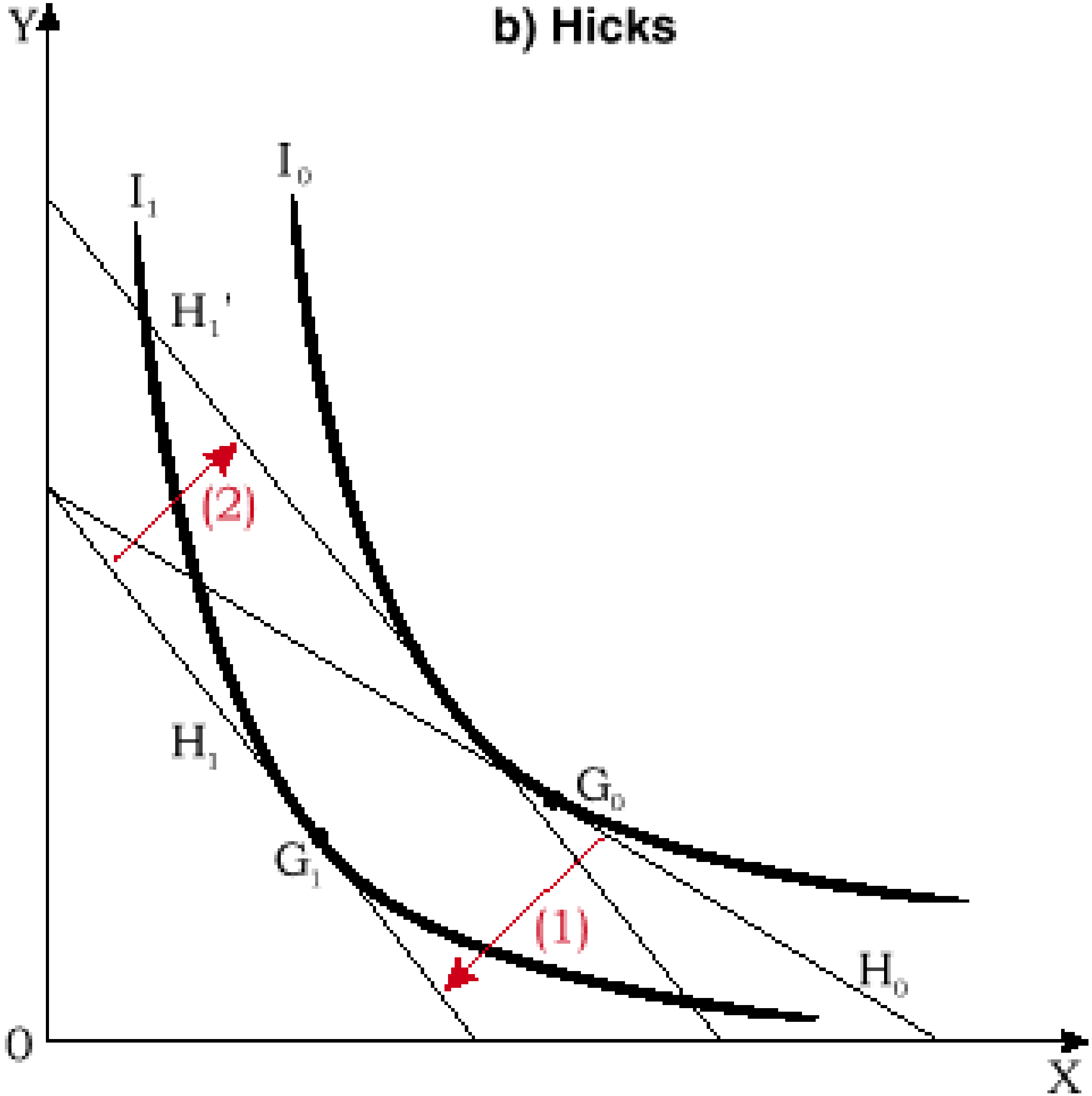

a) Slutzky
Y
X
0
I_0
I_1
H_0'
H_1
H_0
G_0
G_1
(1)
(2)



b) Hicks
Y
X
0
I1
I0
H1'
(2)
H1
G0
G1
(1)
H0

Superlative indices

- Fixed base indices that mimic coli
- *Exact index* – equals a constant-utility index for a specific utility function *U*
- *Superlative index* – is exact for a ”flexible” class of utility functions (Erwin Diewert’s teori)
- Examples:
  Fisher, Walsh, Törnqvist indices

