PRICE INDEX THEORY

Course lectures within Economic Statistics
at Stockholm University
[Partial set of slides]

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Fixed basket price index 1

$$I = \frac{\sum_{i=1}^{n} q_i \; p_{1,i}}{\sum_{i=1}^{n} q_i \; p_{0,i}} = \frac{q_1 \; p_{1,1} + q_2 \; p_{1,2} + \ldots + q_n \; p_{1,n}}{q_1 \; p_{0,1} + q_2 \; p_{0,2} + \ldots + q_n \; p_{0,n}}$$

- **Variables**

  q = quantity (volume)

  p = price

- **Objects and times**

  i = Product (good/service from given seller)

  0 = Base period (alias: reference period)

  1 = Current period

Fixed basket price index 2

$$I = \frac{\sum_{i=1}^{n} q_i \; p_{1,i}}{\sum_{i=1}^{n} q_i \; p_{0,i}} = \frac{q_1 \; p_{1,1} + q_2 \; p_{1,2} + \ldots + q_n \; p_{1,n}}{q_1 \; p_{0,1} + q_2 \; p_{0,2} + \ldots + q_n \; p_{0,n}}$$

Exempel

$$I = \frac{50 \times 98 + 100 \times 49 + 20 \times 195}{50 \times 88 + 100 \times 48 + 20 \times 195} \times 100 = 104.6$$

Price and volume indices

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Price index

$$I = \frac{\sum_i q_{0,i}\ p_{1,i}}{\sum_i q_{0,i}\ p_{0,i}}$$

Laspeyres

$$I = \frac{\sum_i q_{1,i}\ p_{1,i}}{\sum_i q_{1,i}\ p_{0,i}}$$

Paasche

Volume index

$$I = \frac{\sum_i q_{1,i}\ p_{0,i}}{\sum_i q_{0,i}\ p_{0,i}}$$

Laspeyres

$$I = \frac{\sum_i q_{1,i}\ p_{1,i}}{\sum_i q_{0,i}\ p_{1,i}}$$

Paasche

Factors of a value index

$$\frac{\sum_i q_{1,i}\, p_{0,i}}{\sum_i q_{0,i}\, p_{0,i}} \cdot \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{1,i}\, p_{0,i}} = \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} = \frac{\text{Total value}(1)}{\text{Total value}(0)}$$

Volume index × Price index = Value index

Laspeyres Paasche

$$\frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{1,i}} \cdot \frac{\sum_i q_{0,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} = \frac{\sum_i q_{1,i}\, p_{1,i}}{\sum_i q_{0,i}\, p_{0,i}} = \frac{\text{Total value}(1)}{\text{Total value}(0)}$$

Paasche Laspeyres

Practical uses

- ***Deflating* is to compute**

$$\text{Volume index} = \frac{\text{Value index}}{\text{Price index}}$$

↳ ***Eliminates price change***

- ***Implicit price index* is computed as**

$$\text{Price index} = \frac{\text{Value index}}{\text{Volume index}}$$

'Laspeyres type' (Lowe-index)

$$I_{2006,\mathrm{Dec}}^{2007,\mathrm{April}} = \frac{\sum_i q_{2005;i}\ p_{2007,\mathrm{April};i}}{\sum_i q_{2005;i}\ p_{2006,\mathrm{Dec};i}}$$

- A useful generalisation of Laspeyres index
- Example: Annual link in HICP (Harmonised index of consumer prices)
- Price base period = Dec 2006
- Weight base period = entire year 2005

Laspeyres in another form

$$I = \frac{\sum_i q_{0,i}\ p_{1,i}}{\sum_i q_{0,i}\ p_{0,i}} = \sum_i \frac{q_{0,i}\ p_{0,i}}{\sum_k q_{0,k}\ p_{0,k}} \cdot \frac{p_{1,i}}{p_{0,i}} = \sum_i w_i \cdot \frac{p_{1,i}}{p_{0,i}}$$

$$\text{with weights} \quad w_i = \frac{q_{0,i}\ p_{0,i}}{\sum_k q_{0,k}\ p_{0,k}}, \quad \text{satisfying} \quad \sum_i w_i = 1$$

