PRICE INDEX THEORY

Statistiska centralbyrån Statistics Sweden

Course lectures within Economic Statistics
at Stockholm University
Part 3

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Cost Of Living Index (COLI)

- Pertains to unchanged standard of living
- ***Ideal solution:***
  Konüs index compares two baskets
- Both baskets yield the same *utility* – at minimal cost
  - ↳ ***Substitutions alter the basket***
- ***Practical solution:***
  A fixed basket of a ”compromise” kind
  - ↳ ***Yields index that approximates coli!***

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Target and accuracy of CPI

- *Target* of CPI is coli
- *Practical computation* is based on a suitable fixed basket
- *Statistical accuracy:* How closely the computation hits the target

Theory of COLI

- Simplified assumption: 1 consumer
- In each period the consumer maximises her/his utility within a budget constraint
  - ↳ *Theoretical utility function*
    $U(q_1, \ldots, q_G) = \max!$
- Index should reflect the development of cost for retaining a constant utility in the most cost-efficient way

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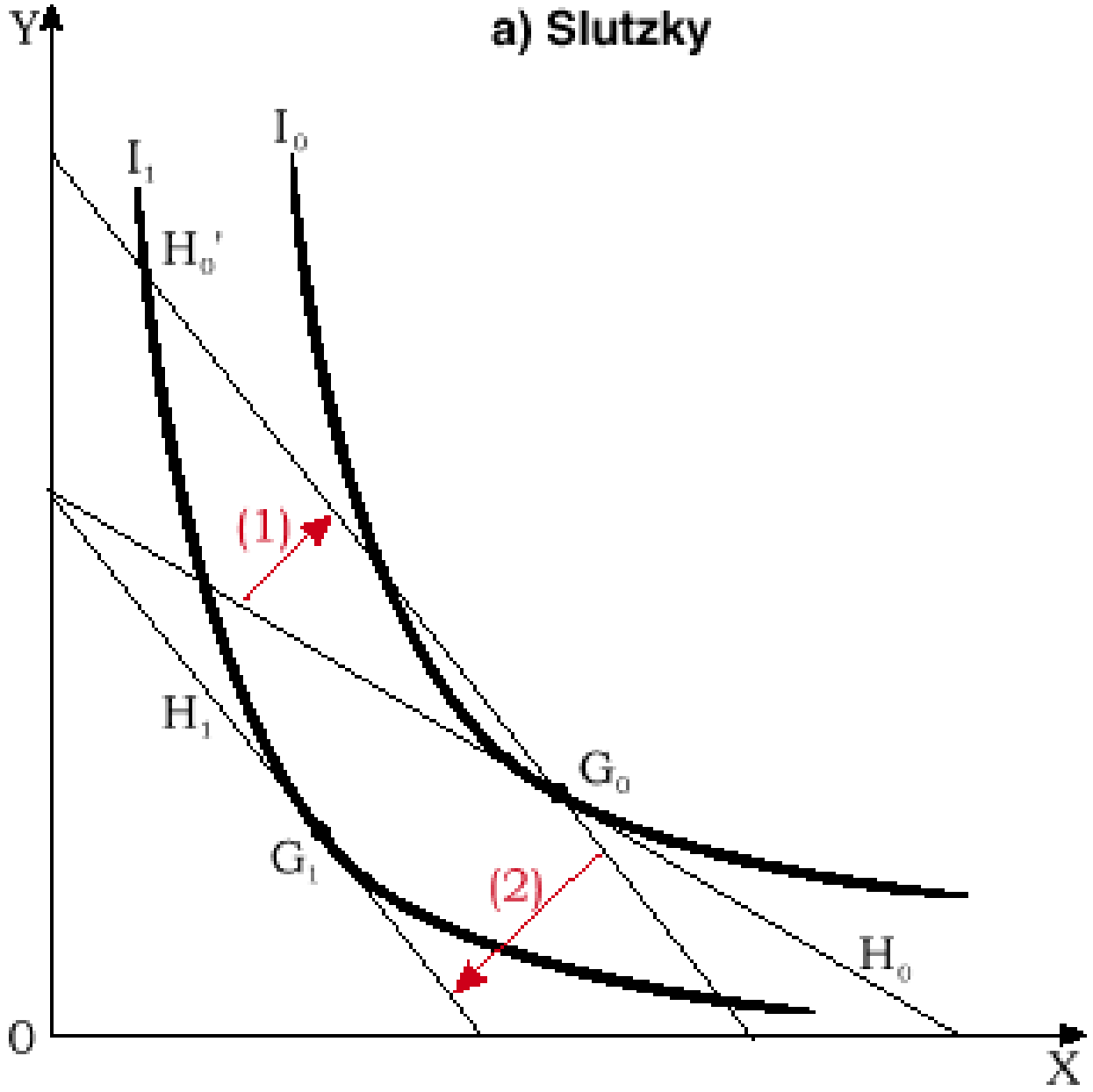
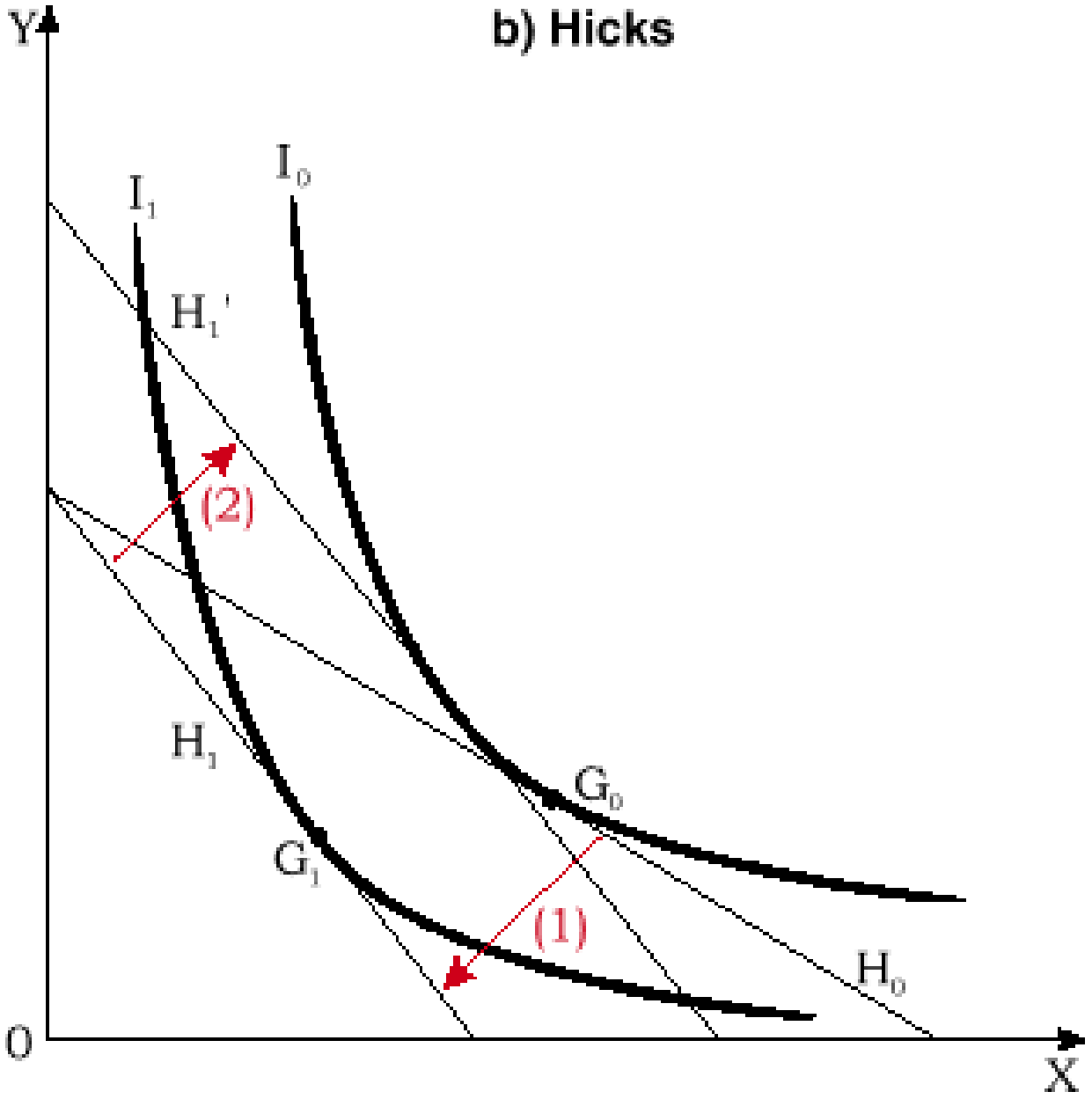