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”Fisher’s ideal index”

$$I = \sqrt{\frac{\sum_i q_{0,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} \cdot \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{1,i}\, p_{0,i}}}$$

Laspeyres Paasche

- ***Symmetry between q_0 and q_1***

- **Variant: *Walsh index***

$$I = \frac{\sum_i \sqrt{q_{0,i}\, q_{1,i}}\; p_{1,i}}{\sum_i \sqrt{q_{0,i}\, q_{1,i}}\; p_{0,i}}$$

Walsh link over full year

$$I_{2004}^{2005} = \frac{\sum_i P_i^{2005} \times \sqrt{Q_i^{2004} \times Q_i^{2005}}}{\sum_i P_i^{2004} \times \sqrt{Q_i^{2004} \times Q_i^{2005}}} = \sum_g W_g \times I_{2004;g}^{2005}$$

where $$W_g = \frac{\sqrt{U_g^{2004} \times U_g^{2005}} \Big/ I_{2004;g}^{2005}}{\sum_{g'} \sqrt{U_{g'}^{2004} \times U_{g'}^{2005}} \Big/ I_{2004;g'}^{2005}}$$

Expenditure

Final Laspeyres link

$$I_{2005}^{2007,\,\text{Jan}} = \frac{\sum_i P_i^{2007,\,\text{Jan}} \times Q_i^{2005}}{\sum_i P_i^{2005} \times Q_i^{2005}} = \sum_g W'_g \times I_{2005;g}^{2007,\text{Jan}}$$

During 2007 weighting with expenditures of 2005.

More time for weight preparation ⇒
Improved accuracy, smoother process

Alternative annual links 1

Year	Lasperes	Paasche	Itix dec	Walsh approx.
1993	104,483	104,141	103,911	104,312
1994	102,177	102,006	102,291	102,088
1995	102,470	102,194	102,168	102,329
1996	100,945	100,579	99,823	100,757
1997	100,673	100,333	101,269	100,505
1998	100,129	99,844	99,555	99,989
1999	100,480	100,286	100,785	100,329
2000	100,942	100,731	101,152	100,848
2001	102,524	102,479	102,658	102,505
2002	102,245	101,987	102,168	102,124
Mean	101,707	101,458	101,578	101,579

Alternative annual links 2

Year	Walsh approx.	Walsh alt.	Edge-worth	Törn-qvist
1993	104,312	104,312	104,316	104,313
1994	102,088	102,089	102,093	102,088
1995	102,329	102,329	102,334	102,330
1996	100,757	100,755	100,764	100,754
1997	100,505	100,505	100,503	100,505
1998	99,989	99,988	99,988	99,989
1999	100,329	100,328	100,383	100,392
2000	100,848	100,847	100,837	100,843
2001	102,505	102,504	102,502	102,501
2002	102,124	102,123	102,118	102,127
Mean	101,579	101,578	101,584	101,584

Sub-indices by product group

$$I_{2004;g}^{2005} = \frac{I_{2003,\text{dec};g}^{2004,\text{dec}} \times \frac{1}{12}\sum_{m=1}^{12} I_{2004,\text{dec};g}^{2005,m}}{\frac{1}{12}\sum_{m=1}^{12} I_{2003,\text{dec};g}^{2004,m}}$$

- ***Transforms December base to year base***

$$I_{2005;g}^{2007,\text{jan}} = \frac{I_{2004,\text{dec};g}^{2005,\text{dec}}}{\frac{1}{12}\sum_{m=1}^{12} I_{2004,\text{dec};g}^{2005,m}} \times I_{2005,\text{dec};g}^{2006,\text{dec}} \times I_{2006,\text{dec};g}^{2007,\text{jan}}$$

New products come in soon

- *Treatment of group g that is new in2007:*

$$I_{2005;g}^{2007,\text{jan}} = \frac{I_{2004,\text{dec};g}^{2005,\text{dec}}}{\frac{1}{12}\sum_{m=1}^{12} I_{2004,\text{dec};g}^{2005,m}} \times I_{2005,\text{dec};g}^{2006,\text{dec}} \times I_{2006,\text{dec};g}^{2007,\text{jan}}$$