Problems with fixed baskets

- Laspeyres > Paasches price index
  - ↳ *True almost always*
    – due to altered consumption pattern

- Fixed basket gets out of date – at new prices, new choices give better value for money
  - ↳ *Products with larger price rises are "substituted away" by buyers*

Ex.: Petrol price up → car use down

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Chaining in Swedish CPI

$$I_{1980}^{2007,\text{Jan}} = I_{1980}^{1980,\text{Dec}} \times I_{1980,\text{Dec}}^{1981,\text{Dec}} \times I_{1981,\text{Dec}}^{1982,\text{Dec}} \times \dots$$

$$\dots \times I_{2002,\text{Dec}}^{2003,\text{Dec}} \times I_{2003,\text{Dec}}^{2004} \times I_{2004}^{2005} \times I_{2005}^{2007,\text{Jan}}$$

Link of Previous type

Special link

Link of New type

Final link

Price indices (in Sweden) 1

- *CPI* – *Consumer Price Index*
  KPI – Konsumentprisindex
- *HICP* – *Harmonised Index for*
  HIKP *Consumer Prices*
- NPI – *Net Price Index*
- KPIX – *Underlying Inflation (Core Inflation)*

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Price indices (in Sweden) 2

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- **PPI** – ***Producer Price Index (goods)***
- ***SPPI*** – ***Producer Price Index for Services***
  TPI – **Tjänsteprisindex**
- **BPI** – ***Building Price Index***
- ***Real Estate Price Index***
- ***CCI*** – ***Construction Cost Index for***
  E84 ***Buildings***
  (building materials, labour)

International classification standards for breakdown

- **COICOP – Classification of Individual Consumption by Purpose – *in CPI***
- **NACE – Industry classification standard / Nomenclature statistique des Activités économiques dans la Communauté Européenne – *in PPI, SPPI***

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Classification levels in CPI

- 00 CPI overall (all items index)
- 01 Food and non-alcoholic beverages
- 01.1 Food
- 01.1.8 Sugar, jam, chocolate etc.
- 1819 Ice cream
- 1819-80 Ice cream brand X, type Y

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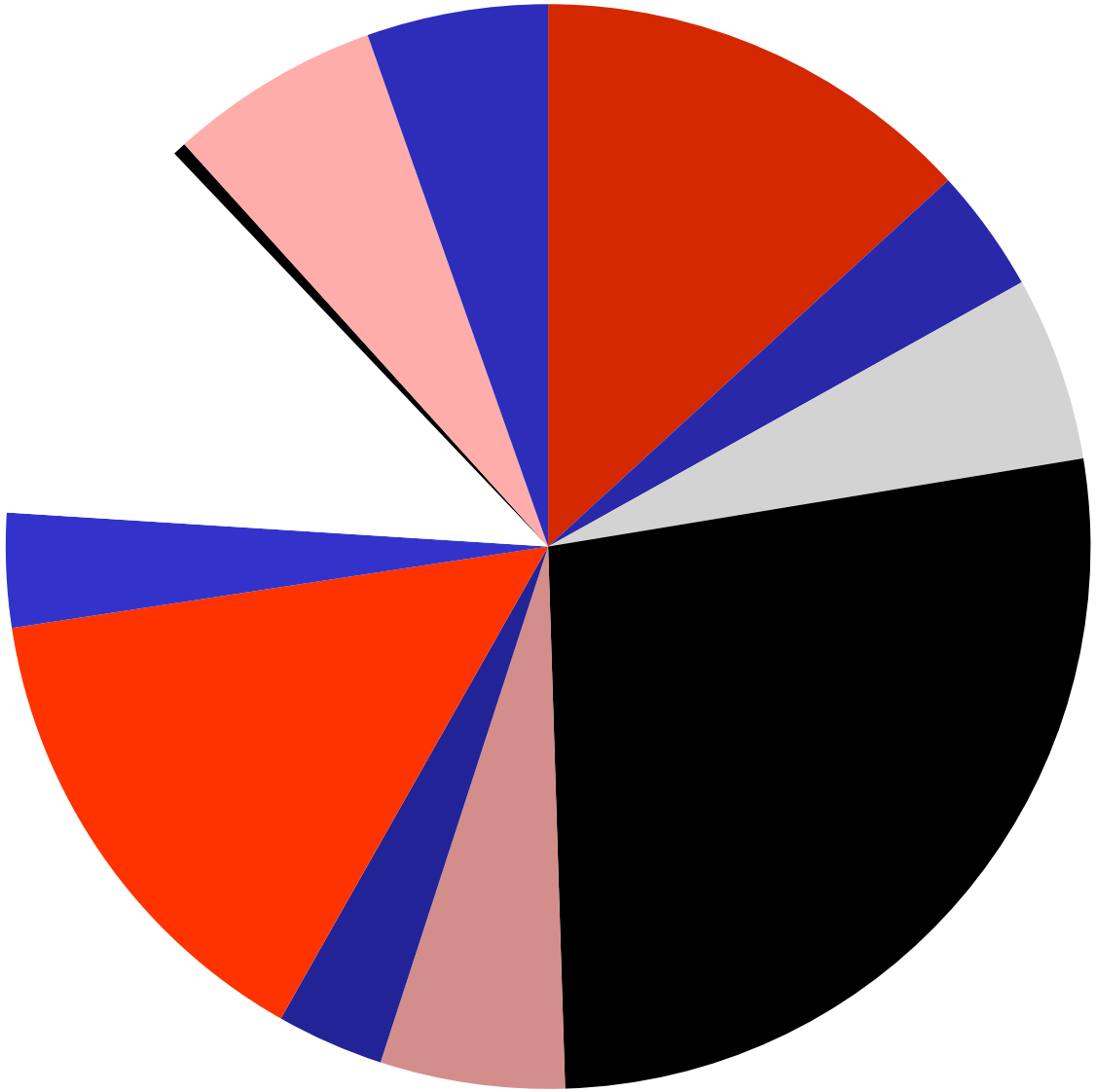
Swedish CPI basket in 2010



