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# Characterization of Stationary properties on Macroeconomic time series

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Abstract

In today's economic research there have been discussions on whether macroeconomic time series can be characterized by a trend-stationary property or not. It is common that applicants confuse the trend-stationary model with the difference-stationary model; statistically they are quite similar but the economic interpretation is very different. The growth component, i.e. trend-component in a time series is usually assumed to be a deterministic process in empirical work when analyzing macroeconomic data.

In this paper we analyze and discuss whether there is enough evidence to conclude if one can model macroeconomic variables with a deterministic model or a stochastic model. Two hypotheses are constructed in a two-model fashion, that is, we build two models where one represents the null hypothesis and the other represents the alternative hypothesis. The null hypothesis states the difference-stationary model and the alternative states the trend-statonary model. The parameters to include in the models are determined via the ARIMA procedure. Various statistical tests are performed in this study with the purpose of maximizing the confidence of our results.

The results attained are that we cannot discriminate the different approaches of characterizing a macroeconomic time series. However, a unit root was detected for both the difference-stationary model and the trend-stationary model, implying that the series can be modeled in a stochastic fashion.

# Table of Contents

# 1. Background

It has long been discussed whether macroeconomic time series can be characterized by the trend-stationary property. That is, if the variable of interest follow a long-run deterministic trend. Alternatively, economic time series can be modeled by a difference-stationary approach where we assume that the model does not follow a deterministic trend. The two different models are

$$\text{TS model:} \qquad y_t = \delta + \beta t + \epsilon_t \tag{1.1}$$

$$\text{DS model:} \qquad y_t = \delta + y_{t-1} + \epsilon_t \tag{1.2}$$

where $t$ is time, $\beta$ the slope and $\epsilon_t$ is assumed to be identically and independently normally distributed with mean 0 and variance $\sigma^2$, henceforth abbreviated, $\epsilon_t \sim IN(0, \sigma^2)$. We will continue to specify the residuals throughout the paper with this specification unless otherwise stated. We will also denote difference-stationary by “DS” and trend-stationary by “TS”.

The equations [1.1] and [1.2] differ vastly in terms of economic theory; if the series is assumed to be a TS model, it has a long-run equilibrium of constant growth. This is a strong assumption when analyzing economic time series. However, in the DS model, we have the implicit assumption of a stochastic process which appears more suitable when researching about economic variables as they usually are stochastic.

Taylor (1979), Kydland and Prescott (1980) and Bodkin (1969) are some authors that remove the trend component of their applied models when they research about business cycle theory. They later regard the residuals, attained from the fitted trend, as appropriate data for their target research question. This is also where the TS hypothesis shows its strength.

The probably most discussed paper when analyzing properties of macroeconomic time series is written by Nelson and Plosser (1982). They concluded that there was no evidence of a non-stationary deterministic process of the economic series analyzed. The methodology used was to include two models that represent a hypothesis for each possible modeling procedure where one is the alternative of the other. The null hypothesis was: the macroeconomic variable, $y_t$, follows a DS model. The alternative hypothesis was: the variable follows a TS model. The conclusions from Nelson and Plosser were that macroeconomic variables are more consistent with the DS model than the TS model. We will re-examine this conclusion brought by the authors and see whether the results can be extended to Swedish macroeconomic data.

Other approaches have also been used to characterize economic variables: Harvey (1985) modeled economic times series by their cyclical component, trend component and random error component. He then estimated the components via the Kalman filter. The conclusions drawn on the research was very similar to Nelson and Plosser’s technique even though the methodology between the two papers was different.  A re-examination of Nelson and Plosser has been made by Rudebrusch (1992) where he used the same data but the methodology was slightly different.

The aim of this report is to investigate if the following Swedish time series – Inflation, lnGDP and lnWage follow the same pattern as found in Nelson and Plosser (1982). A full description of the variables is found in appendix 1.

In Section 2 we give the theoretical framework used in our analysis. In Section 3 we perform our analysis of the three series. In Section 4 we bring the conclusions and discussions of this research and a brief overview of some extensions that can be applied for future research.

## 1.1 Description of the applied time series

The data have been collected from Worldbank. A brief review is made below.

**Inflation**: The GDP deflator reflects the prices of goods and services produced within Sweden between the years 1961-2010, with the base year 2000. The variable is defined as

$$\text{GDP deflator} = \frac{\text{GDP in } \textbf{current} \text{ local currency}}{\text{GDP in } \textbf{constant} \text{ local currency}}$$

The total number of data points for this variable is 50. The definition of the variable is taken from Worldbank, see the reference labeled “Worldbank(Inflation)”.

**lnGDP**: This variable is measured annually in (current) US dollars with the base year 1961 for the period 1961-2010 in Sweden. These data give us 50 data points. The definition of the variable is taken from Worldbank, see the reference labeled “Worldbank(lnGDP)”. The GDP variable will be transformed into its natural logarithm as is usually done with economic data. The reason for this transformation is that it is easier to interpret the results and also that GDP data are generally characterized by an exponentially increasing function.

**lnWage**: This variable is measured in (current) US dollars with the base year 1970 for the period 1970-2010 in Sweden. Thus we have 41 data points. The definition of the variable is taken from Worldbank, see the reference labeled “Worldbank(lnWage)”. This variable will also be transformed into its natural logarithm for convenience in interpretation and visualization.

We will apply univariate time series analysis for each of the macroeconomic variables above and see if we can detect any indication of either the DS property or the TS property. The applied modeling technique is the ARIMA technique. The tests used in this research are the augmented Dickey Fuller test, Phillips Perron test and the normalized bias test.

# 2. Theoretical framework

In this section we explain the difference between a TS and DS model. Then we derive the models in the ARIMA class, explain how to find the proper model and lastly how to evaluate the goodness of fit for found model. We end this section with a discussion of tools for testing the null and alternative hypothesis below,

$$H_0: \text{DS model} \qquad\qquad H_a: \text{TS model}$$

## 2.1 Two kinds of stationarity

In this report we will allow our time series, $\{y_t\}$, to be weakly stationary (henceforth we use the term stationary), with this we mean

(I) $E[y_t] = \mu$, i.e. a constant mean

(II) $Cov(y_t, y_{t-k}) = f(k)$, that is, the covariance only depends on the time difference.

A consequence of (II) is that the variance is constant for all t

$$Var(y_t) = Cov(y_t, y_{t-0}) = f(0)$$

### 2.1.1 Trend-stationary

If the mean of a time series is constantly growing around a deterministic trend, we characterize the series as a TS model. This implies that stationarity would be attained if we remove the trend component from the model. Consider equation [1.1], the TS model

$$y_t = \delta + \beta t + \epsilon_t$$

where $\epsilon_t \sim IN(0, \sigma^2)$.

This model is not stationary as the mean depends on $t$. Thus, if we were to remove the trend, $\delta + \beta t$ from this series, then stationarity would be attained.

By estimating the trend and replace the true value with the estimated value, ${y_t}^*$, we can rearrange [1.1] as

$$y_t - {y_t}^* = \delta + \beta t + \epsilon_t - \delta^* - \beta^* t,$$

and taking the expected value we find

$$E[y_t - {y_t}^*] = E[(\delta - \delta^*) + E(\beta t - \beta^* t) + E(\epsilon_t)] = 0,$$

because $\delta^*$and $\beta^*$ are unbiased estimates.

It may be shown that the covariance

$$Cov[y_t - {y_t}^*, y_{t-k} - {y_{t-k}}^*]$$

is a function of the time difference, k. Proving this statement is beyond the scope of this paper.

### 2.1.2 Difference-stationary

Consider equation [1.2], the DS model

$$y_t = \delta + y_{t-1} + \epsilon_t$$

The expected value is attained as

$$\begin{aligned} E[y_t] &= E[\delta + y_{t-1} + \epsilon_t] \\ &= E[\delta + \delta + y_{t-2} + \epsilon_t + \epsilon_{t-1}] \\ &= E[\delta t + y_0 + \textstyle\sum_{i=0}^{t} \epsilon_{t-i}] = \delta t + E[y_0] \qquad t = 1, 2, 3 \dots i \text{ and } 0 < i < \infty \end{aligned}$$

Hence the series is not weak stationary. However, the first difference, $\Delta y_t = y_t - y_{t-1}$ of equation [1.2] is

$$\Delta y_t = \delta + \epsilon_t \qquad [2.1]$$

which is obviously stationary since we only have the residual and the constant left.