Imputed in years before

Priced in 2007

Index construction change for Swedish CPI from 2005

- *Previous construction – before 2005:*
  - **<u>Lower level</u>: ’RA-formula’**
  - **<u>Upper level</u>: ’Updated basket’ +Laspeyres type**
  - **A<u>nnual chaining</u>:**

- *New construction – from 2005:*
  - **<u>Lower level</u>: Geometric mean**
  - **<u>Upper level</u>: Walsh + Laspeyres**
  - **A<u>nnual chaining</u>:**

Some scope issues

<u>*Universes of purchase transactions*</u>

- ***Domestic* concept – purchases within the country (also by foreign visitors)**
- ***National* concept – purchases by residents of the country (also those made abroad)**

<u>*Aggregation principles*</u>

- ***Plutocratic* – weight by expenditure (usual)**
- ***Democratic* – weight by households/people**

<u>*Conditional coli*</u>

- **Constant environment assumed – heating cost raise by colder winter shall not be shown**

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Sources of expenditure data for weight computation

- Household Budget Survey (HBS)
  - *Suits National concept*
  - *Sampling errors*
  - *Often low response rate due to respondent burden*
- National Accounts
  - *Based on HBS, retail statistics etc.*
- Various complementary sources, such as industry organisation data

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Price updating of weights

$$I_{2006,\text{dec}}^{2007,\text{april}} = \frac{\sum_i q_{2005;i}\, p_{2007,\text{april};i}}{\sum_i q_{2005;i}\, p_{2006,\text{dec};i}} = \sum_i w_{2005;i} \cdot \frac{p_{2007,\text{april};i}}{p_{2006,\text{dec};i}}$$

Lowe index (HICP)

$$där \quad w_{2005;i} = \frac{q_{2005;i}\, p_{2005;i} \cdot \dfrac{p_{2006,\text{dec};i}}{p_{2005;i}}}{\sum_j q_{2005;j}\, p_{2005;i} \cdot \dfrac{p_{2006,\text{dec};j}}{p_{2005;j}}}$$

$$= \frac{U_{2005;i} \cdot I_{2005,i}^{2006,\text{dec}}}{\sum_j U_{2005;j} \cdot I_{2005,j}^{2006,\text{dec}}}$$

Value amount SEK

Price updating

Price updating questioned (?)

$$I_{2006,\text{dec}}^{2007,\text{april}} = \sum_i w_{2005;i} \cdot \frac{p_{2007,\text{april};i}}{p_{2006,\text{dec};i}}$$

$$= \sum_i w_{2005;i} \; I_{2006,\text{dec},i}^{2007,\text{april}}$$

- **Lowe-index:**
  - ***Follows a basket***
  - ***Conforms to HICP rules***

$$w_{2005;i} = \frac{U_{2005;i} \; I_{2005,i}^{2006,\text{dec}}}{\sum_j U_{2005;j} \; I_{2005,j}^{2006,\text{dec}}}$$

- **Young-index:**
  - ***Smaller bias (?)***

$$w_{2005;i} = \frac{U_{2005;i}}{\sum_j U_{2005;j}}$$

Missing prices

<u>*Causes:*</u>

- Non-response (refusal etc.)
- Seasonal product
- Model temporarily unavailable or not sold
- (Model permanently unavailable: replace)

<u>*Remedies, main alternatives:*</u>

1) Use preceding price ('carry forward')
   - *May currently miss price change*
2) Skip observation
   - *May yield volatility in index*

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Methods for seasonal products – ideas

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- Seasonal basket / Rothwell index
  - *Out-of-season products excluded*
- Counter-seasonal imputation
  - *Out-of-season products represen-ted by in-season seasonal products*
- All-seasonal imputation
  - *Out-of-season products represen-ted by available products*

Methods for seasonal products – properties

- **Seasonal basket index *and* Counter-seasonal imputation index *tend to have similar outcome – under condition of similarity in price curves for seasonal products***
- ***On the other hand, vast differences may occur without the condition***

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Axiomatic index theory 1

Index = function $P(p^0, p^1, q^0, q^1)$ *of price & volume vectors* p, q *given for times (periods)* 0 & 1