- Food etc.
- Alkohol, tobacco
- Clothing etc.
- Housing
- Furnishings
- Health
- Transport
- Communication
- Recreation, culture
- Educaton
- Restaurants, hotels
- Misc.

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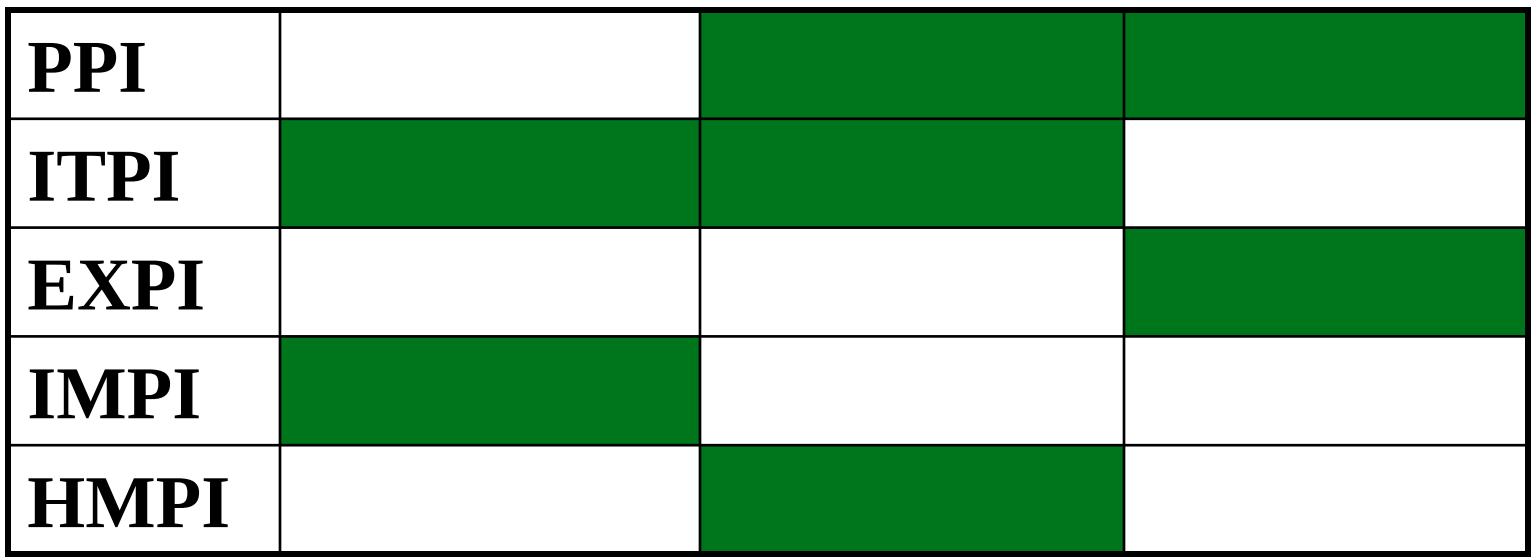
Producer and Import Price Indices (PPI)