## 2.2 ARIMA models

Since we will use the ARIMA methodology we here give a short introduction. A hypothetical ARIMA model can be written as

$$\begin{aligned} y_t = \alpha &+ \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} \qquad [2.2] \\ &+ \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q} \end{aligned}$$

where $\epsilon_t \sim IN(0, \sigma^2)$, $\phi_i$ is a weight parameter for the i:th component, i = 1, 2, 3 … p and $\theta_j$ is a weight parameter for the j:th component, j = 1, 2, 3 … q, where $0 \le p < \infty$ and $0 \le q < \infty$. This model is extensive as seen by its display. To simplify the model descriptions we introduce B, the backshift operator, defined as $By_t = y_{t-1}$, then model [2.2], can be expressed in terms of this operator, B. We get

$$y_t = \alpha + \left(\textstyle\sum_{i=1}^{p} \phi_i B^i\right) y_t + \left(1 - \textstyle\sum_{j=1}^{q} \theta_j B^j\right) \epsilon_t$$

If we introduce the functions $\phi$ and $\Theta$

$$\phi(B) = \left(1 - \textstyle\sum_{i=1}^{p} \phi_i B^i\right), \text{and } \Theta(B) = \left(1 - \textstyle\sum_{j=1}^{q} \theta_j B^j\right)$$

then further simplification can be applied

$$\phi(B) y_t = \alpha + \Theta(B) \epsilon_t$$

which is much more simplified than [2.2] but of course we pay a price for higher abstraction.

### 2.2.1 Statistical dependence measures

During the identification process of the ARIMA model we use the auto-correlation function and the partial auto-correlation function.

**Auto-correlation function:** The auto-correlation is a correlation within the time series itself. E.g. the auto-correlation at lag k is defined by

$$\rho_k = \frac{\mathrm{Cov}(y_{t,} y_{t-k})}{\mathrm{Var}(y_t)}$$

Here we estimate $\rho_k$ by $r_k$

$$r_k = \frac{\sum_{t=1}^{T-k}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

Note that the variance in the denominator is for $y_t$ and not $\sqrt{\mathrm{Var}(y_t)\mathrm{Var}(y_{t-k})}$ as generally used when estimating a correlation between two phenomena. The reason is the assumption of weak stationarity from which follows constant variance, i.e. $\mathrm{Var}(y_t) = \mathrm{Var}(y_{t-k})$.

**Partial auto-correlation function:** The auto-correlation measures the connection between $y_t$ and $y_{t-k}$ without filtering out the influence that lays between these two data points. However, the partial auto-correlation filters out the influence carried by the data points that lie between the two elements of interest. The mathematics for the partial auto-correlation is complex, and will not be derived in our application. The formula is given as

$$r_{kk} = \begin{cases} r_1 & \text{if } k = 1 \\ \dfrac{r_{k-\sum_{t=1}^{k-1} r_{k-1,t} r_{k-t}}}{1 - \sum_{t=1}^{k-1} r_{k-1,t} r_t} & \text{if } k = 2,3 \ldots \end{cases}$$

where $r_{kt} = r_{k-1,t} - r_{kk} r_{k-1,k-t}, \quad t = 1,2 \ldots k-1$

In order to detect whether the two functions follows certain patterns such as “cut-offs” (more on this in the coming sub-sections), one needs to address the standard deviation of the measure as to obtain the significance bounds. This is discussed in Box et al. (1994) at page 188 for both the auto-correlation function and the partial auto-correlation function.

Hereafter we denote the auto-correlation function as “ACF” and the partial auto-correlation function as “PACF”.

### 2.2.2 The Moving Average (MA) process

A moving average process of order q is defined as

$$y_t = \nu + \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q} \qquad [2.3]$$

where $\epsilon_t \sim \mathrm{IN}(0, \sigma^2)$, $\theta_i$ is the weight for $\epsilon_{t-i}$ and $\nu$ is an arbitrary constant. One can express equation [2.3] as

$$y_t = \nu + \Theta(B)\epsilon_t$$

To better understand the MA(q) we calculate the ACF for MA(1) at lags 1 and 2.

The MA(1) model can be defined by

$$y_t = \nu + \epsilon_t - \theta_1 \epsilon_{t-1},$$

with the variance

$$\begin{aligned} \mathrm{Var}[y_t] &= \mathrm{Var}[\nu + \epsilon_t - \theta_1 \epsilon_{t-1}] \\ &= \mathrm{Var}[\nu] + \mathrm{Var}[\epsilon_t] - \mathrm{Var}[\theta_1 \epsilon_{t-1}] \\ &= \sigma^2{}_\epsilon + {\theta_1}^2 \sigma^2{}_\epsilon = \sigma^2{}_\epsilon \left(1 + {\theta_1}^2\right) \end{aligned}$$

The auto-covariance for this process for lag $k = 1$ is given by

$$\begin{aligned} \mathrm{Cov}(y_t, y_{t-1}) &= \mathrm{E}[y_t, y_{t-1}] \\ &= \mathrm{E}[(\epsilon_t - \theta_1 \epsilon_{t-1})\,(\epsilon_{t-1} - \theta_1 \epsilon_{t-2})] \\ &= \mathrm{E}\left[\left(\epsilon_t \epsilon_{t-1} - \epsilon_t \theta_1 \epsilon_{t-2} - \theta_1 {\epsilon_{t-1}}^2 + {\theta_1}^2 \epsilon_{t-1} \epsilon_{t-2}\right)\right] \\ &= \mathrm{E}[\epsilon_t \epsilon_{t-1}] - \mathrm{E}[\epsilon_t \theta_1 \epsilon_{t-2}] - \mathrm{E}[\theta_1 {\epsilon_{t-1}}^2] + \mathrm{E}[{\theta_1}^2 \epsilon_{t-1} \epsilon_{t-2}] \\ &= -\mathrm{E}[\theta_1 {\epsilon_{t-1}}^2] = -\theta_1 \sigma^2{}_\epsilon \end{aligned}$$

since $\mathrm{E}[y_t] = 0$ and $\mathrm{E}[\epsilon_m, \epsilon_n] = 0,$ where $m \neq n$. Thus the ACF for lag 1 is

$$\rho_1 = \frac{-\theta_1 \sigma^2{}_\epsilon}{\sigma^2{}_\epsilon (1 + {\theta_1}^2)} = \frac{-\theta_1}{(1 + {\theta_1}^2)}$$

The auto-covariance for this process at lag $k = 2$ is given by

$$\mathrm{Cov}(y_t, y_{t-2}) = \mathrm{E}[(\epsilon_t - \theta_1 \epsilon_{t-1})(\epsilon_{t-2} - \theta_1 \epsilon_{t-3})] = 0$$

as we have different time indices, hence, the numerator for the ACF formula is zero. The result for $k \geq 3$ is similarly the same.

The conclusion from these calculations is that the ACF "cuts-off" after lag 1 since a calculation for $r_2$ for an MA(1) process becomes 0 as the time indices of the residuals are different. Consequently, if we observe the sample ACF that "cuts-off" after lag 1 it indicates that the data is fit for an MA(1) model. However we must also observe the PACF to see the other dimension that may indicate an MA(1) process. It can be shown that the PACF for an MA(1) process decays exponentially, for a proof of this statement see Box et al. (1994) page 73.

It can also be shown that the theoretical ACF for an MA(q) process is significant up to lag q and insignificant otherwise, see Montgomery et al. (2008) page 236. The PACF for the MA(q) process is exponentially decaying in absolute value as proven in Box et al. (1994).

### 2.2.3 The Autoregressive (AR) process

The general autoregressive process, AR(p), is given by the following equation

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \epsilon_t, \text{ or}$$

$$\phi(B) y_t = \delta + \epsilon_t$$

where $\epsilon_t \sim IN(0, \sigma^2)$ and $\phi_i$ are weights

To better understand the AR(p) process we calculate the ACF for AR(1) at lag 1.

An AR(1) process is said to be stationary if $|\phi_1| < 1$. The proof for this statement can be found in Montgomery et al. (2008) page 241. We assume that the condition holds. We have already derived the case when $|\phi_1| = 1$, see equation [2.1].

The AR(1) process is written as

$$y_t = \delta + \phi_1 y_{t-1} + \epsilon_t \qquad [2.4]$$

The expected value of the process is calculated as

$$E[y_t] = E[\delta + \phi_1 y_{t-1} + \epsilon_t] = \delta + \phi_1 \mu,$$

further rearrangements implies

$$\mu = \frac{\delta}{(1 - \phi_1)} \qquad [2.5]$$

The variance for $y_t$ is

$$Var[y_t] = Var[\delta + \phi_1 y_{t-1} + \epsilon_t] = Var[\delta] + Var[\phi_1 y_{t-1}] + Var[\epsilon_t]$$

$$= {\phi_1}^2 Var[y_{t-1}] + \sigma^2 = {\phi_1}^2 Var[y_t] + \sigma^2$$

further rearrangement implies

$$Var[y_t] = \frac{\sigma^2}{(1 - {\phi_1}^2)} \qquad [2.6]$$

where $\sigma^2$ is the variance for the error term.