Axioms state desirable properties of P

Examples of axioms (tests):

- $P > 0$, **continuous function**
- **Identity test (unchanged prices)**

$$P(p, p, q^0, q^1) = 1$$

Axiomatic index theory 2

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***Further tests*:**

- **Proportionality in current prices**

$$P(p^0, \lambda p^1, q^0, q^1) = \lambda P(p^0, p^1, q^0, q^1)$$

- **Invariance under proportional volume changes**

$$P(p^0, p^1, q^0, \lambda q^1) = P(p^0, p^1, q^0, q^1)$$

Axiomatic index theory 3

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Further tests (continued):

- **Invariance in units of measurement**
- **Time reversal test**

$$P(p^0, p^1, q^0, q^1) = 1 / P(p^1, p^0, q^1, q^0)$$

- **Volume symmetry test**

$$P(p^0, p^1, q^0, q^1) = P(p^0, p^1, q^1, q^0)$$

- **Monotonicity test**

$$P(p^0, p^1, q^0, q^1) < P(p^0, p^2, q^0, q^1) \text{ if } p^1 < p^2$$

Axiomatic index theory 4

Even more tests:

- Fixed basket test

$$P(p^0, p^1, q, q) = \text{Lowe index, or}$$
$$= q\,p^1 \,/\, q\,p^0 \quad \text{(vector notation)}$$

- Consistency in aggregation

Stepwise aggregation should yield equal index number as direct aggregation

Axiomatic index theory 5

- Lots of reasonable axioms can be posed – choice among them may be considered arbitrary
- Impossible to pass all desirable tests
- ”Number of tests passed” is not really a valid quality score for an index

Axiomatic index theory 6

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- **Axioms are useful as whistle-blowers on drawbacks of index formulas**
  - ***Example: Carli index fails time reversal test in a severe way – this reveals bias!***

Actually, for Carli index,

$$P\,(p^0, p^1) \times P\,(p^1, p^0) \geq 1$$

with equality only exceptionally

Quality Adjustment, QA *(Kvalitetsvärdering)*

- To be made at product replacement in price collection
- Generally a difficult task
- Fashion variation is not quality change
- QA may have great impact on index
- Particularly difficult for unique products

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Value of quality difference

- Value of quality change shall not be shown as price change in index
  – shall be adjusted away
- *Consumer perspective* (CPI):
  Value of quality change is value of change in consumer utility
- *Producer perspective* (PPI, SPPI):
  Value of quality change is change in production cost at unchanged technology