- PPI – Producer Price Index
- ITPI – Price Index for Domestic Supply
- EXPI – Export Price Index
- IMPI – Import Price Index
- HMPI – Producer Price Index of Home Sales



Actual prices: CPI

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CPI follows:

- **Price on price tag (shown to consumer)**
- **After any sales deduction**
- **After deduction of general discounts**
- **But before deduction of individual discounts, loyalty rebates etc.**
  - ***Not quite ideal, e.g. for cars***
- **Inluding VAT and other indirect taxes**
- **After deduction of subventions**

Actual prices: PPI, SPPI

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PPI, SPPI follow:

- **Invoiced price – transaction (ideally)**
- **After deduction of any discounts**
- **Excluding taxes, VAT**
- **List price rather not, maybe as ”proxy”**
- **Ex. *chargeout rate* (charged hour rate) for consultant services in SPPI – not ideal but practically feasible solution**

Indices – aims – targets

- CPI – Main aim is compensation
  Target is coli
- HICP – Main aim is monetary politics
  Aim is Laspeyres type (?)
- TPI – Main aim is deflating
  Ideal target is Paasche
  - ↳ *Deflating with Paasche price index yields volyme index series i base period prices*
  - ➔ *But take Laspeyres i practice*

Cost Of Living Index (COLI)

- Pertains to unchanged standard of living
- ***Ideal solution:***
  Konüs index compares two baskets
- Both baskets yield the same *utility* – at minimal cost
  - ↳ ***Substitutions alter the basket***
- ***Praktical solution:***
  A fixed basket of a ”compromise” kind
  - ↳ ***Yields index that approximates coli!***

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Target and accuracy of CPI

- *Target* of CPI is coli
- *Practical computation* is based on a suitable fixed basket
- *Statistical accuracy:* How closely the computation hits the target

Theory of COLI

- Simplified assumption: 1 consumer
- In each period the consumer maximises her/his utility within a budget constraint
  - ↳ *Theoretical utility function*
    $U(q_1, \ldots, q_G) = \max!$
- Index should reflect the development of cost for retaining a constant utility in the most cost-efficient way