We can use equation [2.5] to solve for $\delta$ and attain

$$\delta = \mu(1 - \phi_1) \qquad [2.7]$$

substituting equation [2.7] into equation [2.4], the model can be expressed as

$$y_t = \mu(1 - \phi_1) + \phi_1 y_{t-1} + \epsilon_t$$

simplification implies

$$y_t = \mu - \mu\phi_1 + \phi_1 y_{t-1} + \epsilon_t$$

Rearranging and multiplying both sides with $(y_{t-k} - \mu)$ gives

$$(y_t - \mu)(y_{t-k} - \mu) = \phi_1 (y_{t-1} - \mu)(y_{t-k} - \mu) + \epsilon_t (y_{t-k} - \mu)$$

Taking the expected value we get the auto-covariance function

$$\mathrm{E}[(y_t - \mu)(y_{t-k} - \mu)] = \phi_1 \mathrm{E}[(y_{t-1} - \mu)(y_{t-k} - \mu)] + \mathrm{E}[\epsilon_t(y_{t-k} - \mu)]$$
$$= \phi_1 \mathrm{E}[(y_{t-1} - \mu)(y_{t-k} - \mu)] \qquad [2.8]$$

dividing [2.8] by [2.6] we get for $\rho_k$

$$\rho_k = \phi_1 \rho_{k-1}$$
$$= \phi_1{}^2 \rho_{k-2}$$
$$= \cdots$$
$$= \phi_1{}^k \rho_0$$
$$= \phi_1{}^k$$
$$\text{since } \rho_0 = 1$$

Hence the ACF for the AR(1) model decays exponentially as $|\phi_1| < 1$. Thus when we identify an ARIMA model we can observe the ACF to see whether the data resembles an exponentially decaying pattern. This would indicate that the data is suitable for an AR(1) model given the PACF. The PACF for the process is significant at lag 1 and insignificant otherwise. The derivation of this statement can be found in Box et al. (1994) page 66.

The theoretical ACF for an AR(p) model is a combination of exponential decay and damped sinusoid visualizations, see Box et al. (1994) page 56. The PACF for the AR(p) process is also shown in the same literature at page 66; it is significant at lag p and insignificant otherwise.

### 2.2.4 Mixed Autoregressive-Moving Average (ARMA) processes

The general autoregressive-moving average process, ARMA(p,q), is given as

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p}$$
$$+\epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q}$$

or alternatively

$$\phi(B) y_t = \delta + \Theta(B) \epsilon_t$$

This special case of ARIMA models is a mixture of components that puts weight on past errors and past data points.

The derivation of the ACF and PACF for this type of process is complex and left to references; see Box et al. (1994) page 80.

It is shown that both the ACF and PACF exhibit exponentially decay and/or dampened sinusoid patterns. Thus if the data follows these types of patterns we can try fit an ARMA(p,q) model.

As both the ACF and PACF follow the same pattern it is difficult to assess on how many lags to include. We can however consider different evaluation criteria between the models. The criteria's are the Akaike's and Schwarz's information criteria (details on these criteria are discussed in the next sub-section).

### 2.2.5 Autoregressive Integrated Moving Average (ARIMA) processes

It is common that time series are not stationary. The ARIMA process includes another component, d, which is the number of differences needed to make the time series stationary.

The general ARIMA(p,d,q) process is

$$\phi(B)(1-B)^d y_t = \delta + \Theta(B)\epsilon_t$$

where $(1-B)^d$ is the differencing argument.

### 2.2.6 ARIMA identification for the DS and TS models

The DS model will be expressed as an ARIMA(p,1,q) processes where p and q are unknown. The reason for this is the hypothesis of attaining a stationary model by differencing the time series. Formally,

$$\phi(B)(1-B)y_t = \delta + \Theta(B)\epsilon_t$$

The TS model will be represented as an ARIMA(p,0,q) process with a trend component since stationarity will be attained by removing the trend component from series. Formally,

$$\phi(B)y_t = \delta + \beta t + \Theta(B)\epsilon_t$$

## 2.3 Model evaluation

In order to evaluate the "goodness" of the applied models we rely on different indicators.

**Ljung-Box test:** The Ljung-Box statistic test whether the estimated residuals are correlated or not. If this test fails to reject the hypothesis that all auto-correlations are zero and if the residuals are found to be normally distributed, then the residuals can be regarded as independent. The hypotheses are

$$H_0: \rho_1 = \rho_2 = \cdots = \rho_k \qquad H_a: \text{at least one } \rho_i \neq 0$$

As test statistic we have

$$Q_{LB} = T(T+2)\sum_{k=1}^{K} \frac{1}{T-K} r_k^2 \sim \chi^2{}_{(K-p-q)},$$

$$\text{where } r_k = \left(\sum_{t=1}^{n} \hat{\epsilon}_t{}^2\right)^{-1} \left(\sum_{t=k+1}^{n} \hat{\epsilon}_t \hat{\epsilon}_{t-k}\right)$$

The $\hat{\epsilon}$ 's are estimated residuals for the fitted ARIMA model, k is the lag between the residuals, K is the number of lags being tested, and T is the number of observations subtracted by the number of differences applied for the model. The test statistic follows a Chi-square distribution under the null hypothesis with $K-p-q$ degrees of freedom, where p is the number of auto regressive lags and q is the number of moving average lags for the applied ARIMA model. For further details, see Ljung and Box (1978).

**The normality assumption:** To see if the residuals are normally distributed we observe the QQ-plot of the residuals and the histogram for the distribution of the residuals.

The QQ-plot is graphed by the normal quantiles on one axis and the normalized residuals on the other axis with a 45 degree line going through the origin. If the residuals have a normal distribution then the QQ-plot will have the estimated residuals very close to the 45 degree line.

The histogram is graphed by the residual estimate on the horizontal axis and the percentage of residuals on the vertical axis. If the residuals have a normal distribution the histogram will have a bell shaped curve, but the main benefit is to understand the tails.

**Two information criteria:** When choosing between competing models, we observe Akaike's (AIC) and Schwarz's (SIC) information criteria

$$\mathrm{AIC} = \ln\left(\frac{\sum_{t=1}^{T} \epsilon_t^2}{T}\right) + \frac{2p}{T} \qquad \mathrm{SIC} = \ln\left(\frac{\sum_{t=1}^{T} \epsilon_t^2}{T}\right) + \frac{p\ln(T)}{T}$$

These two statistics differ since $2p < p\ln(T)$ when the sample size $T > e^2 \approx 7.3$, hence we have that AIC gives lesser penalty than SIC. Another remark is that SIC is consistent whereas AIC is not. For further details, see Akaike (1974) and Schwartz (1978).

## 2.4 Testing for unit root

In this sub-section we discuss tests that can discriminate between the DS model and the TS model. The tests are the augmented Dickey Fuller (ADF) test, the normalized bias (NB) test and the Phillips Perron (PP) test.

The ADF test is an extension of the Dickey Fuller test. The difference between these two is that in the Dickey Fuller test we ignore any of the underlying auto-correlation existing in the data, while in the ADF test we allow for this interaction to occur. Nelson and Plosser (1982) used the ADF test and since we wish to re-examine their conclusions on Swedish economic data we follow their example and use the same test.

The ADF test assumes a pure autoregressive process, i.e. an AR(p) process. Hence it may cause complications when the ARIMA modeling dimension is applied. Fortunately it has been shown that testing for unit roots can be done conventionally by the general AR(p) process even if the fitted model is an ARIMA(p,d,q) under the condition that as long as the sample size gets large, so will the number of lags in the unit root regression, see Dickey (Stationarity Issues in Time Series Model) and Said and Dickey (1984). Also, Phillips and Perron (1988) writes at page 336: "…Dickey-Fuller regression t test for a unit root may still be used in an ARIMA(p,1,q) model provided the lag length in the autoregression increases with the sample size, T, at a controlled rate less than $T^{\frac{1}{3}}$", where T is the sample size of the data.

The next test is called the NB test which also has been created by Dickey and Fuller. It has been noted that this test has more power than the ADF test if we are evaluating an AR(1) model, see Dickey and Fuller (1979).

The last test that we will use in this paper is the PP test. The test accommodates models with fitted drift (constant) and time trend (deterministic trend) such that the models can be discriminated between the DS model and TS model.

The null and alternative hypotheses are identical for each of the three tests

$$H_0\text{: a unit root exist,} \quad H_a\text{: no unit root}$$

If we fail to reject $H_0$, we conclude that the DS model is appropriate and if we reject $H_0$, we conclude that the TS model is appropriate.