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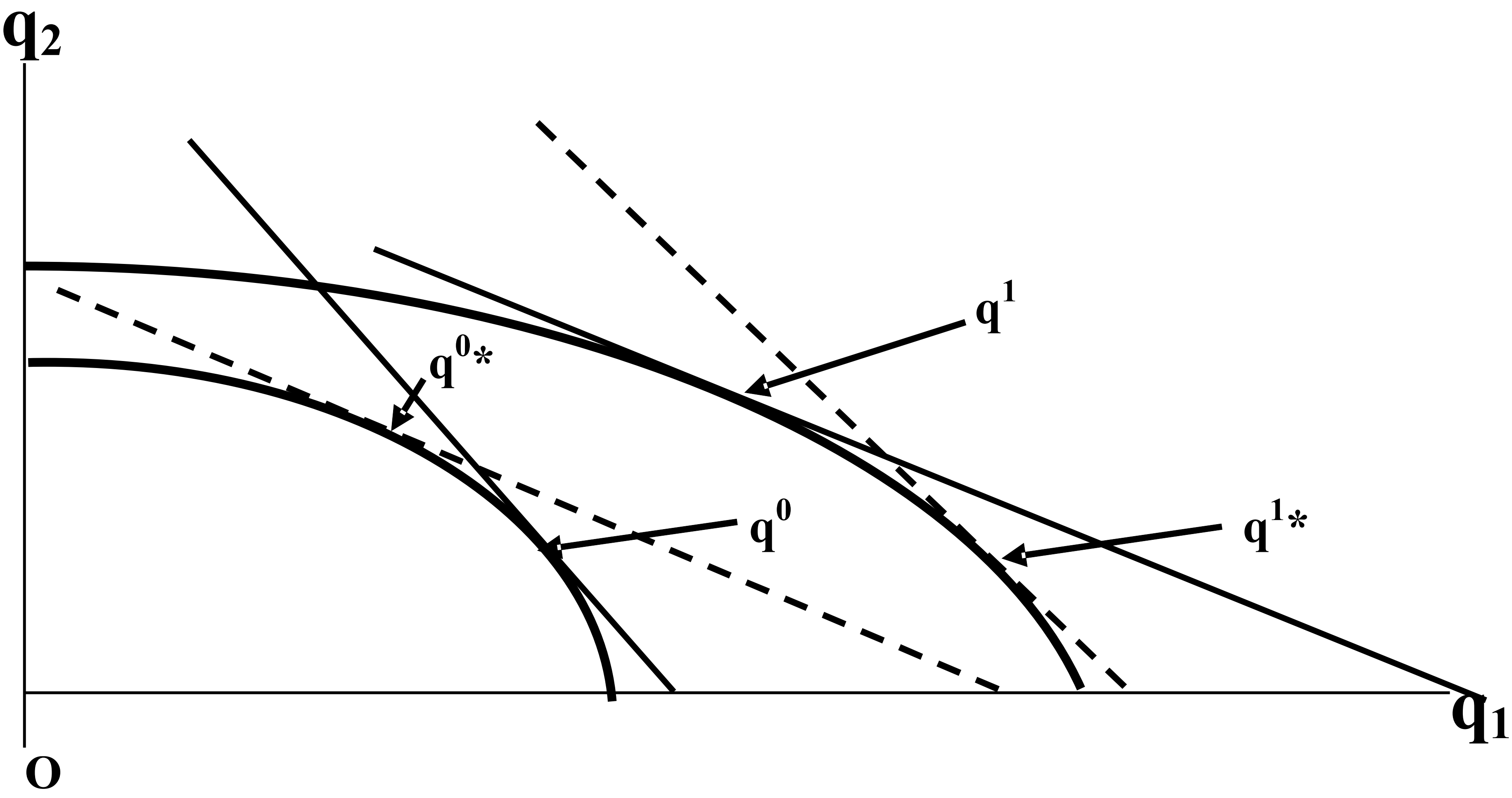
Output index



QA methods 1: "Explicit" methods

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- *These methods evaluate quality-related characteristics of products*

- Direct price comparison (same quality)
- Judgmental QA
- Quantity adjustment
- Production cost adjustment (suits PPI)
- "Option pricing"
- Hedonic regression
  - *Presently highly regarded method*

QA methods 2: ”Implicit” methods

- *These methods take value of quality difference as a diference in price*
- *Rely on ”revealed preference”*
- *”Objective” yet controversial*

- ”Bridged overlap”/Form of imputation
- ”Class mean imputation”
- ”Link to show no price change”
  - *”Banned” metod!*

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Judgmental QA – issues

☺ *Flexible – applicable in various areas*

☺ *Consumer perspective (though not ideal)*

☹ *"Subjective" – lacking control*

- Support for judgments is essential
  - *Criteria for appropriate support?*
- Empirical issue – how the method performs

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Product areas with Price Collector QA in Sweden

- Clothing material etc.
- Furniture, furnishings
- ”Other medical” goods
- Bicycles, car accessories
- Tv, radio, cameras, sports equipmt. etc.
- Canteen services etc. (some)
- ”Other effects” etc.

QA impact overall (per cent)

Year	Judg-mental	Bridged overlap	“Autom. linking”
1997	-0.69	0.08	-0.68
1998	-0.70	-0.44	-1.44
1999	-1.89	-1.24	-2.09
2000	-1.53	-2.33	-1.91
2001	-2.23	-2.50	-3.03
2002	-1.49	-0.79	-1.82

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Hedonic example 1

t = 1			t = 2			
Price	Size	Trait_A	Price	Size	Trait_A	Price relative
390	23	0	290	23	0	74,36
480	39	0	519	39	0	108,13
700	51	1	700	51	1	100,00
550	39	0	550	39	0	100,00
520	35	1	520	35	1	100,00
490	43	0	698	53	1	142,45

A replacement

Hedonic example 2

- **Regression equation (fitted for t = 1)**

$$\ln Price = 5.604 + $$
$$+ 0.0155 \times \text{Size} + 0.1331 \times \text{Trait\_A} + \varepsilon$$