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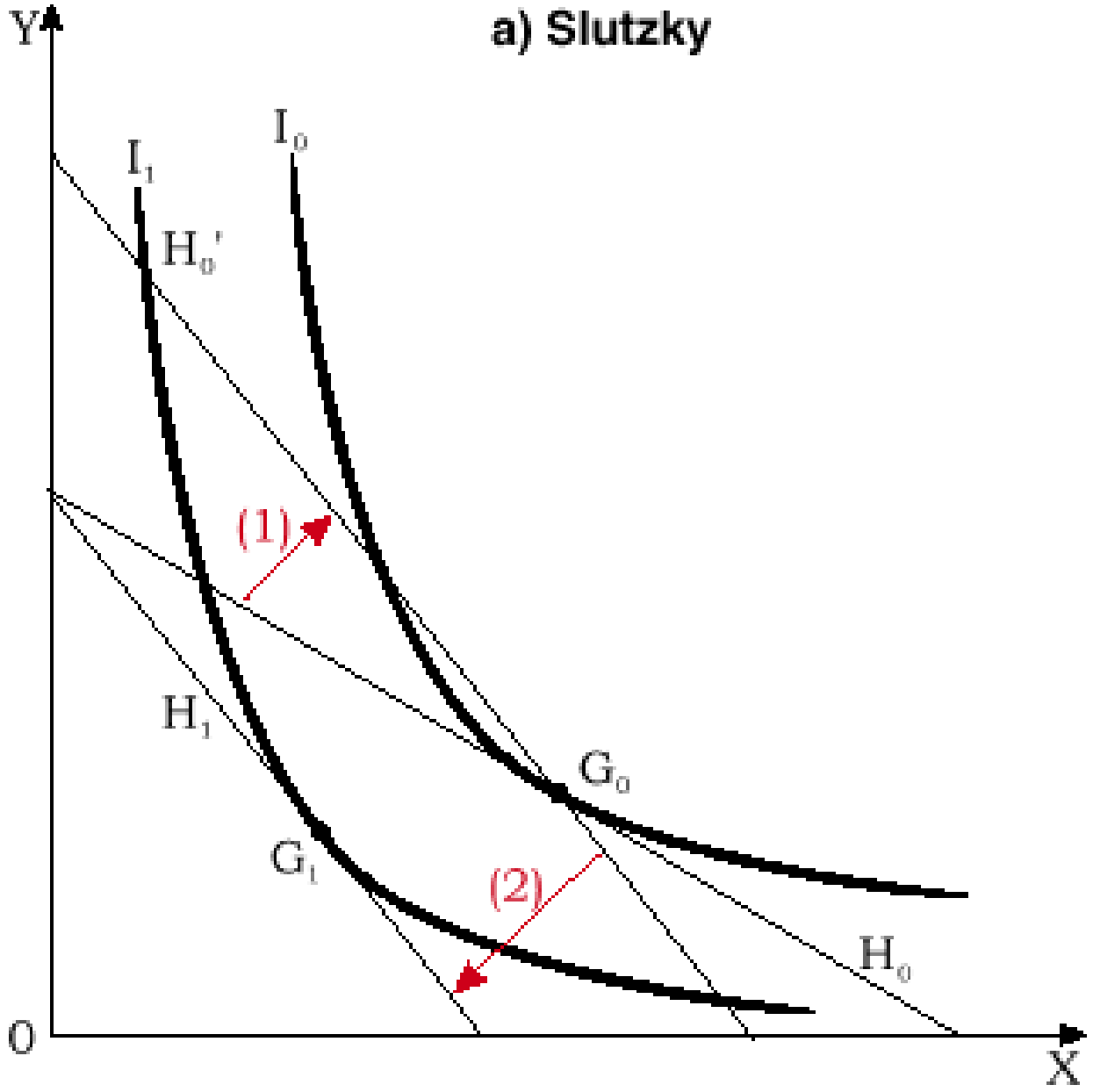
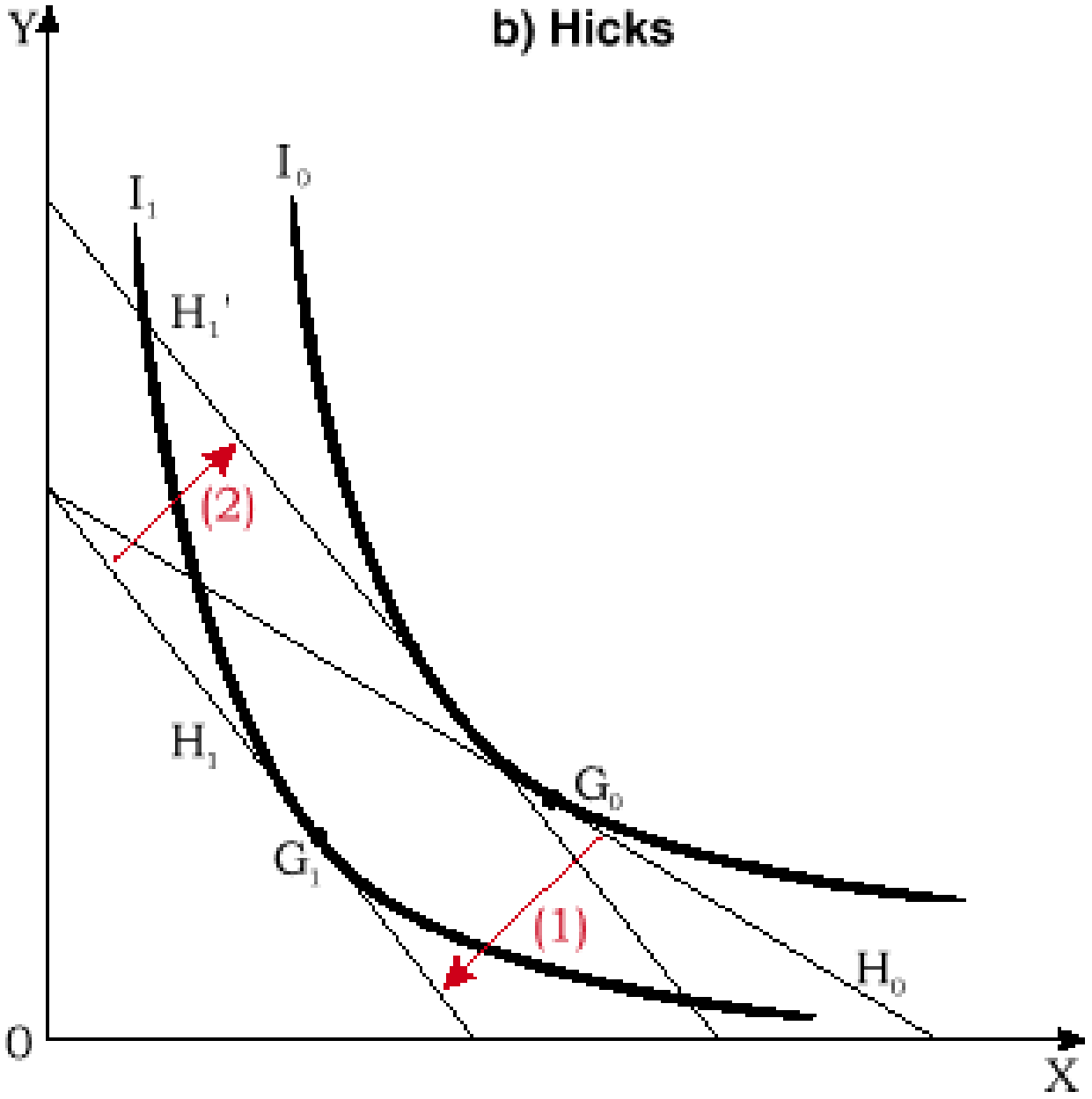