### 2.4.1 The augmented Dickey Fuller (ADF) test

For the DS model we have a special case of testing the hypothesis of a unit root. Consider the general AR(p) model expressed in first differences

$$\Delta y_t = \mu + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + \epsilon_t \qquad [2.9]$$

Rudebusch (1992) argues that a trend coefficient should be included in the ADF regression when we test the DS null while having the TS alternative hypothesis. Also, West (1987) shows that the null hypothesis has low power against trend alternatives if we do not include a time trend in the regression. Hence we include a time trend and estimate the equation

$$y_t = \mu + \delta y_{t-1} + \beta t + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + \epsilon_t \qquad [2.10]$$

If we find that $\delta = 1$, the model is not stationary. The test statistic is

$$\tau = \frac{(\delta - 1)}{SE(\delta)}$$

where $SE(\delta)$ is the standard error of the estimate, $\delta$.

We can obtain the p-value simply by estimating model [2.10] by a least squares regression; see Ismail E. Mohamed (The Augmented Dickey-Fuller (ADF) Test). However, it has been shown that the statistic does not follow the usual t-distribution but is skewed towards negative values; see Dickey and Fuller (1979). For the DS models we will manually regress [2.10] and attain a t-value for the parameter of interest, namely, $\delta$, which we then manipulate for the purpose of obtaining a true p-value by a macro in SAS. The reason for this is that SAS will automatically test [2.9], not [2.10] which give us misleading values. For the TS models we will not manually estimate regressions and convert them to the true p-value since our TS model is not defined in differences.

### 2.4.2 The normalized bias (NB) test

It has been shown that the NB test possesses more power than the ADF test for autoregressive models of order 1, but this is not true for higher orders. SAS provides the output of the normalized bias test together with the ADF test. Consequently, we will emphasize the normalized bias test if we have a model that consists of one autoregressive component and if the model of interest is a TS model.

The test statistic is

$$\rho = n(\delta - 1)$$

This statistic also suffers from the non-normal property; see Dickey (1979) for further details. The true p-value of the test statistic will be reported in SAS.

### 2.4.3 The Phillips Perron (PP) test

The difference between the PP test and the ADF test is that the ADF test includes additional lagged terms of an AR(p) process to countermeasure the possibility that the true model is more complicated than the applied model. The PP test solution for this possible misspecification is to use nonparametric techniques to make a correction of the standard error that provides a consistent estimator of the variance.

There is problem with this test; it only has power in autoregressive models; see Phillips and Perron (1988). The authors argue that (page 345) "…For models with moving average errors and negative serial correlation the Z tests suffer appreciable size distortions and are not recommended. In such cases the Said-Dickey procedure of using a long autoregression seems preferable." Thus I will only apply this test in the cases when we have a pure autoregressive model of the applied series. The regression that is applied is defined as an ARIMA(p,0,0) model with a trend coefficient.

### 2.4.4 Power

When applying testing procedures it is important to recognize the test's ability to reject a false null, specifically; the power of the tests needs to be addressed. See Park, Hun Myoung (2008) for further details regarding power.

The powers of the tests applied in this problem have been criticized by Dejong et al. (1989) where they write (page 432) "…inferences based exclusively on tests for integration may be fragile". Also, Schwert (1987) and Blough (1988) have shown that testing for unit roots can have low power. Cochrane (1991) warns that (page 283) "…application of unit root tests without consideration for their low power and for the restrictions that they inevitably impose in a finite sample can be misleading".

However, the tests we apply were already chosen prior to the research. Also, Nelson and Plosser (1982) only introduced one formal test for unit root in order to test whether the null model could be rejected, namely, the ADF test. As the purpose of this paper is to extend their work and see whether the same behavior can be detected in Swedish data, we stick to the tests presented.

# 3. Results

The estimation technique for the applied models is the conditional least squares method. This estimation differs from ordinary least squares by assuming that the past unobserved observations have a residual equal to zero. For more information regarding this topic, see the reference labeled “SAS(CLS)”.

## 3.1 Overview of the data

We start off by observing the structure of our data for the relevant variables which is done by plotting the variable against time.

As seen in figure Inflation, the series has a clear trend but we also see that the exponential growth is broken at 1990. After this year the trend is linear. This implies that the series is not stationary. From 1990 and forward the series seem to change its pattern. One reason could be because of the Swedish crisis that occurred at that time.

**Inflation**

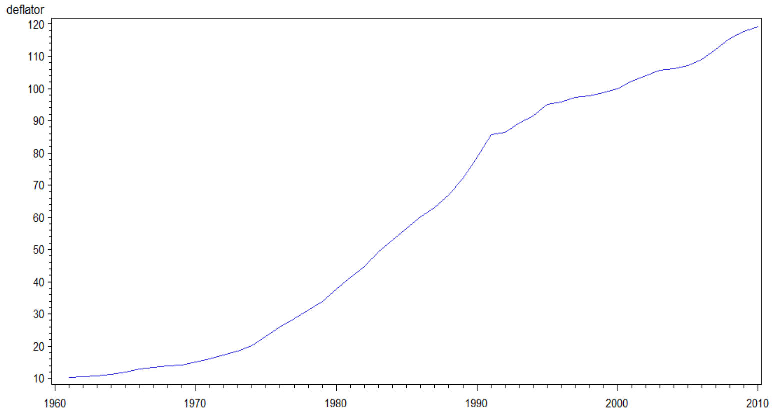


*Figure 3.1.1*

The next variable of interest is lnGDP. From the plot we can see a clear trend. The mean of this series is not constant; it changes with respect to the dependent variable. The Swedish crisis that occurred in the 90’s seems to react rather slowly with respect to lnGDP. We also see a clear intervention at 1980. This could be because of the recession in the US that spread across the world.

**lnGDP**

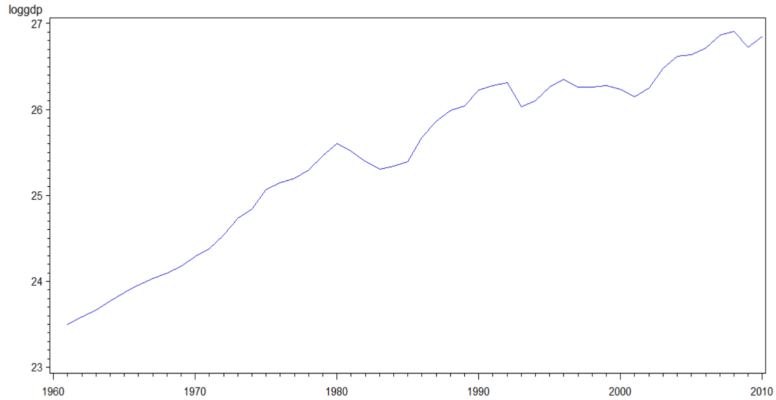


*Figure 3.1.2*

The final variable is lnWage. A clear trend is detected. Observing the series more closely we see in the year 1990 an intervention occurs. The reason could be because of the Swedish crisis in the 90’s. We also see that the effect of the crisis takes effect immediately. However in the lnGDP series there seem to be a lag of two years before we see the effect.

**lnWage**

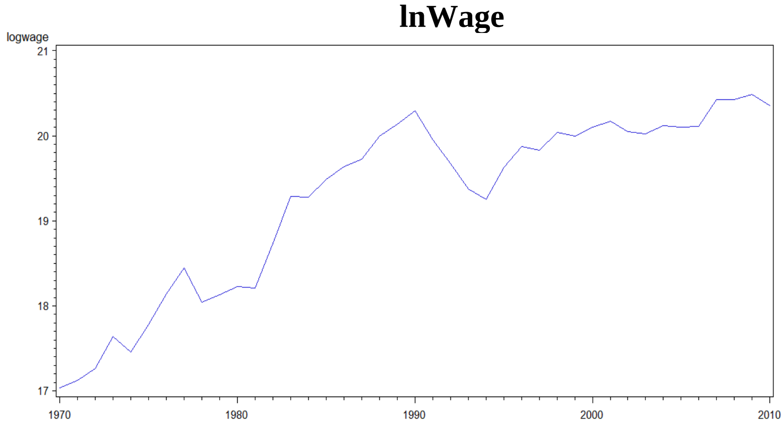


*Figure 3.1.3*

We alert the reader that Inflation and lnWage are not linear as seen by the figures. This causes problems to our research as we need to introduce transformation techniques which would take us beyond the scope of this paper. We will however continue working with the report as intended.

## 3.2 Model identifications

We will now find ARIMA models for the three different economic time series.

The procedure we follow is

1) Calculate the ACF and PACF to decide the order of p,d and q.
2) Estimate the ARIMA(p,d,q) model to obtain the residual serie.
3) Observe whether the Ljung-box test statistic is significant or not. If significant, then go back to 1. If insignificant, then move on to 4.
4) Observe the normal probability plot and histograms to see if the residuals follow a normal distribution.

We create two models for each variable where one represents the null hypothesis and the other represents the alternative hypothesis. The null model will always be a DS model and the alternative model will always be a TS model. There will be a total of six models to include in this research as we have three variables that will be represented by two models each.