- **Hedonic function**

$$Price = h(\text{Size}, \text{Trait\_A}) + r$$
$$= e^{5.604 + 0.0155 \times \text{Size} + 0.1331 \times \text{Trait\_A}} + r$$

Hedonic example 3

- **Quality change factor for replacement:**

$$g = \frac{h\,(\text{Size of replacement model, Trait\_A of replacement model})}{h\,(\text{Size of replaced model, Trait\_A of replaced model})}$$

$$= e^{0.0155 \times (53 - 43) + 0.1331 \times (1 - 0)} = 1.3339$$

Hedonic example 4

- **Index computation with hedonic quality adjustment:**

$$g = e^{0.0155 \times (53-43) + 0.1331 \times (1-0)} = 1.3339$$

$$I = \left(\frac{290}{390} \times \frac{519}{480} \times \frac{700}{700} \times \frac{550}{550} \times \frac{520}{520} \times \frac{698}{490 \times 1.3339} \right)^{1/6} \times 100$$

$$= \mathbf{97.49}$$

Hedonic equation (”model”)

- **Example – ”semi-logarithmic” form**

$$\ln P = b_0 + b_1 z_1 + b_2 z_2 + \ldots + b_k z_k + \varepsilon$$

Hedonic Regression
obs. (n), # regressors (p)

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- *Heuristics*

$$\operatorname{var} \hat{y}_i = \sigma^2 h_i$$

$$\text{where } h_i = x_i^T (X^T X)^{-1} x_i$$

  - **Fact:** $\frac{1}{n} \sum_{i=1}^{n} h_i = p / n$

- *Rule of thumb (?)*
  - **Demand ≥20 obs. / regressor (or so, effectively)**

Insurance: Adjustment for excess

- *Actuarial risk premium at excess b is*

$$r(0) \int_b^\infty (x - b)\, dF(x)$$

Rate of damages > 0

Damage distribution

- *If the excess is raised from b to c then the risk premium falls by*

$$r(b')\,(c - b)\,, \qquad b \le b' \le c$$

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Insurance: Gross vs net principle 1

Gross premium
\+ Premium supplements (yield on reserves)
– Claims
– Changes in actuarial provisions

= Service charge (Net premium)

Insurance: Gross vs net principle 2

- Gross premium
  - *Adequate for compensation index*
- Service charge (Net premium)
  - *Prescribed for NA & HICP*
  - *Can be used only for weights*
  - *Then acceptable proxy also for compensation index*

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Banking services: Delineation of coverage

- Exclusion of FISIM (Financial Inter-mediation Services Indirectly Measured)
  - *Only part of price is seen*
    - *Could give artificial index changes*
- Currency exchange is implicitly charged
  - *Is FISIM by HICP rules*

