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Optimal pricing and quantity of products with two offerings

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Abstract

This paper analyzes the decision of a firm offering two versions of a product, a deluxe and a regular. While both products satisfy the same market, the deluxe version is sold at a high price relative to its cost and is aimed at the high end of the demand curve. The regular version is sold at a low price relative to its cost and is targeted to customers at the low end of the demand curve. This two-offering strategy is especially popular with book publishers where a paperback book is introduced some time after the hardbound version is introduced. The time between the introduction of the two versions of the product is accompanied by a downward shift in the demand curve due to customers losing interest in the product or satisfying their demand from a secondary used market. We solve a profit maximization model for a firm using a two-offering strategy. The model is solved for linear and exponential deterioration in demand, which is assumed to be deterministic. Also, a model with linear deterioration in demand, which is assumed to be stochastic, is solved. The results indicate that substantial improvements in profit can be obtained by using the two-offering strategy. Numerical sensitivity analysis and examples are used to illustrate the results.

Keywords: Marketing; Pricing strategy; Decision analysis

1. Background

Pricing is among the most important decisions a manger can make. The marketing literature is rich in terms of delineating the different pricing strategies a firm can follow under different conditions (Noble and Gruca, 1999). These conditions include degree of product differentiation, the competitive situation, and the nature of demand. Skimming and penetration are the classic strategies for pricing new products (Noble and Gruca, 1999; Simon, 1992). A skimming strategy is one in which a firm sets a high introduction price and then systematically reduces that price. This strategy is most appropriate when products are highly differentiated products, a segment of the market is price-insensitive, and there is limited economy of scope or learning curve effects.

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Many companies follow a specific type of a skimming pricing strategy in which essentially the same product is offered at two prices at different periods of time. The first offering has a higher price and is aimed at customers at the high end of the price range of the demand curve. For these customers, obtaining the product early is important and they are willing to pay a premium for early ownership. The second offering is aimed at customers at the lower price range of the demand curve. For example, book publishers first introduce books in hardbound version. The publishers skim the market on average for a year before re-introducing the book in paperback form (Burleson, 2002). This market is quite large with hardbound books total sales of $2.63 billion in 2001 (Platt and Gwiazdowski, 2002). The question one must ask is how are prices for both hardbound and paperback books established for potential bestsellers by the publishing industry.

This paper analyzes the pricing decisions of a firm using a two-offerings pricing strategy such as in the publishing business. Of particular interest is the ability of a firm to skim the market for some duration by introducing a higher price deluxe version of the product (e.g. hardbound books) before re-introducing the product in regular version (e.g. paperback books) (Burleson, 2002). Firms that can use such a strategy must have some sort of patent or copyright which protects them from competitive actions. Publishers for example are able to use such a strategy because they have the publication copyright and therefore can act as monopolists. The decisions facing a firm are how many units of deluxe version of the product to offer and at what price and how many units of the regular version to offer and at what price. Offering a large quantity of the deluxe version at relatively low price means the firm must wait longer before re-introducing the regular version, which increases the number of customers who will lose interest in the product or satisfy their demand from a secondary used market. In the case of books, consumers may borrow a hardbound copy from a friend or a library or buy a hardbound copy in the secondary used market. The loss of customers implies a downward shift in the demand function which is proportional to the quantity of hardbound copies sold in the first offering.

In spite of the importance of pricing, prices in many firms are still determined by ad hoc methods that are based more on opinion and intuition rather than using a systematic approach (Simon, 1992). In this paper, we formulate and solve a profit maximization model for a firm using a two-offering strategy. The model is solved for linear and exponential deterioration in demand, which is assumed to be deterministic. Also, a model with linear deterioration in demand, which is assumed to be stochastic, is solved. The remainder of this paper is organized into five sections: In Section 2 we present a brief review of related literature. In Section 3, we formulate and solve the deterministic models. In Section 4, we formulate and solve the stochastic model. Section 5 contains a discussion and numerical sensitivity analysis and Section 6 presents the conclusion and suggestions for future research. The numerical examples are only for illustration and are not for providing general conclusions. Such conclusions are left for sensitivity analysis.

2. Literature review

Product pricing have a direct impact on profits and therefore is among the most important decisions a manager can make. A popular approach to improving profits is to skim the market by charging a higher price initially and then reducing the price systematically. This approach requires a company to have at least some sort of a "temporary monopoly position" (Noble and Gruca, 1999). This requirement makes this approach ideal for companies enjoying a monopoly position for the entire life cycle of the product, which is typical of copyrighted products such as books, movies, and software packages, and companies with patent protection. These firms enjoy a natural monopoly position and therefore can use different pricing schemes to skim the market. These companies use minor variations in the product to avoid charging different prices for identical products which can be annoying to customers. Skimming the market is accomplished by introducing a deluxe version of the product which has small extra features such as hardbound books and movies with extra clips. These features incur a minor additional cost relative to the total production and royalty costs.