The general DS model is written as

$$\phi(B)(1-B)y_t = \delta + \Theta(B)\epsilon_t$$

see page 12. Also, the order of p and q are chosen given the sample ACF and PACF of the data. The SAS code for identifying this model is

```
proc arima data=(data set);
identify var=(variable)(1);
run;
```

As the data is defined in first differences, we write (1) after the (variable).

The general TS model is written as

$$\phi(B)y_t = \delta + \beta t + \Theta(B)\epsilon_t,$$

see page 12. Also, the order of p and q are chosen given the sample ACF and PACF of the data. The SAS code for identifying this model is

```
proc arima data=(data set);
identify var=(variable);
run;
```

As the data is not defined in first differences, we ignore the (1) after the (variable). Observe that we simply identify the raw data by applying this code.

On the following pages we present the procedure on how to identify the order of p and q for the DS and TS models for our variables.

### 3.2.1 Identifying Inflation

**DS model:** The DS model is identified as

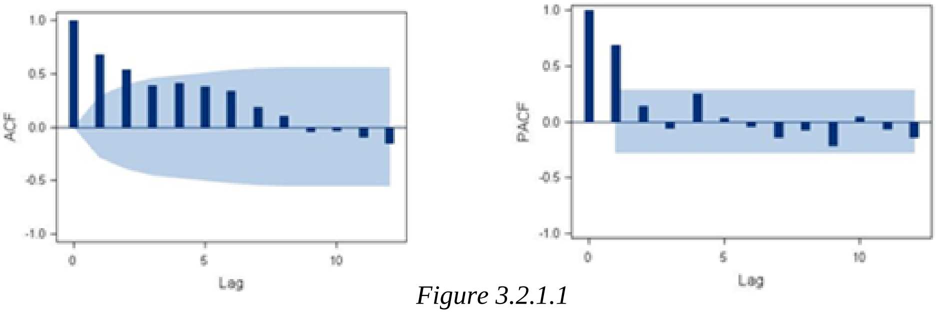


*Figure 3.2.1.1*

The ACF decays somewhat exponentially. The PACF is significant at lag 1 and insignificant otherwise, suggesting an ARIMA(1,1,0) model. Hence we choose the model

(Inflation 1): $$(1-\phi_1 B)(1-B)y_t = \nu + \epsilon_t$$

The evaluation statistics show that the auto-correlation for the residuals can be regarded as zero. The normality assumption in the residuals is not reasonable with respect to the QQ plot see figure 2.1 in appendix 2. This might be a consequence of the fact that our data needs transformation techniques as discussed on the previous page.

**TS model:** Now we identify the TS model for Inflation, we get the ACF and PACF as

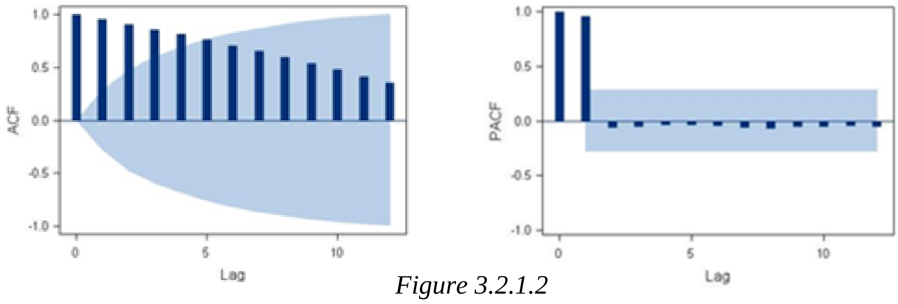


*Figure 3.2.1.2*

We see that the ACF is decaying exponentially and the PACF is significant at lag 1, suggesting an ARIMA(1,0,0).

Hence we fit the TS model below to our Inflation time series:

(Inflation 2): $$(1-\phi_1 B)y_t = \nu + \beta t + \epsilon_t$$

We retained poor evaluation statistics; see table 2.2 in appendix 2 where the auto-correlation check for residuals are all significant at the conventional levels. To solve this problem, I will follow the Nelson and Plosser (1982) model where they identify all of their macroeconomic variables with an AR(2) model. The evaluation statistics from this model is much better, see table 2.3 in appendix 2 where we have a table for the auto-correlation check on the residuals; there are no significant autocorrelation of the residuals at any lag. The normality check for residuals can be seen in figure 2.2 in appendix 2; the residuals do not seem to follow a normal distribution in strict sense which violates the assumption of normality in the residuals.

The same procedure is be made for the remaining variables. A summary of the identifications and estimations are shown on the next page.

### 3.2.2 Summary of the identifications and estimations

A summary for the conclusions are presented in the tables below.

*Table 3.2.2.1: Evaluation statistics for the DS models*

| Model | AIC | SIC | Significant(Yes/No) |
|---|---|---|---|
| (Inflation 1) ARIMA(1,1,0) | 156.22 | 160 | No |
| (lnGDP 1) ARIMA(1,1,0) | -82.06 | -78.27 | Yes |
| (lnGDP 1) ARIMA(1,1,1) | -80.92 | -75.25 | Yes |
| (lnGDP 1) ARIMA(2,1,2) | -81.67 | -72.21 | Yes |
| (lnGDP 1) ARIMA(3,1,3) | -83.06 | -69.81 | No |
| (lnWage 1) ARIMA(1,1,0) | -5.17 | -1.79 | No |
| (lnWage 1) ARIMA(1,1,1) | -3.36 | 1.71 | No |
| (lnWage 1) ARIMA(2,1,2) | 0.64 | 9.09 | No |

See tables 2.1, 2.4-2.7 and 2.10-2.12 in appendix 2.

*Table 3.2.2.2: Evaluation statistics for the TS models*

| Model | AIC | SIC | Significant(Yes/No) |
|---|---|---|---|
| (Inflation 2) ARIMA(1,0,0) | 192 | 197.737 | Yes |
| (Inflation 2) ARIMA(2,0,0) | 160 | 167.65 | No |
| (lnGDP 2) ARIMA(1,0,0) | -79.22 | -73.48 | Yes |
| (lnGDP 2) ARIMA(1,0,1) | -83.51 | -75.86 | No |
| (lnWage 2) ARIMA(1,0,0) | -4,51 | 0.62 | No |

See tables 2.2-2.3, 2.8-2.9 and 2.13 in appendix 2.

In the tables 3.2.2.1 and 3.2.2.2 we have all the prescribed ARIMA models given the ACF and PACF for each time series. We indicate the shaded rows as the chosen DS and TS models. The “Significant(Yes/No)” column tells us whether the estimated model had significant auto-correlation between the residuals. See figures 2.1-2.6 in appendix 2 for the normal probability plots and histograms attained.

*Table 3.2.2.3: The estimated time series for the DS models*

| Model | Cons. | $\Delta y_{t-1}$ | $\Delta y_{t-2}$ | $\Delta y_{t-3}$ | $\Delta \epsilon_{t-1}$ | $\Delta \epsilon_{t-2}$ | $\Delta \epsilon_{t-3}$ |
|---|---|---|---|---|---|---|---|
| (Inflation 1) ARIMA(1,1,0) | 1.96** | 0.7** | - | - | - | - | - |
| (lnGDP 1) ARIMA(3,1,3) | 0.07** | -1.14** | -0.85** | -0.71** | -1.58** | -1.35** | -0.71** |
| (lnWage 1) ARIMA(1,1,0) | 0.08* | 0.16 | - | - | - | - | - |

See tables 3.1, 3.3 and 3.5 in appendix 3.

*Table 3.2.2.4: The estimated time series for the TS models*

| Model | Cons. | $y_{t-1}$ | $y_{t-2}$ | $\epsilon_{t-1}$ | t |
|---|---|---|---|---|---|
| (Inflation 2) ARIMA(2,0,0) | −4198.6** | 1.69** | −0.71** | - | 2.15** |
| (lnGDP 2) ARIMA(1,0,1) | -120.25** | 0.92** | - | -0.36* | 0.07** |
| (lnWage 2) ARIMA(1,0,0) | −159.72** | 0.94** | - | - | 0.09** |

See tables 3.2, 3.4 and 3.6 in appendix 3.

Moving on to tables 3.2.2.3 and 3.2.2.4 we show the six estimated models. The column “Cons.” refers to the value of the constant in the applied model, $y_t$ accounts for the dependent variable which is Inflation, lnGDP or lnWage. $\epsilon_t$ is the error term for the dependent variable and t is the estimation of the trend. Some cells are left empty as the number of p and q differ between them. Also, * indicates that the p-value is <0.05 or close to 0.05, ** means that the p-value is <0.01 or close to 0.01.

We see almost all estimated parameters have p-values that are less than 0.01 indicating that the models have predictive power with respect to the components.

### 3.3 Unit root tests

Here we construct the ADF test, PP test and the NB test. Tables are found in appendix 4.