a) Slutzky
Y
X
0
I_0
I_1
H_0'
(1)
H_1
G_0
G_1
(2)
H_0


b) Hicks
Y
X
0
I_0
I_1
H_1'
H_1
H_0
G_0
G_1
(1)
(2)

Superlative indices

- **Fixed base indices that mimic coli**
- ***Exact index* – equals a constant-utility index for a specific utility function *U***
- ***Superlative index* – is exact for a ”flexible” class of utility functions (Erwin Diewert’s teori)**
- **Examples:**
  Fisher, Walsh, Törnqvist indices

”Fisher’s ideal index”

$$I = \sqrt{\frac{\sum_i q_{0,i}\ p_{1,i}}{\sum_i q_{0,i}\ p_{0,i}} \cdot \frac{\sum_i q_{1,i}\ p_{1,i}}{\sum_i q_{1,i}\ p_{0,i}}}$$

Laspeyres Paasche

- ***Symmetry between q_0 and q_1***

- **Variant: *Walsh index***

$$I = \frac{\sum_i \sqrt{q_{0,i}\ q_{1,i}}\ p_{1,i}}{\sum_i \sqrt{q_{0,i}\ q_{1,i}}\ p_{0,i}}$$

Walsh link over full year

$$I_{2004}^{2005} = \frac{\sum_i P_i^{2005} \times \sqrt{Q_i^{2004} \times Q_i^{2005}}}{\sum_i P_i^{2004} \times \sqrt{Q_i^{2004} \times Q_i^{2005}}} = \sum_g W_g \times I_{2004;g}^{2005}$$

where $$W_g = \frac{\sqrt{U_g^{2004} \times U_g^{2005}} \Big/ I_{2004;g}^{2005}}{\sum_{g'} \sqrt{U_{g'}^{2004} \times U_{g'}^{2005}} \Big/ I_{2004;g'}^{2005}}$$