The economic literature suggests that the optimal pricing strategy of a monopolist facing uncertain demand depends on his/her production capacity. Harris and Raviv (1981) analyzed a case in which there is a single monopolist selling to N potential customers. The monopolist has a homogeneous product which has constant marginal costs. Each of the N potential buyers is willing to buy up to one unit at or below his/her own reservation price. If capacity limits exceeds potential demand, or when those limits can be chosen by the monopolist at low cost, then a single price scheme is shown to be optimal. The rational for this result is that when capacity exceeds potential demand the monopolist cannot offer multiple prices because all buyers will choose to pay the lowest price. In order to be able to charge different prices, the seller must create scarcity by making clear that he/she will not sell more than some quantity which is below N. While this may be true for some products, the time aspect which some firms exploit enables them to create scarcity for an initial period of time before the introduction of the lower cost version of the product.

An important factor to consider in pricing is the saturation effect which describes the shrinkage in the potential number of customers with increased market penetration (Raman and Chatterjee, 1995). The saturation effect may be attributed to several factors which include loss of interest by customers over time, the ability to satisfy demand using other sources such as borrowing the product (e.g. books from library, movies from video rental stores), or purchasing a used product in the secondary used market (e.g. used book stores). This saturation effect can be modeled as a downward shift in the demand function.

A related new area of research in which pricing is used to improve profits is the perishable asset yield management (PARM) or simply yield management (Weatherford and Bodily, 1992) in which two or more prices are used to sell the same product. PARM subsumes penetration, skimming, and other pricing strategies. Much of the research in this approach have focused on a firm rationing lower priced units sold first (Dana, 2001), which is opposite to skimming. This approach is heavily used in the airline industry. In general, the goal of yield management is to maximize revenue (Weatherford and Bodily, 1992) by attempting to synthesize a range of optimal prices from a static set of prices in response to a shifting demand function (Gallego and van Ryzin, 1994). Weatherford and Bodily (1992) define yield management as the optimal revenue management of perishable assets through price segmentation. Feng and Gallego (1995) developed a model for deciding the optimal timing of a single price change from a given initial price to either a given lower or higher second price. The firm in this case has a fixed stock of items to sell over a finite horizon which is applicable to airlines selling seats before planes depart, hotels renting rooms before midnight, and retailers selling seasonal items with long procurement lead times. Feng and Xiao (2000) extended Feng and Gallego's (1995) model to deal with more than two prices. In a more recent paper, Feng and Gallego (2000) analyzed the problem of deciding on the optimal timing of price changes within a given a set of permissible price paths with markovian time dependent demand intensities. The papers by Feng and Xiao (2000) and Feng and Gallego (2000) contain a review of yield management research.

The problem addressed in this paper differs from the classic yield management problem in two main aspects: First, the product sold at the two different prices does not have the same cost. There is a cost, albeit small, to giving an image of a deluxe version. Second, the product is not perishable at a certain point in time like an airline seat or a hotel room, rather it can be used to satisfy the demand of other customers once the customer who purchased it is finished with it. Therefore, there is a saturation effect which can be viewed as a downward shift in the demand function that is proportional to the total quantity sold at the first price.

It is noteworthy to point out that the type of products dealt with in this paper have some characteristics similar to perishable products. However, there are some differences which merit a separate treatment. Perishable products whose management is addressed in the single-period news-vendor model (Khouja, 1999) have a defined point in time (or a time interval) when interest in the product quickly vanishes. Examples of these products include newspapers, magazines, holiday products, style goods, and many food products. These products are single-period products because at some point in time (end of day for the newspaper or expiration date for food) the product is no longer sellable, or interest in the product quickly deteriorates once the season (in case of style goods) draws to an end. The type of products we address in this paper, such as books, music compact disks, and movies do not have such a well defined point in time when the product is no longer sellable or usable. Furthermore, the selling firm in this paper is assumed to enjoy a monopoly position for the product.

3. Deterministic deteriorating demand

The delay between introducing the deluxe version of the product and the re-introduction of the regular version allows some customers at the low end of the demand curve to satisfy their demand by obtaining a copy of the deluxe product as a loan or by purchasing it in the secondary used market. The larger the quantity of the deluxe product sold, the longer it takes to sell it before re-introducing the regular version and the longer the time the deluxe version can circulate causing a downward shift in the demand curve as shown in Fig. 1. The magnitude of the shift can be assumed to be linear in the quantity of the deluxe version introduced or, because of the additional loss of interest due to the passage of time, exponential. For both linear and exponential deterioration in demand, the following notation is needed (a summary of all three models and their notation is shown in Tables 1 and 2):