### 3.3.1 ADF test

For (Inflation 1) we attained the estimated parameter “deflator_1st_LAG” and used it in the test statistic for the ADF unit root test,

$$\tau = \frac{(0.948 - 1)}{0.025} = -2.08,$$

see output 4.1 for the p-value obtained and table 4.1 for the regression.

The same procedure is made for the remaining DS models. However we have special case for the (lnGDP 1) model. Recall that the model chosen for (lnGDP 1) was an ARIMA(3,1,3), thus we get the lags (see page 13),

$$n^{\frac{1}{3}} = 49^{\frac{1}{3}} \approx 3{,}66$$

The tables below summarize the results for the ADF test on the DS and TS models.

*Table 3.3.1.1: ADF test for the DS models*

| Model | Test statistic | P-value |
|---|---|---|
| (Inflation 1) | -2.08 | 0.25 |
| (lnGDP 1) | -1.88 | 0.34 |
| (lnWage 1) | -2 | 0.29 |

See tables 4.1, 4.5 and 4.7 and outputs 4.1, 4.2 and 4.3 in appendix 4 for the regression tables and the attained p-values.

*Table 3.3.1.2: ADF test for the TS models*

| Model | Test statistic | P-value |
|---|---|---|
| (Inflation 2) | -2,14 | 0,51 |
| (lnGDP 2) | -1,85 | 0,67 |
| (lnWage 2) | -1,99 | 0,59 |

See tables 4.3, 4.6 and 4.9 in appendix 4.

From the tables above it is clear that we fail to reject any of the null hypotheses that the series has a unit root for every variable for both the DS and the TS definition. All the p-values are high which indicates that there exists a unit root in these models.

### 3.3.2 PP test

The table below summarizes the results for the PP test on the DS and TS models.

*Table 3.3.2.1: PP test for the DS models*

| Model | Test statistic | P-value |
|---|---|---|
| (Inflation 1) | -2.07 | 0.55 |
| (lnGDP 1) | - | - |
| (lnWage 1) | -1.75 | 0.71 |

See tables 4.2 and 4.8 in appendix 4.

*Table 3.3.2.2: PP test for the TS models*

| Model | Test statistic | P-value |
|---|---|---|
| (Inflation 2) | -2.08 | 0.55 |
| (lnGDP 2) | - | - |
| (lnWage 2) | -1.75 | 0.71 |

See tables 4.4 and 4.10 in appendix 4.

The findings are that we fail to reject all variables null hypothesis of an existence of a unit root for both the DS and the TS definition. These p-values are high considering the conventional significance levels.

We also constructed a NB test. This test was only appropriate for the (lnWage 2) model as no differencing is applied and that it has one autoregressive lag. The p-value of the test-statistic is 0.58 which further strengthens the evidence of a unit root, see table 4.9 in appendix 4.

Final comments are that we cannot distinguish the DS model from the TS alternative since both models show similar results.

# 4) Conclusions and discussions

The aim of this report is to see if three Swedish economic time series are best described as difference-stationary or trend-stationary in the spirit of Nelson and Plosser (1982).

In addition to the ADF unit root test, we also used the PP test and the NB test (when appropriate) which Nelson and Plosser did not use. The ARIMA procedure was discussed in including evaluation statistics and tests for adequacy. When modifications were necessary, we made adjustments in a way to build more reliable models.

The results obtained from the analysis are that we failed to reject the assumption of a DS model. We attained similar results in the TS model, which makes the two different approaches indistinguishable. We also found evidence that the TS models provided very similar results in terms of the unit root tests which implies that the TS model can be modeled as a DS process since the existence of a unit root could not be denied within that specification.

In terms of assumptions, a DS model should be considered the more appropriate model. It is too artificial to believe that the growth of macroeconomic variables is to be linearly presumed. Also, it would seem more suitable to have a stochastic model than a deterministic model since the natures of the series are stochastic. Even if there are obvious arguments that the power of the tests used in this analysis are generally low, we cannot ignore the results that were shown in the previous section; all the p-values were very high when testing the different unit root tests.

We should also take into account that the scope of this paper was to re-examine Nelson and Plosser's work and see if their conclusions could be generalized into the Swedish macroeconomic data, for certain specified variables. The point is: the power of the test can be argued to be weak without invalidating our work with respect to the research questions made prior to the analysis, but to warn the readers of the fact that the tests applied are fragile.

As discussed at the beginning of section 3, the structure of Inflation and lnWage do not seem to have a linear trend (see figures 3.1.1 and 3.1.3) which is problematic as this research has an underlying assumption of a linear trend. However, the variables were chosen prior to the research. Also, if we make transformations of the data we take this report out of scope and ignoring its purpose.

This analysis has shown similar results as Rudebrusch (1992). However, in this study we also considered the modeling dimension of ARMA models, not only AR models as displayed by Rudebrusch. He concluded that macroeconomic data should be characterized by a DS model, not TS model. We agree with this conclusion as of the **belief** that macroeconomic variables are more stochastic in their features than deterministic. But our results show otherwise.

Further extensions of this paper can be employed. One can for instance analyze quarterly data and see whether results obtained changes versus the case when we have yearly data. This would however complicate the analysis severely as the cyclical variations would be clearer and seasons would surely be detected in the series. The models applied to quarterly data would be more complex as seasonal ARIMA models would probably be of use and the interpretations would be harder to assess.

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# Appendix 1: Definitions of the applied variables

**GDP deflator (Inflation):** “The GDP implicit deflator is the ratio of GDP in current local currency to GDP in constant local currency. The base year varies by country”, see Worldbank(Inflation).

**Nominal GDP (lnGDP):** “GDP at purchaser's prices is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources. Data are in current U.S. dollars. Dollar figures for GDP are converted from domestic currencies using single year official exchange rates. For a few countries where the official exchange rate does not reflect the rate effectively applied to actual foreign exchange transactions, an alternative conversion factor is used”, see Worldbank(lnGDP).

**Workers' remittances and compensation of employees, paid (lnWage):** “Workers' remittances and compensation of employees comprise current transfers by migrant workers and wages and salaries earned by nonresident workers. Remittances are classified as current private transfers from migrant workers resident in the host country for more than a year, irrespective of their immigration status, to recipients in their country of origin. Migrants' transfers are defined as the net worth of migrants who are expected to remain in the host country for more than one year that is transferred from one country to another at the time of migration. Compensation of employees is the income of migrants who have lived in the host country for less than a year. Data are in current U.S. dollars”, see Worldbank(lnWage).

# Appendix 2: Diagnostic outputs

*Table 2.1*

(Inflation 1) – ARIMA(1,1,0)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 7.19 | 5 | 0.2070 |
| 12 | 10.53 | 11 | 0.4839 |
| 18 | 12.55 | 17 | 0.7658 |
| 24 | 16.37 | 23 | 0.8389 |
| AIC | | | 156.2229 |
| SBC | | | 160.0065 |

*Table 2.2*

(Inflation 2) – ARIMA(1,0,0)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 74.82 | 5 | <.0001 |
| 12 | 80.16 | 11 | <.0001 |
| 18 | 119.88 | 17 | <.0001 |
| 24 | 174.32 | 23 | <.0001 |
| AIC | | | 192.001 |
| SBC | | | 197.737 |

*Table 2.3*

(Inflation 2) – ARIMA(2,0,0)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 7.86 | 4 | 0.0968 |
| 12 | 11.54 | 10 | 0.3168 |
| 18 | 13.44 | 16 | 0.6406 |
| 24 | 17.03 | 22 | 0.7615 |
| AIC | | | 159.9993 |
| SBC | | | 167.6474 |

*Table 2.4*

(lnGDP 1) – ARIMA(1,1,0)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 8.93 | 5 | 0.1119 |
| 12 | 19.09 | 11 | 0.0595 |
| 18 | 32.80 | 17 | 0.0119 |
| 24 | 36.72 | 23 | 0.0347 |
| AIC | | | -82.0578 |
| SBC | | | -78.2742 |

*Table 2.5*

(lnGDP 1) – ARIMA(1,1,1)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 7.78 | 4 | 0.0999 |
| 12 | 17.33 | 10 | 0.0674 |
| 18 | 29.29 | 16 | 0.0221 |
| 24 | 33.07 | 22 | 0.0609 |
| AIC | | | -80.9215 |
| SBC | | | -75.246 |

*Table 2.6*

(lnGDP 1) – ARIMA(2,1,2)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 4.91 | 2 | 0.0860 |
| 12 | 11.19 | 8 | 0.1913 |
| 18 | 22.55 | 14 | 0.0679 |
| 24 | 26.44 | 20 | 0.1518 |
| AIC | | | -81.672 |
| SBC | | | -72.2129 |

*Table 2.7*

(lnGDP 1) – ARIMA(3,1,3)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | . | 0 | . |
| 12 | 4.68 | 6 | 0.5851 |
| 18 | 12.34 | 12 | 0.4191 |
| 24 | 16.56 | 18 | 0.5536 |
| AIC | | | -83.0581 |
| SBC | | | -69.8154 |

