PRICE INDEX THEORY

Statistiska centralbyrån Statistics Sweden

Course lectures within Economic Statistics
at Stockholm University
Part 4

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Autumn 2011

Missing prices

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Causes:

- **Non-response (refusal etc.)**
- **Seasonal product**
- **Model temporarily unavailable or not sold**
- **Model permanently unavailable: replace**

Remedies, main alternatives:

1) **Use preceding price (’carry forward’)**
   - ***May currently miss price change***
2) **Skip observation**
   - ***May yield volatility in index***

Methods for seasonal products – ideas

- **Seasonal basket / Rothwell index**
  - ***Out-of-season products excluded***
- **Counter-seasonal imputation**
  - ***Out-of-season products represented by in-season seasonal products***
- **All-seasonal imputation**
  - ***Out-of-season products represented by available products***

Methods for seasonal products – properties

- **Seasonal basket index *and* Counter-seasonal imputation index *tend to have similar outcome – under condition of similarity in price curves for seasonal products***
- ***On the other hand, vast differences may occur without the condition***

Axiomatic index theory 1

Index = function $P(p^0, p^1, q^0, q^1)$ *of price & volume vectors* p, q *given for times (periods)* 0 & 1

Axioms state desirable properties of P

Examples of axioms (tests):

- $P > 0$, continuous function
- **Identity test (unchanged prices)**

$$P(p, p, q^0, q^1) = 1$$

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Axiomatic index theory 2

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Further tests:

- **Proportionality in current prices**

$$P(p^0, \lambda p^1, q^0, q^1) = \lambda P(p^0, p^1, q^0, q^1)$$

- **Invariance under proportional volume changes**

$$P(p^0, p^1, q^0, \lambda q^1) = P(p^0, p^1, q^0, q^1)$$

Axiomatic index theory 3

Further tests (continued):

- **Invariance in units of measurement**
- **Time reversal test**

$$P(p^0, p^1, q^0, q^1) = 1 / P(p^1, p^0, q^1, q^0)$$

- **Volume symmetry test**

$$P(p^0, p^1, q^0, q^1) = P(p^0, p^1, q^1, q^0)$$

- **Monotonicity test**

$$P(p^0, p^1, q^0, q^1) < P(p^0, p^2, q^0, q^1) \text{ if } p^1 < p^2$$

Axiomatic index theory 4

Even more tests:

- **Fixed basket test**

$$P(p^0, p^1, q, q) = \text{Lowe index, or}$$
$$= q\,p^1 \,/\, q\,p^0 \quad \text{(vector notation)}$$

- **Transitivity (in full form – too demanding)**

$$P(p^0, p^2, q^0, q^2) = P(p^0, p^1, q^0, q^1) \times P(p^1, p^2, q^1, q^2)$$

- **Consistency in aggregation**

Stepwise aggregation should yield equal index number as direct aggregation

Axiomatic index theory 5

- Lots of reasonable axioms can be posed – choice among them may be considered arbitrary
- Impossible to pass all desirable tests
- ”Number of tests passed” is not really a valid quality score for an index

Axiomatic index theory 6

- **Axioms are useful as whistle-blowers on drawbacks of index formulas**
  - ***Example: Carli index fails time reversal test in a severe way – this reveals bias!***