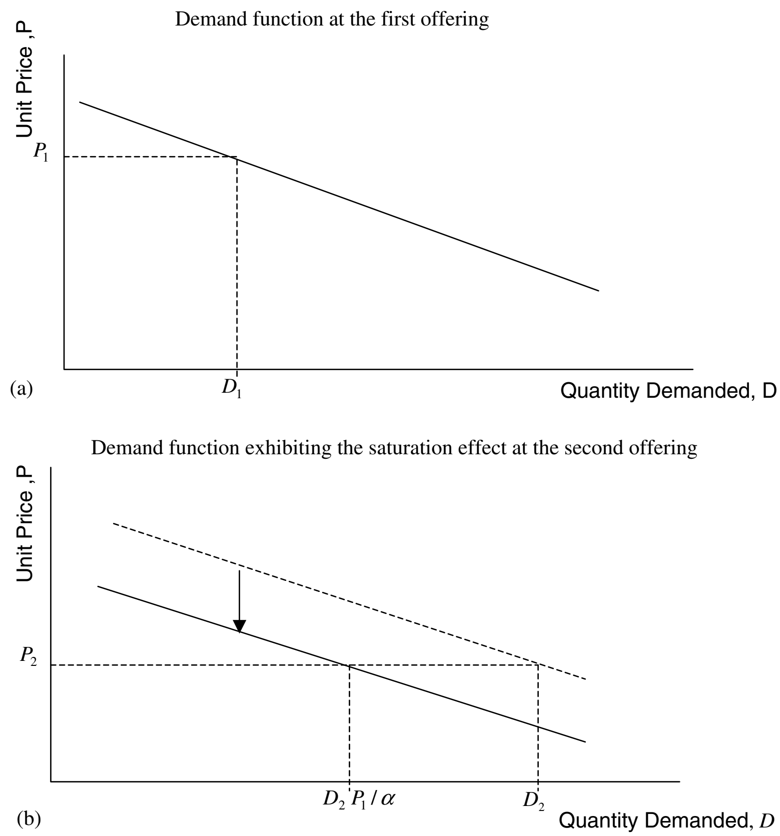

Fig. 1. Demand at two-offering pricing strategy: (a) demand function at the first offering and (b) demand function exhibiting the saturation effect at the second offering.

Table 1
Summary of the three models

	Model		
	I	II	III
Demand as function of unit price	Linear	Linear	Linear
Nature of demand	Deterministic	Deterministic	Stochastic (uniform)
Deterioration in demand function due to saturation	Linear	Exponential	Linear
Incremental demand at second offering deteriorates according to	$\alpha P_1/P_0$	$e^{\beta(P_1-P_0)}$	$\alpha P_1/P_0$
Assumptions/justifications	Loss of interest is slow and products circulate at low speed	Lost of interest accelerates and products circulate at high speed	Same as linear
Profitability of two-offerings is increasing in	α		α
Profitability of two-offering strategy is decreasing in	b, C_2	b, C_2, β	b, C_2

C_1 per unit cost of the deluxe version of the product,
C_2 per unit price of the regular version of the product,
P_1 per unit selling price of the deluxe version of the product,
Q_1 the quantity of the deluxe version of the product to offer (at price of P_1),
P_2 per unit selling of the regular version of the product,
Q_2 the quantity of the regular version of the product to offer (at price of P_2),
$D(P)$ demand as a function of price,
D_0 the quantity demanded at unit price of zero,
P_0 the per unit price at which demand becomes zero, and
b the decrease in demand due to a one dollar increase in price.

3.1. Model I: Linear deterioration of deterministic demand

Similar to Harris and Raviv (1981) we analyze a single monopolist selling to N potential customers. Each of the N potential customers is willing to buy one unit at or below his/her own reservation price and the reservation prices are uniformly distributed on a continuum. This results in a linear demand function given by

$$D(P) = D_0 - bP. \tag{1}$$

Potential customers reservation prices decrease with time and the amount of the deluxe product sold in the market (this is similar to saying some customers lose interest or satisfy their demand by some other means). Assuming the firm will produce exactly the quantity demanded at each price, then the quantity produced at the first price (P_1) is

$$Q_1 = (D_0 - bP_1). \tag{2}$$

Ignoring the saturation effect, the additional quantity demanded at the second price (P_2) is $[(D_0 - bP_2) - (D_0 - bP_1)]$. However due to the saturation effect, not all of that demand is realized. The proportion of the demand realized decreases the larger the penetration of the market (i.e. the smaller the value of P_1). We assume that the proportion of the incremental demand realized is given by $\alpha P_1/P_0$, where α is an empirically determined constant which can be estimated using past data for similar products. We assume $\alpha \leqslant 1$ which implies that any initial offering of the deluxe product will have negative effect on the demand of the regular product. Therefore, the quantity sold at price P_2 is