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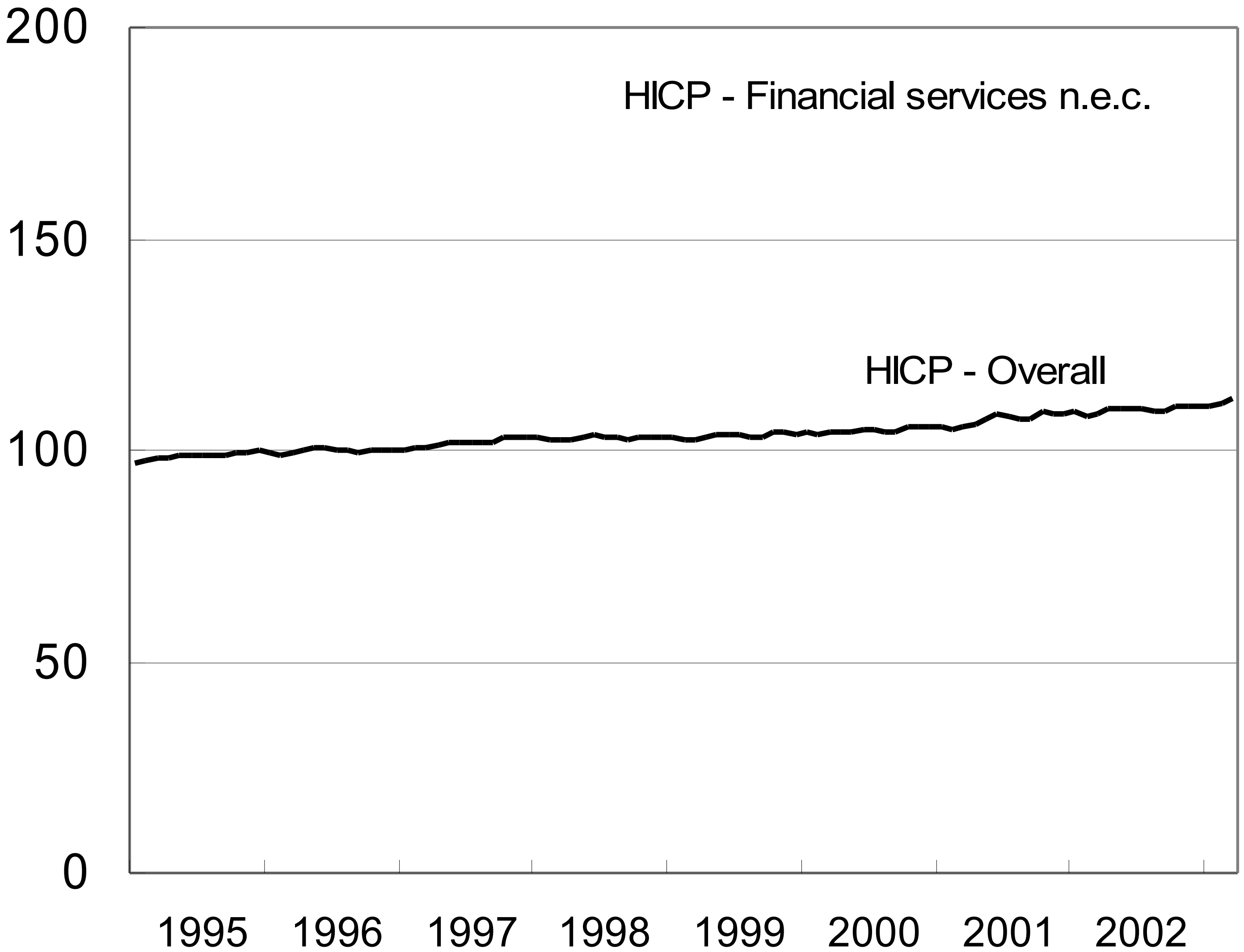
Banking services: HICP outcome



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Owner Occupied Housing: Alternative approaches

- Exclusion of capital part (the house)
- (Net) Aquisition Approach
  - *”Houses like potatoes”*
- Rental Equivalent Approach
  - *Appealing, but depends on rents*
- User Cost Approach
  - *Variants: partial cost*
- Payment Approach

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Owner Occupied Housing

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- *Swedish CPI:*
  - Depreciation
  - Interest cost
  - Real estate tax
  - Site rent
  - Repairs
  - Insurance
  - Water, etc.
  - Oil, Electricity

- *HICP – plan:*
  - Purchase of new houses
  - Repairs
  - Insurance
  - Water, etc.
  - Oil, Electricity

Interest cost

- Interest on mortgage + equity
  - *On mortgage = Interest payment*
    On equity = Opportunity cost
- Rates of interest on mortgages of different types
- Based on a capital equal to present owner's purchase price
- Interest cost deducted in underlying inflation

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Interest cost index

$$I = RS \cdot KS$$

Interest rate index (RS) — *Capital stock index* (KS)

$$RS_{01} = \frac{\sum_i w_i^{RS} \overline{R}_i^{1}}{\sum_i w_i^{RS} \overline{R}_i^{0}}$$

Average rate, mortgage type i ($\overline{R}_i$)

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Depreciation

- Loss of value due to wear etc.
- Weight = 1,4 % of market value
- *Before 1999*: Building Price Index (BPI), updated by a Factor Price Index
- *From 1999*: Price index for ”major” repairs
  = 0,7 × (price index for material) +
  0,3 × (price index for labour)
  ↳ *A wage index, adjusted*

Re-considerations

- How to find the true cost of having your own home?