Actually, for Carli index,

$$P(p^0, p^1) \times P(p^1, p^0) \geq 1$$

with equality only exceptionally

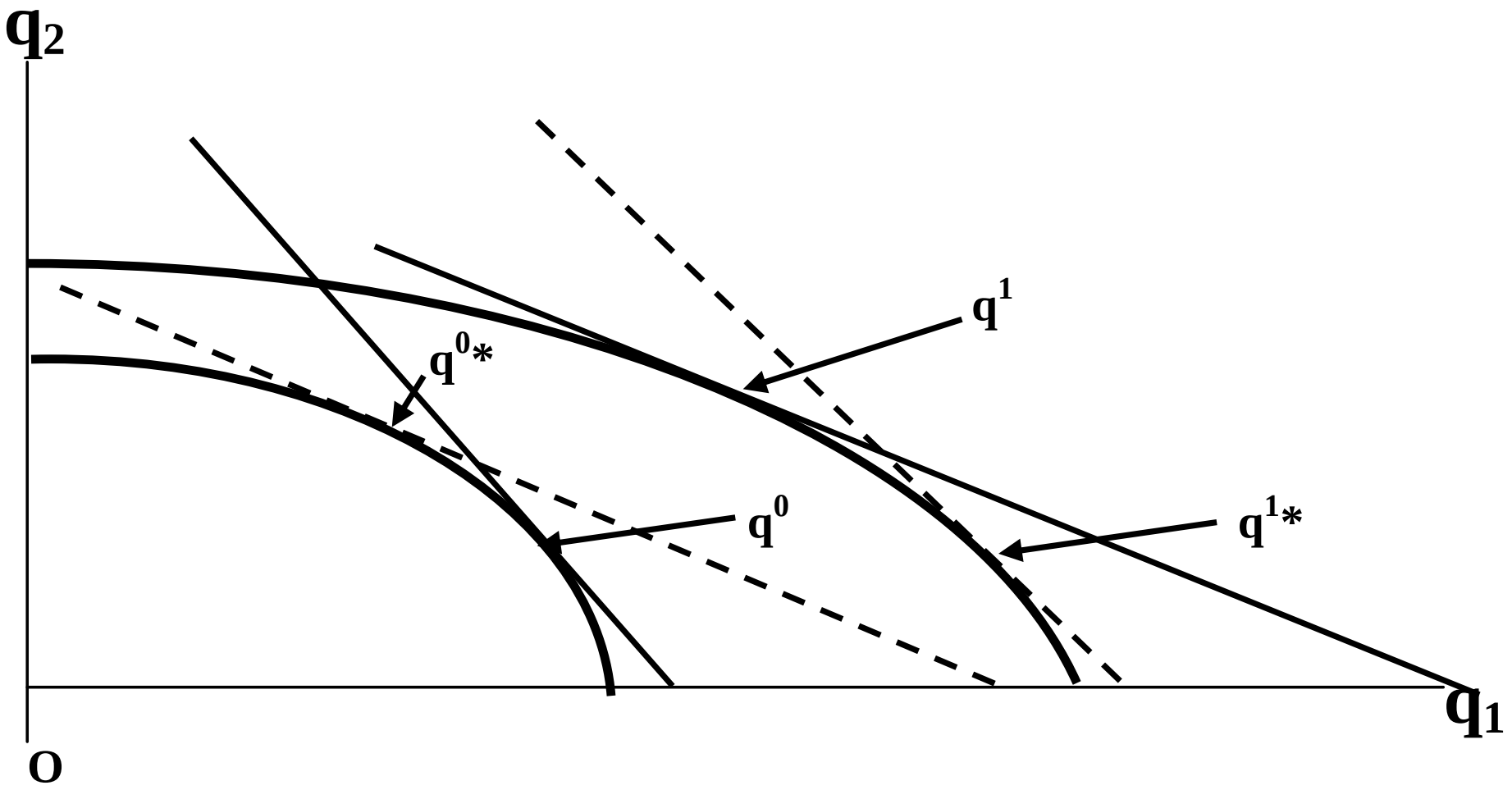
Quality Adjustment, QA *(Kvalitetsvärdering)*

- To be made at product replacement in price collection
- Generally a difficult task
- Fashion variation is not quality change
- QA may have great impact on index
- Particularly difficult for unique products

Value of quality difference

- Value of quality change shall not be shown as price change in index – shall be adjusted away
- *Consumer perspective* (CPI): Value of quality change is value of change in consumer utility
- *Producer perspective* (PPI, SPPI): Value of quality change is change in production cost at unchanged technology