Table 2
Definition of notation

Symbol	Definition
C_1	Per unit cost of the deluxe version of the product
C_2	Per unit price of the regular version of the product
P	Per unit price of the product
P_1	Per unit selling price of the deluxe version of the product
P_2	Per unit selling of the regular version of the product
Q_1	The quantity of the deluxe version of the product to offer (at price of P_1)
Q_2	The quantity of the regular version of the product to offer (at price of P_2)
$D(P)$	Demand as a function of price
P_0	The per unit price at which demand becomes zero
b	The decrease in demand due to a one dollar increase in price
D_0	The quantity demanded at unit price of zero
α	A parameter of deterioration in demand for the linear deterioration case
β	A parameter of deterioration in demand for the exponential deterioration case
V_1	Per unit discount price of the deluxe version of the product
V_2	Per unit discount price of the regular version of the product
S_1	Per unit penalty shortage cost of the deluxe version of the product
S_2	Per unit penalty shortage cost of the regular version of the product
G	The range of the uniform distribution of demand for the stochastic demand model
Z	Profit

$$Q_2 = [(D_0 - bP_2) - (D_0 - bP_1)]\alpha P_1/P_0 \tag{3}$$

and the total profit from the two offerings is

$$Z = (P_1 - C_1)(D_0 - bP_1) + (P_2 - C_2)[(D_0 - bP_2) - (D_0 - bP_1)]\alpha P_1/P_0. \tag{4}$$

The partial derivative of Z with respect to P_2 is

$$\frac{\partial Z}{\partial P_2} = \frac{b\alpha P_1(P_1 + C_2 - 2P_2)}{P_0}. \tag{5}$$

A necessary condition for optimality of P_2 is to set $\partial Z/\partial P_2 = 0$ which gives

$$P_2^* = (C_2 + P_1)/2. \tag{6}$$

Eq. (6) can be re-written as $P_1^* - P_2^* = P_2^* - C_2$. Adding and subtracting C_2 from the left hand side gives $P_1^* - C_2 - (P_2^* - C_2) = P_2^* - C_2$ which can be re-written as $P_1^* - C_2 = 2(P_2^* - C_2)$. Suppose that the cost of giving the image of a deluxe version of the product is small (in other words, C_1 and C_2 are very close), then Eq. (2) can be interpreted as the profit margin from selling the deluxe version of the product is almost twice the profit margin from selling the regular version of the product.

Substituting from (6) into (4) and taking the partial derivative with respect to P_1 gives

$$\frac{\partial Z}{\partial P_1} = \frac{\alpha bC_2^2 + 4P_0(bC_1 + D_0) - bP_1[4\alpha C_2 + 8P_0 - 3\alpha P_1]}{4P_0}. \tag{7}$$

A necessary condition for optimality of P_1 is to set $\partial Z/\partial P_1 = 0$ which gives

$$P_1^* = \frac{2}{3}C_2 + \frac{4P_0}{3\alpha} - \frac{\sqrt{16(\alpha C_2 b + 2bP_0)^2 - 12\alpha b[\alpha bC_2^2 + 4(bC_1 + D_0)P_0]}}{6b\alpha}. \tag{8}$$

Substituting from (8) back into (6) gives

$$P_2^* = \frac{5}{6}C_2 + \frac{2P_0}{3\alpha} - \frac{\sqrt{16(\alpha C_2 b + 2bP_0)^2 - 12\alpha b[\alpha b C_2^2 + 4(bC_1 + D_0)P_0]}}{12b\alpha}. \tag{9}$$

Substituting for P_1^* and P_2^* in Eqs. (2) and (3) gives the optimal quantities at the first and second offerings as

$$Q_1^* = D_0 + \frac{\sqrt{16(\alpha C_2 b + 2bP_0)^2 - 12\alpha b[\alpha b C_2^2 + 4(bC_1 + D_0)P_0]}}{6\alpha} - \frac{2}{3}b\left(C_2 + \frac{2P_0}{\alpha}\right) \tag{10}$$

and

$$Q_2^* = \frac{64bP_0^2 + 8\alpha(5C_2b - 3D_0 - 3bC_1) - 2b\alpha^2C_2^2 - (\alpha C_2 + 8P_0)\sqrt{16(b\alpha C_2 + 2bP_0)^2 - 12\alpha b[\alpha b C_2^2 + 4(bC_1 + D_0)P_0]}}{36\alpha P}, \tag{11}$$

respectively.

3.2. Numerical example 1

Consider an example of a publisher about to publish a book with estimated demand function $D(P) = 1{,}600{,}000 - 75{,}000P$ which implies a 75,000 units drop in demand for each \$1.00 increase in price. Also, $C_1 = \$5.00$, $C_2 = \$2.00$, and $\alpha = 1$. Using Eq. (1) gives $P_0 = \$21.33$ and using Eqs. (8) and (9) gives an optimal first offering price for the deluxe version of product of $P_1^* = \$18.10$ per unit and a second optimal offering price for the regular version of product of $P_2^* = \$10.05$ per unit. Using Eqs. (10) and (11) gives optimal quantities of $Q_1^* = 242{,}420$ units and $Q_2^* = 512{,}308$ units for the first and second offerings, respectively. Substituting into Eq. (4) gives an optimal total profit of $Z^* = \$7{,}300{,}000$. If management uses only a single offering of the regular version of the product, then the profit is $Z = (P - C_1)(D_0 - bP)$ which, using the first order conditions, gives an optimal price of $P^* = (D_0 + bC_2)/2b = \$11.67$ per unit, an optimal production quantity of $Q^* = 700{,}000$ units, and an optimal profit of $Z^* = \$7{,}008{,}000$. Using two versions of the product results in an increase of \$292,000 (or 4%) in profit.