a) Slutzky
Y
X
0
I_0
I_1
H_0'
(1)
H_1
G_0
G_1
(2)
H_0


b) Hicks
Y
X
0
I_0
I_1
H_1'
H_1
H_0
G_0
G_1
(1)
(2)

Superlative indices

- Fixed base indices that mimic coli
- *Exact index* – equals a constant-utility index for a specific utility function *U*
- *Superlative index* – is exact for a ”flexible” class of utility functions (Erwin Diewert’s teori)
- Examples:
  Fisher, Walsh, Törnqvist indices

”Fisher’s ideal index”

$$I = \sqrt{\frac{\sum_i q_{0,i}\ p_{1,i}}{\sum_i q_{0,i}\ p_{0,i}} \cdot \frac{\sum_i q_{1,i}\ p_{1,i}}{\sum_i q_{1,i}\ p_{0,i}}}$$

Laspeyres Paasche

- ***Symmetry between q_0 and q_1***

- **Variant: *Walsh index***

$$I = \frac{\sum_i \sqrt{q_{0,i}\ q_{1,i}}\ p_{1,i}}{\sum_i \sqrt{q_{0,i}\ q_{1,i}}\ p_{0,i}}$$

Walsh link over full year

$$I_{2004}^{2005} = \frac{\sum_i P_i^{2005} \times \sqrt{Q_i^{2004} \times Q_i^{2005}}}{\sum_i P_i^{2004} \times \sqrt{Q_i^{2004} \times Q_i^{2005}}} = \sum_g W_g \times I_{2004;g}^{2005}$$

where $$W_g = \frac{\sqrt{U_g^{2004} \times U_g^{2005}} \Big/ I_{2004;g}^{2005}}{\sum_{g'} \sqrt{U_{g'}^{2004} \times U_{g'}^{2005}} \Big/ I_{2004;g'}^{2005}}$$

Expenditure

Final Laspeyres link

$$I_{2005}^{2007,\,\mathrm{Jan}} = \frac{\sum_i P_i^{2007,\,\mathrm{Jan}} \times Q_i^{2005}}{\sum_i P_i^{2005} \times Q_i^{2005}} = \sum_g W'_g \times I_{2005;g}^{2007,\mathrm{Jan}}$$

During 2007 weighting with expenditures of 2005.
More time for weight preparation ⇒
Improved accuracy, smoother process

Alternative annual links 1

Year	Lasperes	Paasche	Itix dec	Walsh approx.
1993	104,483	104,141	103,911	104,312
1994	102,177	102,006	102,291	102,088
1995	102,470	102,194	102,168	102,329
1996	100,945	100,579	99,823	100,757
1997	100,673	100,333	101,269	100,505
1998	100,129	99,844	99,555	99,989
1999	100,480	100,286	100,785	100,329
2000	100,942	100,731	101,152	100,848
2001	102,524	102,479	102,658	102,505
2002	102,245	101,987	102,168	102,124
Mean	101,707	101,458	101,578	101,579

Alternative annual links 2

Year	Walsh approx.	Walsh alt.	Edge-worth	Törn-qvist
1993	104,312	104,312	104,316	104,313
1994	102,088	102,089	102,093	102,088
1995	102,329	102,329	102,334	102,330
1996	100,757	100,755	100,764	100,754
1997	100,505	100,505	100,503	100,505
1998	99,989	99,988	99,988	99,989
1999	100,329	100,328	100,383	100,392
2000	100,848	100,847	100,837	100,843
2001	102,505	102,504	102,502	102,501
2002	102,124	102,123	102,118	102,127
Mean	101,579	101,578	101,584	101,584