Output index



QA methods 1: ”Explicit” methods

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- *These methods evaluate quality-related characteristics of products*
- **Direct price comparison (same quality)**
- **Judgmental QA**
- **Quantity adjustment**
- **Production cost adjustment (suits PPI)**
- **”Option pricing”**
- **Hedonic regression**
  - *Presently highly regarded method*

QA methods 2: ”Implicit” methods

- *These methods take value of quality difference as a diference in price*
- *Rely on ”revealed preference”*
- *”Objective” yet controversial*

- ”Bridged overlap”/Form of imputation
- ”Class mean imputation”
- ”Link to show no price change”
  - *”Banned” metod!*

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Judgmental QA – issues

☺ *Flexible – applicable in various areas*

☺ *Consumer perspective (though not ideal)*

☹ *”Subjective” – lacking control*

- **Support for judgments is essential**
  - ↳ *Criteria for appropriate support?*

- **Empirical issue – how the method performs**

Product areas with Price Collector QA in Sweden

- Clothing material etc.
- Furniture, furnishings
- ”Other medical” goods
- Bicycles, car accessories
- Tv, radio, cameras, sports equipmt. etc.
- Canteen services etc. (some)
- ”Other effects” etc.

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QA impact overall (per cent)

Year	Judg-mental	Bridged overlap	"Autom. linking"
1997	-0.69	0.08	-0.68
1998	-0.70	-0.44	-1.44
1999	-1.89	-1.24	-2.09
2000	-1.53	-2.33	-1.91
2001	-2.23	-2.50	-3.03
2002	-1.49	-0.79	-1.82

Hedonic example 1

t = 1			t = 2			Price
Price	Size	Trait_A	Price	Size	Trait_A	relative
390	23	0	290	23	0	74,36
480	39	0	519	39	0	108,13
700	51	1	700	51	1	100,00
550	39	0	550	39	0	100,00
520	35	1	520	35	1	100,00
490	43	0	698	53	1	142,45

A replacement

Hedonic example 2

- **Regression equation (fitted for t = 1)**

$$\ln Price = 5.604 + $$

$$+ 0.0155 \times \text{Size} + 0.1331 \times \text{Trait\_A} + \varepsilon$$

- **Hedonic function**

$$Price = h(\text{Size}, \text{Trait\_A}) + r$$

$$= e^{5.604 + 0.0155 \times \text{Size} + 0.1331 \times \text{Trait\_A}} + r$$

Hedonic example 3

- **Quality change factor for replacement:**

$$g = \frac{h(\text{Size of replacement model}, \text{Trait\_A of replacement model})}{h(\text{Size of replaced model}, \text{Trait\_A of replaced model})}$$

$$= e^{0.0155 \times (53-43) + 0.1331 \times (1-0)} = 1.3339$$

Hedonic example 4

- **Index computation with hedonic quality adjustment:**

$$g = e^{0.0155 \times (53-43) + 0.1331 \times (1-0)} = 1.3339$$

$$I = \left(\frac{290}{390} \times \frac{519}{480} \times \frac{700}{700} \times \frac{550}{550} \times \frac{520}{520} \times \frac{698}{490 \times 1.3339} \right)^{1/6} \times 100$$

$$= \mathbf{97.49}$$

Hedonic equation (”model”)

- **Example – ”semi-logarithmic” form**

$$\ln P = b_0 + b_1 z_1 + b_2 z_2 + \ldots + b_k z_k + \varepsilon$$

Hedonic Regression
obs. (n), # regressors (p)

- ***Heuristics***

$$\operatorname{var} \hat{y}_i = \sigma^2 h_i$$

$$\text{where } h_i = x_i^T (X^T X)^{-1} x_i$$

Fact: $$\frac{1}{n} \sum_{i=1}^{n} h_i = p/n$$

- ***Rule of thumb (?)***

Demand ≥20 obs. / regressor

(or so, effectively)