3.3. Model II: exponential deterioration of deterministic demand

In this case, we assume that customers lose interest at an increasing rate due to the increased number of units of the deluxe version of product in the market. Exponential deterioration in demand as a function of the quantity sold at the first price is used to reflect this assumption. The demand at the second offering is

$$Q_2 = [(D_0 - bP_2) - (D_0 - bP_1)]e^{\beta(P_1 - P_0)}, \tag{12}$$

where β is an empirically determined constant which can be estimated using past data for similar products. The profit from the two offerings is

$$Z = (P_1 - C_1)(D_0 - bP_1) + (P_2 - C_2)[(D_0 - bP_2) - (D_0 - bP_1)]e^{\beta(P_1 - P_0)}. \tag{13}$$

Using the McClaurin series expansion results in the following approximation:

$$e^{\beta(P_1 - P_0)} = \frac{1}{2}e^{\beta(1 - P_0)}[2 + (\beta - 2)\beta + \beta P_1(2 - 2\beta + \beta P_1)]. \tag{14}$$

Substituting from Eq. (14) into Eq. (13) gives

$$Z = (P_1 - C_1)(D_0 - bP_1) + \frac{1}{2}be^{\beta(1 - P_0)}(P_1 - P_2)(P_2 - C_2)[2 + (\beta - 2)\beta + \beta P_1(2 - 2\beta + \beta P_1)]. \tag{15}$$

Setting the partial derivative of Z with respect to P_2 to zero (i.e. $\partial Z/\partial P_2 = 0$) gives (6) which is then substituted into (15) and used to obtain $\partial Z/\partial P_1$. However, $\partial Z/\partial P_1$ is a third degree polynomial and the solution to $\partial Z/\partial P_1 = 0$ is relatively complex as shown in Appendix A. An iterative solution can be obtained by solving $\partial Z/\partial P_1 = 0$ (without substituting for P_2) which gives

$$P_1 = \frac{e^{-\beta}}{3b\beta^2(C_2 - P_2)}\Bigg[b\beta e^{\beta}(C_2 - P_2)(\beta P_2 - 2\beta - 2) - 2be^{\beta P_0} + e^{\beta P_0}\Bigg[b(4b + 2e^{\beta-\beta P_0}\beta(P_2 - C_2)(4b(\beta - 1) - 3\beta(bC_1 + D_0) + 2b\beta P_2) + be^{-2\beta(P_0-1)}\beta^2(C_2 - P_2)^2((\beta - 2)\beta - 2 + \beta P_2(2 - 2\beta + \beta P_2)))\Bigg]^{1/2}\Bigg]. \tag{16}$$

Eqs. (6) and (16) can be used iteratively until convergence to find P_1^* and P_2^*. Eq. (6) can be initiated with a reasonable estimate of P_1 in the range of $3C_1$ to $5C_1$. The optimal order quantities are obtained using Eqs. (2) and (12).

3.4. Numerical example 2

Returning to example 1, we now use exponential decay in demand with $\beta = 0.0556$ which gives the same deterioration at $P_1 = P_0$ and $P_1 = \$15$ as the linear case. (Deterioration takes exponential shape. The common point at 15 was chosen arbitrarily.) Using Eqs. (6) and (16) and starting with $P_1 = 3C_1$ in Eq. (6) iteratively converges to a first optimal offering price for the deluxe version of the product of $P_1^* = \$16.79$ per unit and a second optimal offering price for the regular version of the product of $P_2^* = \$9.40$ per unit. Eqs. (2) and (12) give a corresponding optimal quantities of $Q_1^* = 340{,}750$ units and $Q_2^* = 430{,}525$ units and Eq. (15) results in optimal total profit of $Z^* = \$7{,}203{,}000$. As in example 1, if management uses only a single offering of the regular version of the product, then the optimal price is $P^* = (D_0 + bC_2)/2b = \$11.67$ per unit, the optimal quantity is $Q^* = 700{,}000$ units, and the optimal profit is $Z^* = \$7{,}008{,}000$ (because all potential customers with reservation price equal to or greater than P^* will purchase the product, there is no difference between linear and exponential deterioration in demand for the single price solution). Using two versions of the product results in an increase of \$195,000 (or 2.8%) in profit.

4. Model III: Stochastic demand case

In this case, demand at any given price is a random variable with an expected value given by Eq. (1). Similar to the classic single-period news-vendor problem (Khouja, 1999), any quantity not sold at the offered price must be discounted and unsatisfied demand may incur a penalty due to the loss of goodwill with customers. The following additional notation is needed:

- V_1 per unit discount price of the deluxe version of the product,
- V_2 per unit discount price of the regular version of the product,
- S_1 per unit penalty shortage cost of the deluxe version of the product, and
- S_2 per unit penalty shortage cost of the regular version of the product.