Sub-indices by product group

$$I^{2005}_{2004;g} = \frac{I^{2004,dec}_{2003,dec;g} \times \frac{1}{12}\sum_{m=1}^{12} I^{2005,m}_{2004,dec;g}}{\frac{1}{12}\sum_{m=1}^{12} I^{2004,m}_{2003,dec;g}}$$

➢ ***Trans-forms December base to year base***

$$I^{2007,jan}_{2005;g} = \frac{I^{2005,dec}_{2004,dec;g}}{\frac{1}{12}\sum_{m=1}^{12} I^{2005,m}_{2004,dec;g}} \times I^{2006,dec}_{2005,dec;g} \times I^{2007,jan}_{2006,dec;g}$$

New products come in soon

- ***Treatment of group g that is new in2007:***

$$I_{2005;g}^{2007,\text{jan}} = \frac{I_{2004,\text{dec};g}^{2005,\text{dec}}}{\frac{1}{12}\sum_{m=1}^{12} I_{2004,\text{dec};g}^{2005,m}} \times I_{2005,\text{dec};g}^{2006,\text{dec}} \times I_{2006,\text{dec};g}^{2007,\text{jan}}$$

Imputed in years before

Priced in 2007

Index construction change for Swedish CPI from 2005

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- ***Previous construction – before 2005:***
  - Lower level: ’RA-formula’
  - Upper level: ’Updated basket’ +Laspeyres type
  - Annual chaining: By December

- ***New construction – from 2005:***
  - Lower level: Geometric mean
  - Upper level: Walsh + Laspeyres
  - Annual chaining: By full year

Some scope issues

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Universes of purchase transactions

- *Domestic* concept – purchases within the country (also by foreign visitors)
- *National* concept – purchases by residents of the country (also those made abroad)

Aggregation principles

- *Plutocratic* – weight by expenditure (usual)
- *Democratic* – weight by households/people

Conditional coli

- Constant environment assumed – heating cost raise by colder winter shall not be shown

Sources of expenditure data for weight computation

- Household Budget Survey (HBS)
  - *Suits National concept*
  - *Sampling errors*
  - *Often low response rate due to respondent burden*
- National Accounts
  - *Based on HBS, retail statistics etc.*
- Various complementary sources, such as industry organisation data

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Price updating of weights

$$I_{2006,dec}^{2007,april} = \frac{\sum_i q_{2005;i}\ p_{2007,april;i}}{\sum_i q_{2005;i}\ p_{2006,dec;i}} = \sum_i w_{2005;i} \cdot \frac{p_{2007,april;i}}{p_{2006,dec;i}}$$

Lowe index (HICP)

$$\text{där}\quad w_{2005;i} = \frac{q_{2005;i}\ p_{2005;i} \cdot \dfrac{p_{2006,dec;i}}{p_{2005;i}}}{\sum_j q_{2005;j}\ p_{2005;i} \cdot \dfrac{p_{2006,dec;j}}{p_{2005;j}}}$$

Value amount SEK

$$= \frac{U_{2005;i} \cdot I_{2005,i}^{2006,dec}}{\sum_j U_{2005;j} \cdot I_{2005,j}^{2006,dec}}$$

Price updating

Price updating questioned (?)

$$I_{2006,\mathrm{dec}}^{2007,\mathrm{april}} = \sum_i w_{2005;i} \cdot \frac{p_{2007,\mathrm{april};i}}{p_{2006,\mathrm{dec};i}}$$

$$= \sum_i w_{2005;i} \; I_{2006,\mathrm{dec};i}^{2007,\mathrm{april}}$$

- **Lowe index:**
  - ***Follows a basket***
  - ***Conforms to HICP rules***

$$w_{2005;i} = \frac{U_{2005;i} \; I_{2005,i}^{2006,\mathrm{dec}}}{\sum_j U_{2005;j} \; I_{2005,j}^{2006,\mathrm{dec}}}$$

- **Young index:**
  - ***Smaller bias (?)***

$$w_{2005;i} = \frac{U_{2005;i}}{\sum_j U_{2005;j}}$$