Final Laspeyres link

$$I_{2005}^{2007,\,\mathrm{Jan}} = \frac{\sum_i P_i^{2007,\,\mathrm{Jan}} \times Q_i^{2005}}{\sum_i P_i^{2005} \times Q_i^{2005}} = \sum_g W'_g \times I_{2005;g}^{2007,\,\mathrm{Jan}}$$

During 2007 weighting with expenditures of 2005.
More time for weight preparation ⇒
Improved accuracy, smoother process

Alternative annual links 1

Year	Lasperes	Paasche	Itix dec	Walsh approx.
1993	104,483	104,141	103,911	104,312
1994	102,177	102,006	102,291	102,088
1995	102,470	102,194	102,168	102,329
1996	100,945	100,579	99,823	100,757
1997	100,673	100,333	101,269	100,505
1998	100,129	99,844	99,555	99,989
1999	100,480	100,286	100,785	100,329
2000	100,942	100,731	101,152	100,848
2001	102,524	102,479	102,658	102,505
2002	102,245	101,987	102,168	102,124
Mean	101,707	101,458	101,578	101,579

Alternative annual links 2

Year	Walsh approx.	Walsh alt.	Edge-worth	Törn-qvist
1993	104,312	104,312	104,316	104,313
1994	102,088	102,089	102,093	102,088
1995	102,329	102,329	102,334	102,330
1996	100,757	100,755	100,764	100,754
1997	100,505	100,505	100,503	100,505
1998	99,989	99,988	99,988	99,989
1999	100,329	100,328	100,383	100,392
2000	100,848	100,847	100,837	100,843
2001	102,505	102,504	102,502	102,501
2002	102,124	102,123	102,118	102,127
Mean	101,579	101,578	101,584	101,584

Levels of aggregation in the Swedish CPI

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Full-year base, Walsh index

Overall index

Coicop classes

350 Product groups

December base, Jevons index

Elementary aggregates

Sub-indices by product group

$$I_{2004;g}^{2005} = \frac{I_{2003,\mathrm{dec};g}^{2004,\mathrm{dec}} \times \frac{1}{12}\sum_{m=1}^{12} I_{2004,\mathrm{dec};g}^{2005,m}}{\frac{1}{12}\sum_{m=1}^{12} I_{2003,\mathrm{dec};g}^{2004,m}}$$

➢ ***Trans-forms December base to year base***

$$I_{2005;g}^{2007,\mathrm{jan}} = \frac{I_{2004,\mathrm{dec};g}^{2005,\mathrm{dec}}}{\frac{1}{12}\sum_{m=1}^{12} I_{2004,\mathrm{dec};g}^{2005,m}} \times I_{2005,\mathrm{dec};g}^{2006,\mathrm{dec}} \times I_{2006,\mathrm{dec};g}^{2007,\mathrm{jan}}$$

New products come in soon

- ***Treatment of group g that is new in2007:***

$$I_{2005;g}^{2007,\text{jan}} = \frac{I_{2004,\text{dec};g}^{2005,\text{dec}}}{\frac{1}{12}\sum_{m=1}^{12} I_{2004,\text{dec};g}^{2005,m}} \times I_{2005,\text{dec};g}^{2006,\text{dec}} \times I_{2006,\text{dec};g}^{2007,\text{jan}}$$

Imputed in years before

Priced in 2007

Index construction change for Swedish CPI from 2005

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- ***Previous construction – before 2005:***
  - Lower level: ’RA-formula’
  - Upper level: ’Updated basket’ +Laspeyres type
  - Annual chaining: By December

- ***New construction – from 2005:***
  - Lower level: Geometric mean
  - Upper level: Walsh + Laspeyres
  - Annual chaining: By full year

Elementary aggregates 0

- Weithting data are available on higher levels of aggregation
- Overall index is practically computed by weighting together of subindices
- *Elementary aggregates* are on lowest level of aggregation – weights usally not available
  - ↳ *Index formulas ”without q” needed*