The profit is a random variable with an expected value:

$$E\{Z\} = \int_{Q_1}^{+\infty}[Q_1P_1 - (D_1 - Q_1)S_1]f(D_1)\,dD_1 + \int_0^{Q_1}[D_1P_1 + (Q_1 - D_1)V_1]f(D_1)\,dD_1 - C_1Q_1 + \int_{Q_2}^{+\infty}[Q_2P_2 - (D_2 - Q_2)S_2]f(D_2)\,dD_2 + \int_0^{Q_2}[D_2P_2 + (Q_2 - D_2)V_2]f(D_2)\,dD_2 - C_2Q_2. \tag{17}$$

Suppose that at any unit price demand is uniformly distributed with an expected value:

$$E\{D(P)\} = D_0 - bP \tag{18}$$

and range:

$$G = 2RE\{D(P)\}. \tag{19}$$

Assuming linear deterioration in the mean demand, the expression for the expected profit ($E\{Z\}$) is derived in Appendix A. Let

$$h_1 = S_1 - V_1, \tag{20}$$

$$k_1 = S_1 + V_1, \tag{21}$$

$$h_2 = S_2 - V_2, \tag{22}$$

$$k_2 = S_2 + V_2, \tag{23}$$

$$R_1 = 1 + R, \tag{24}$$

$$R_2 = 1 - R, \tag{25}$$

$$U_1 = P_1 + S_1 - V_1, \tag{26}$$

$$U_2 = P_2 + S_2 - V_2, \tag{27}$$

$$W_1 = h_1 + RK_1 + R_1P_1, \tag{28}$$

$$W_2 = h_2 + RK_2 + R_1P_2. \tag{29}$$

Setting $\partial E\{Z\}/\partial Q_1 = 0$ gives the following necessary condition for Q_1 to be optimal:

$$Q_1 = \frac{(D_0 - bP_1)[P_0(W_1 - 2RC_1)U_1 - 2RP_1(S_2V_2R + RC_2^2 + P_2(S_2 - V_2R_2 + P_2) - C_2W_2)]}{P_0U_1U_2}. \tag{30}$$

Substituting from Eq. (30) into Eq. (A.8) of Appendix A and setting $\partial E\{Z\}/\partial Q_2 = 0$ gives the following necessary condition for Q_2 to be optimal:

$$Q_2 = \frac{P_1(D_0 - Q_1 - bP_2)(W_2 - 2RC_2)}{P_0U_2}. \tag{31}$$

Closed-form expressions for the optimal P_1 and P_2 require solving fourth degree polynomials and are shown in Appendix A. However, because both models have linear deterioration, we can use the optimal prices from model I in model III to avoid the complex iterative solution to model III. The optimal order quantities which are derived in closed-form for model III are used to adjust for the stochastic nature of demand. Therefore, Eqs. (8) and (9) with $\alpha = 1$ are used to obtain P_1 and P_2, which are substituted into Eq. (30) to obtain Q_1, and then into Eq. (31) to obtain Q_2.

4.1. Numerical example 3

Returning to example 1, the mean demand is $E\{D(P)\} = 1{,}600{,}000 - 75{,}000P$ with a range of 0.2 $E\{D(P)\}$ which implies $R = 0.10$. Also, $S_1 = \$1.00$, $S_2 = \$0.50$, $V_1 = \$2.00$, and $V_2 = \$0.00$. As described above, to avoid a complex iterative process, we solve the problem using the deterministic demand case (which was done in example 1) and obtain price estimates of $P_1 = \$18.10$ and $P_2 = \$10.05$. Substituting in Eqs. (30) and (31) give $Q_1^* = 239{,}181$ and $Q_2^* = 547{,}035$ with a total expected profit given by (A.8) of

$E\{Z^*\} = \$7{,}115{,}490$. Obtaining the exact optimal solution requires using $P_1 = \$18.10$ and $P_2 = \$10.05$ from example in Eqs. (30) and (31) to obtain initial values for Q_1 and Q_2 which are in turn used in Eqs. (A.9)–(A.20) to obtain new values for P_1 and P_2 and this iterative process is continued until convergence. The procedure gives an optimal offering price for the deluxe version of the product of $P_1^* = \$18.37$ per unit with a quantity of $Q_1^* = 218{,}934$ units, an optimal offering price for the regular version of the product of $P_1^* = \$10.27$ per unit with a quantity of $Q_2^* = 559{,}033$ units with a maximum expected total profit of $E\{Z^*\} = \$7{,}118{,}000$. The use of optimal prices from the deterministic model with Eqs. (30) and (31) (without the iterative process) results in a small decrease in profit of 0.036% (less than \$4 per \$10,000) while being less computationally complex.

5. Discussion and sensitivity analysis

We use the case of linear deterioration in deterministic demand to perform sensitivity analysis. Several parameters determine the profitability of using a two-offering strategy of a product. The first is the sensitivity of demand to unit price. The less sensitive the demand to unit price, the higher the profitability from using two offerings. Solving $D_0 - bP_1 = 0$ with Eq. (8) gives the value of b for which it is no longer optimal to use two offerings as

$$b_t = \frac{D_0\left[2\alpha C_2 - 2C_1 + \sqrt{4C_1^2 - 8\alpha C_1 C_2 + \alpha(4+\alpha)C_2^2}\right]}{\alpha C_2^2}. \tag{32}$$

Any value of $b \geqslant b_t$ means a single offering is optimal. The smaller the value of b below b_t the more profitable the two-offering strategy is. Fig. 2 shows the percentage improvement in profit of the two-offering strategy over the single offering strategy (denoted by Z_1) for different values of b up to b_t given by Eq. (32) for numerical example 1 data.