*Table 2.8*

(lnGDP 2) – ARIMA(1,0,0)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 10.97 | 5 | 0.0520 |
| 12 | 16.96 | 11 | 0.1091 |
| 18 | 31.13 | 17 | 0.0193 |
| 24 | 37.84 | 23 | 0.0265 |
| AIC | | | -79.218 |
| SBC | | | -73.4819 |

*Table 2.9*

(lnGDP 2) – ARIMA(1,0,1)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 6.62 | 4 | 0.1571 |
| 12 | 15.79 | 10 | 0.1057 |
| 18 | 26.64 | 16 | 0.0457 |
| 24 | 30.61 | 22 | 0.1044 |
| AIC | | | -83.5126 |
| SBC | | | -75.8645 |

*Table 2.10*

(lnWage 1) – ARIMA(1,1,0)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 4.07 | 5 | 0.5395 |
| 12 | 5.85 | 11 | 0.8834 |
| 18 | 12.56 | 17 | 0.7650 |
| 24 | 15.09 | 23 | 0.8915 |
| AIC | | | -5.17126 |
| SBC | | | -1.7935 |

*Table 2.11*

(lnWage 1) – ARIMA(1,1,1)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 3.49 | 4 | 0.4790 |
| 12 | 4.69 | 10 | 0.8987 |
| 18 | 11.66 | 16 | 0.7668 |
| 24 | 14.21 | 22 | 0.8939 |
| AIC | | | -3.35838 |
| SBC | | | 1.708257 |

*Table 2.12*

(lnWage 1) – ARIMA(2,1,2)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 3.49 | 2 | 0.1747 |
| 12 | 4.88 | 8 | 0.7703 |
| 18 | 11.66 | 14 | 0.6337 |
| 24 | 14.20 | 20 | 0.8201 |
| AIC | | | 0.641602 |
| SBC | | | 9.085999 |

*Table 2.13*

(lnWage 2) – ARIMA(1,0,0)

| To Lag | Chi-Square | DF | Pr > ChiSq |
|---|---|---|---|
| 6 | 4.54 | 5 | 0.4740 |
| 12 | 6.45 | 11 | 0.8421 |
| 18 | 12.87 | 17 | 0.7449 |
| 24 | 15.19 | 23 | 0.8877 |
| AIC | | | -4.5157 |
| SBC | | | 0.625013 |

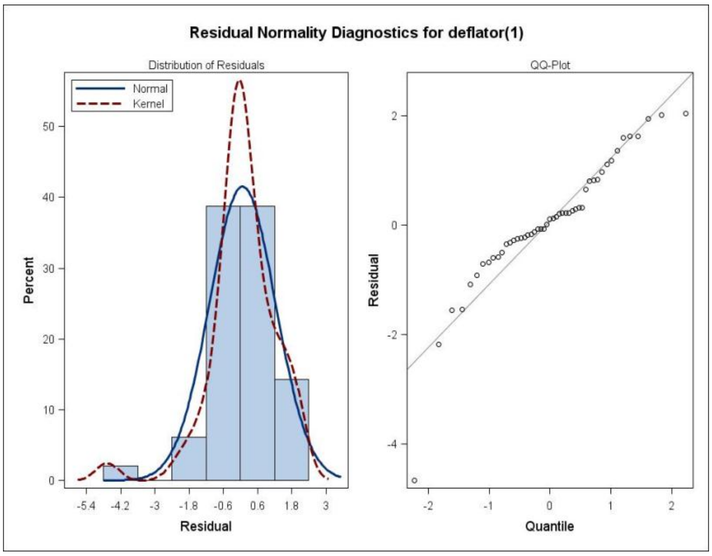


*Figure 2.1*

(Inflation 1) – ARIMA(1,1,0)

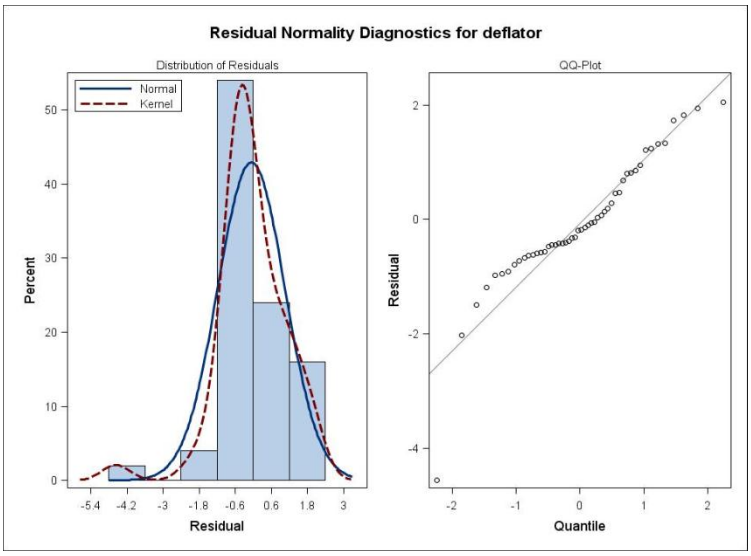


*Figure 2.2*

(Inflation 2) – ARIMA(2,0,0)

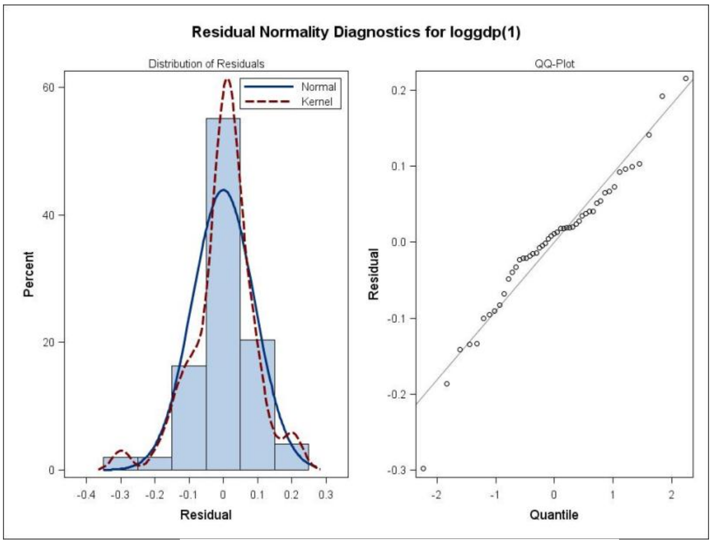


*Figure 2.3*

(lnGDP 1) – ARIMA(3,1,3)

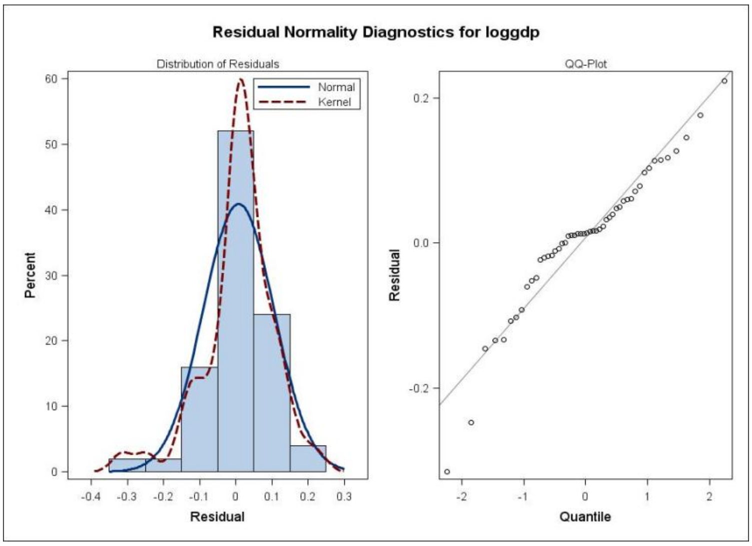


*Figure 2.4*

(lnGDP 2) – ARIMA(1,0,1)

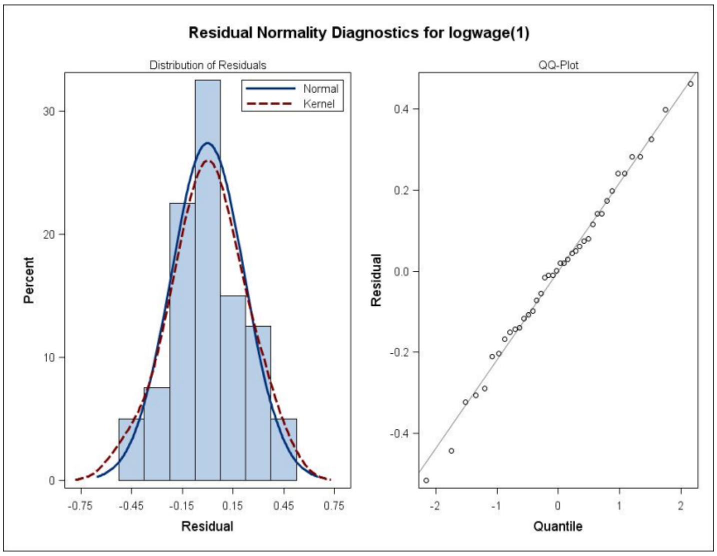


*Figure 2.5*

(lnWage 1) – ARIMA(1,1,0)