- *Recent CPI Commission suggested:*
  Real interest of housing, on market value of house, at interest rate assumed constant
  ↳ *Severely criticised*

- *In Government Budget Proposal 2002:*
  Urgent to improve the computations – the CPI Board should consider the issue

Owner occupied housing: Capital cost

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Present CPI:

- **Depreciation**
- **Interest of mortgages and capital (current market rates)**

Proposal of recent CPI Commission:

- **Depreciation**
- **Real interest of housing, rate taken constant**

Cost prop. to market value of house

A general expression for the capital cost

Market price → P_t

Depreciation rate → d_t

$$C_t = P_t\,(r_t + d_t - \pi_t)$$

Nominal interest rate → r_t

House inflation rate → π_t

Commission Index Proposal

Statistics Sweden

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$$\frac{C_{t+1}}{C_t} = \frac{P_{t+1}(r_t + d_t - \pi_t)}{P_t(r_t + d_t - \pi_t)} = \frac{P_{t+1}}{P_t}$$

Dynamic approach to OOH: Consumer's utility

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Model by A. Klevmarken – consumer's utlility is a function of:

- Consumption of other products
- Housing in rented dwelling
- Owned dwelling at period start
- Owned dwelling at period end
- Financial assets & debts, per. end

Dynamic approach to OOH: Consumer's budget

Income components:

- Labour income
- Capital income
- Net savings withdrawals
- Net new loans

Income is to cover:

- Cost for other consumption (than housing)
- Cost for rents
- Cost for repairs / maintenance
- Cost for loan interest
- Cost for new construction, extensions etc.

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Dynamic approach to OOH: Components concerned

- *Present approach*
  - *Interest cost*
  - *Depreciation*
  - *Repairs, goods*
  - *Repairs, services (– year 2000)*
- *New approach*
  - *Interest cost – new form*
  - *Repairs – new form*
  - *New construction*

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Dynamic approach to OOH: Interest cost alternatives

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- A – At constant nominal loan
- B – At constant real loan
- C – At constant duration of ownership & constant loan share

Dynamic approach to OOH: Interest cost units

- A – $ interest per $ loan
- B – $ interest per house unit with current value covered by loan
- C – $ interest per house unit with purchase value covered by loan

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Swedish core inflation (underlying inflation)

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- Alternative measures of inflation for use in monetary policy
- General idea: To capture price change except changes of temporary/transitional or exogenous kind
- *KPIX / CPIX* measure of core inflation – defined by Sveriges Riksbank and produced monthly by Statistics Sweden

Core inflation measures

- *KPIX / CPIX (formerly called UND1X)*
  - shows price change *except* changes in:
    - Owner occupiers' interest cost
    - Indirect taxes & subsidies
- *UNDINHX (recently discontinued)*
  - shows price change *except* changes in:
    - Owner occupiers' interest cost
    - Indirect taxes & subsidies
    - Prices of mainly imported products

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Index of a tax *j*

$$I^{1}_{S^{1:1}_{j}} = \frac{t^{1}_{j} \times (1 + K^{1})}{t^{0}_{j} \times (1 + K^{0})}$$

Tax rate as tax per unit

VAT rate

- Used for Net Price Index (NPI) and CPIX

Year-to-year link of CPIX

$$\mathrm{CPIX}_{2003}^{2004} = \mathrm{CPI}_{2003;\mathrm{excl.interestcost}}^{2004} -$$

$$- \sum_{k \in T\&S} W_k^{2004} \times \Delta I_{2003;k}^{2004}$$

Taxes & Subsidies

Change in index of tax/subsidy *k*

Walsh weight of a tax *k*

Tax revenues

$$W_k^{2004} = \frac{\sqrt{U_k^{2003} \times U_k^{2004} / I_{2003;k}^{2004}}}{\sum_{g \neq \text{i.c.}} \sqrt{U_g^{2003} \times U_g^{2004} / \text{CPI}_{2003;g}^{2004}}}$$

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Year-to-month link of CPIX

$$\mathrm{CPIX}_{2004}^{2006;\mathrm{May}} = \mathrm{CPI}_{2004;\mathrm{excl.i.c.}}^{2006,\mathrm{May}} -$$

$$- \sum_{k \in T\&S} W_k^{2006} \times \Delta I_{2004;k}^{2006;\mathrm{May}}$$