The second parameter is the unit cost of the deluxe product. The less costly the deluxe version of the product is, the higher the profitability from using a two-offering strategy. Solving $D_0 - bP_1 = 0$ with Eq. (8) gives the value of C_1 for which it is no longer optimal to use two offerings as

$$C_{1t} = \alpha C_2 + \frac{(4-3\alpha)D_0}{4b} - \frac{b\alpha C_2^2}{4D_0}. \tag{33}$$

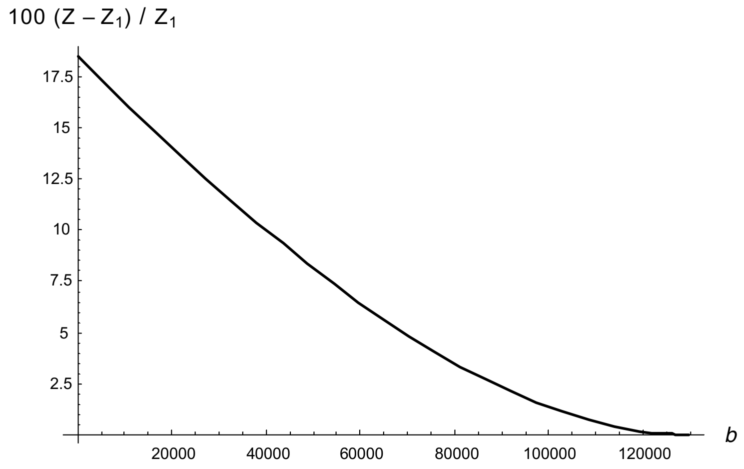

Fig. 2. Percent increase in profit of the two-offering strategy over the one-offering strategy vs. demand sensitivity to unit price.

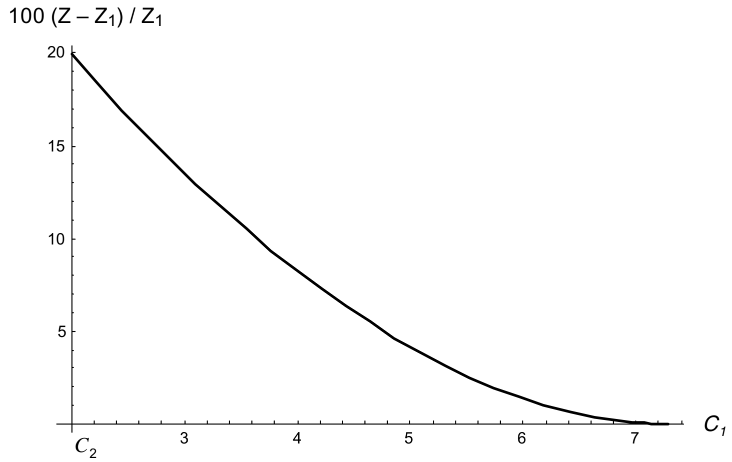

Fig. 3. Percent increase in profit of the two-offering strategy over the one-offering strategy vs. unit cost of deluxe product.

Any value of $C_1 \geqslant C_{1t}$ means a single offering is optimal. The smaller the value of C_1 below C_{1t} the more profitable the two-offering strategy is. Fig. 3 shows the percentage improvement in profit of the two-offering strategy over the single-offering strategy for different value of C_1 up to C_{1t} given by Eq. (33) for numerical example 1 data.

The third parameter is the linear deterioration parameter. The larger the deterioration parameter (i.e. smaller deterioration in demand), the higher the profitability from using a two-offering strategy. Solving $D_0 - bP_1 = 0$ with Eq. (8) gives the value of α for which it is no longer optimal to use two offerings as

$$\alpha_c = \frac{4D_0(D_0 - bC_1)}{(bC_2 - 3D_0)(bC_2 - D_0)}. \tag{34}$$

Any value of $\alpha < \alpha_c$ means that a single offering is optimal. The larger the value of α above α_c the more profitable the two-offering strategy is. Fig. 4 shows the percentage improvement in profit of the two-