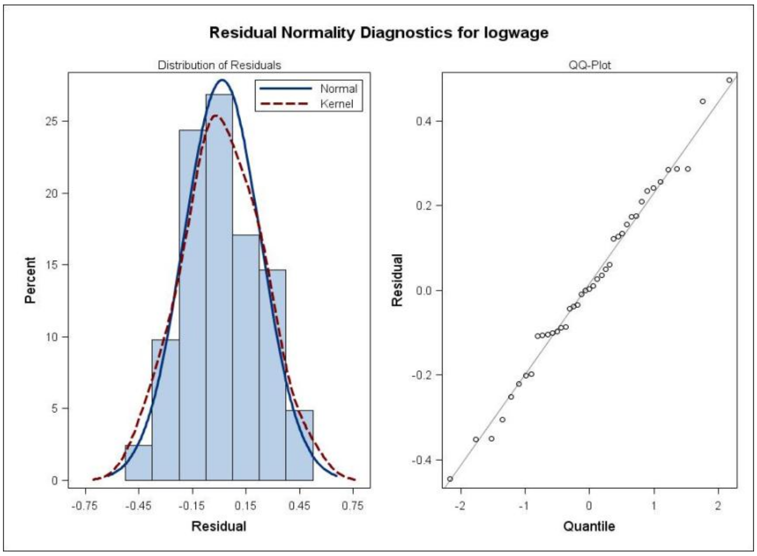


*Figure 2.6*

(lnWage 2) – ARIMA(1,0,0)

# Appendix 3: Estimation for the preferred models

*Table 3.1*

(Inflation 1) – ARIMA(1,1,0)

| Parameter | Estimate | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|
| MU | 1.96040 | 0.51016 | 3.84 | 0.0004 |
| AR1,1 | 0.69862 | 0.10587 | 6.60 | <.0001 |

Note: MU is the estimate for the constant in the model.

*Table 3.2*

(Inflation 2) – ARIMA(2,0,0)

| Parameter | Estimate | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|
| MU | -4198.6 | 390.43229 | -10.75 | <.0001 |
| AR1,1 | 1.68748 | 0.10546 | 16.00 | <.0001 |
| AR1,2 | -0.70771 | 0.10588 | -6.68 | <.0001 |
| NUM1 | 2.14663 | 0.19910 | 10.78 | <.0001 |

Note: the NUM1 is the deterministic trend parameter. This is true for all the following outputs.

*Table 3.3*

(lnGDP 1) – ARIMA(3,1,3)

| Parameter | Estimate | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|
| MU | 0.07016 | 0.01738 | 4.04 | 0.0002 |
| MA1,1 | -1.58110 | 0.30358 | -5.21 | <.0001 |
| MA1,2 | -1.35405 | 0.39918 | -3.39 | 0.0015 |
| MA1,3 | -0.71397 | 0.26326 | -2.71 | 0.0096 |
| AR1,1 | -1.14267 | 0.29317 | -3.90 | 0.0003 |
| AR1,2 | -0.84925 | 0.29339 | -2.89 | 0.0060 |
| AR1,3 | -0.70658 | 0.19947 | -3.54 | 0.0010 |

*Table 3.4*

(lnGDP 2) – ARIMA(1, 0,1)

| Parameter | Estimate | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|
| MU | -120.24445 | 13.88135 | -8.66 | <.0001 |
| MA1,1 | -0.35521 | 0.15535 | -2.29 | 0.0269 |
| AR1,1 | 0.91675 | 0.07208 | 12.72 | <.0001 |
| NUM1 | 0.07331 | 0.0070650 | 10.38 | <.0001 |

*Table 3.5*

(lnWage 1) – ARIMA(1,1,0)

| Parameter | Estimate | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|
| MU | 0.08203 | 0.04138 | 1.98 | 0.0547 |
| AR1,1 | 0.15808 | 0.16211 | 0.98 | 0.3357 |

*Table 3.6*

(lnWage 2) – ARIMA(1, 0,0)

| Parameter | Estimate | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|
| MU | -159.72351 | 38.62589 | -4.14 | 0.0002 |
| AR1,1 | 0.93928 | 0.07096 | 13.24 | <.0001 |
| NUM1 | 0.08975 | 0.01958 | 4.58 | <.0001 |

# Appendix 4: Unit root test outputs and tables

*Table 4.1: (Inflation 1)*

Parameter Estimates

| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| Intercept | Intercept | 1 | -275.76949 | 134.35244 | -2.05 | 0.0461 |
| deflator_1st_LAG | | 1 | 0.94812 | 0.02516 | 37.68 | <.0001 |
| deflator_1st_DIFF_1st_LAG | | 1 | 0.66102 | 0.10595 | 6.24 | <.0001 |
| t | t | 1 | 0.14078 | 0.06839 | 2.06 | 0.0455 |

Note: “deflator” is Inflation, deflator_1st_LAG is $y_{t-1}$ which is the parameter of interest, deflator_1st_DIFF_1st_LAG is $\Delta y_{t-1}$ and t is the trend coefficient. The same notations hold for the remaining regression tables.

*Table 4.2: (Inflation 1)*

Phillips-Perron Unit Root Tests

| Type | Lags | Tau | Pr < Tau |
|---|---|---|---|
| Trend | 1 | -2.07 | 0.5484 |

Note: “Tau” is the test statistic.

*Table 4.3: (Inflation 2)*

Augmented Dickey-Fuller Unit Root Tests

| Type | Lags | Tau | Pr < Tau |
|---|---|---|---|
| Trend | 2 | -2.14 | 0.5096 |

*Table 4.4: (Inflation 2)*

Phillips-Perron Unit Root Tests

| Type | Lags | Tau | Pr < Tau |
|---|---|---|---|
| Trend | 2 | -2.08 | 0.5458 |

*Table 4.5: (lnGDP 1)*

Parameter Estimates

| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| Intercept | Intercept | 1 | -10.63920 | 7.77754 | -1.37 | 0.1790 |
| loggdp_1st_LAG | | 1 | 0.87304 | 0.06876 | 12.70 | <.0001 |
| loggdp_1st_DIFF_1st_LAG | | 1 | 0.35987 | 0.15983 | 2.25 | 0.0299 |
| loggdp_1st_DIFF_2nd_LAG | | 1 | -0.13654 | 0.17640 | -0.77 | 0.4435 |
| loggdp_1st_DIFF_3rd_LAG | | 1 | -0.02626 | 0.17024 | -0.15 | 0.8782 |
| t | t | 1 | 0.00701 | 0.00476 | 1.47 | 0.1483 |

*Table 4.6: (lnGDP 2)*

Augmented Dickey-Fuller Unit Root Tests

| Type | Lags | Tau | Pr < Tau |
|---|---|---|---|
| Trend | 3 | -1.85 | 0.6663 |

*Table 4.7: (lnWage 1)*

Parameter Estimates

| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| Intercept | Intercept | 1 | -13.85526 | 12.85959 | -1.08 | 0.2887 |
| logwage_1st_LAG | | 1 | 0.84067 | 0.07995 | 10.52 | <.0001 |
| logwage_1st_DIFF_1st_LAG | | 1 | 0.18057 | 0.16302 | 1.11 | 0.2756 |
| t | t | 1 | 0.00854 | 0.00715 | 1.19 | 0.2403 |

*Table 4.8: (lnWage 1)*

Phillips-Perron Unit Root Tests

| Type | Lags | Tau | Pr < Tau |
|---|---|---|---|
| Trend | 1 | -1.75 | 0.7106 |

*Table 4.9: (lnWage 2)*

Augmented Dickey-Fuller Unit Root Tests

| Type | Lags | Rho | Pr < Rho | Tau | Pr < Tau |
|---|---|---|---|---|---|
| Trend | 1 | -7.5830 | 0.5787 | -1.99 | 0.5874 |

Note: “Rho” is the test statistic for the NB test.

*Table 4.10: (lnWage 2)*

Phillips-Perron Unit Root Tests

| Type | Lags | Tau | Pr < Tau |
|---|---|---|---|
| Trend | 1 | -1.75 | 0.7106 |

*Output 4.1 (Inflation)*

p=0.2533529811

*Output 4.2* (lnGDP)

p=0.3387744193

*Output 4.3(lnWage)*

p=0.2858581237