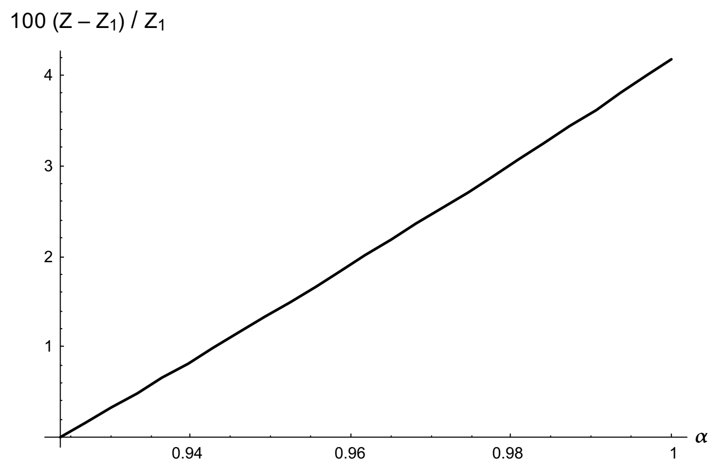

Fig. 4. Percent increase in profit of the two-offering strategy over the one-offering strategy vs. deterioration parameter.

offering-strategy over the single-offering strategy for different value of α up to α_c given by Eq. (34) for numerical example 1 data.

6. Conclusion and suggestions for future research

In this paper, we formulated and solved a model to determine the prices and quantities which maximize the profit of a firm using a two-offering strategy for a product. The product is introduced first in a deluxe version aimed at the high end of the demand curve and re-introduced a while later in a regular version targeted to customers at the low end of the demand curve. The time between the introduction of the two versions of the product is accompanied by a downward shift in the demand curve due to customers losing interest in the product or satisfying their demand from the secondary used market. The results indicate that substantial improvements in profit can be obtained by using the two-offering strategy. The profitability of the two-offering strategy is highest for products with demand that is not very sensitive to price, has a low cost of giving an image of a deluxe version, and customers do not lose interest in it quickly over time.

The proposed model can be applied to a wide variety of industries where firms enjoy, at least temporarily, a monopoly position resulting from patent protection or copyright. Patent protection is often enjoyed by pharmaceuticals and firms in high-tech industries. The number of prices a firm can use to skim the market can exceed two and the profit function would have to be modified to reflect the multiple prices and the associated saturation effect. We limited our model to two prices because firms frequently avoid using many prices since they can be annoying to customers.

Several interesting research questions remain. Among them is what is the impact of new technology on the profitability of the two-offering strategy? For example, the availability of low cost duplication technology may make the strategy of offering a deluxe/collectors edition DVD of a movie less profitable because it may encourage more customers to resort to duplication (in spite of it being illegal). Another question is how does customer interest in a product decrease over time and what role can marketing play in lessening the severity of this phenomenon? Finally, can interest in a product be re-kindled? For example, the release of a movie-version of a book may re-kindle interest in the book and lead to a new round of buying.

Appendix A

A.1. Optimal first offering price derivation—exponential deterioration

Let

$$a_0 = D_0 + bC_1 - bC_2 e^{\beta(1-P_0)}[2 + (\beta - 2)\beta + C_2\beta(\beta - 1)]/4, \tag{A.1}$$

$$a_1 = e^{-\beta P_0}[be^{\beta}(2 + (\beta - 2)\beta + \beta C_2(\beta C_2 + 4\beta - 4)) - 8be^{\beta P_0}]/4, \tag{A.2}$$

$$a_2 = -3be^{\beta(1-P_0)}(\beta C_2 + \beta - 1)/4, \tag{A.3}$$

$$a_3 = be^{\beta(1-P_0)}\beta^2/2 \tag{A.4}$$

and

$$a_4 = 9a_1a_2a_3 - 2a_2^3 - 27a_0a_3^2 + \sqrt{(2a_2^3 + 27a_0a_3^2 - 9a_1a_2a_3)^2 - 4(a_2^3 - 3a_1a_3)^2}. \tag{A.5}$$

Setting $\partial Z/\partial P_1 = 0$ gives

$$P_1^* = -\frac{2^{4/3}(a_2^2 - 3a_1a_3) + 4a_2a_4^{1/3} + 2^{2/3}a_4^{2/3}}{12a_3a_4^{1/3}} - \frac{\mathrm{i}(6a_1a_3 - 2a_2^2 + 2^{1/3}a_4^{2/3})}{2^{5/3}\sqrt{3}a_3a_4^{1/3}}. \tag{A.6}$$

A.2. Derivation of optimal prices—stochastic demand

Assuming linear deterioration in the mean demand and substituting from Eqs. (18) and (19) into Eq. (17) gives

$$\begin{aligned} E\{Z\} = {} & \int_{Q_1}^{(1+R)E\{D(P_1)\}} \frac{Q_1P_1 - (D_1 - Q_1)S_1}{2RE\{D(P_1)\}}\,\mathrm{d}D_1 + \int_{(1-R)E\{D(P_1)\}}^{Q_1} \frac{D_1P_1 + (Q_1 - D_1)V_1}{2RE\{D(P_1)\}}\,\mathrm{d}D_1 - C_1Q_1 - C_2Q_2 \\ & + \int_{Q_2}^{(1+R)(E\{D(P_2)-Q_1\})P_1/P_0} \frac{Q_2P_2 - (D_2 - Q_2)S_2}{2R(E\{D(P_2) - Q_1\})P_1/P_0}\,\mathrm{d}D_2 \\ & + \int_{(1-R)(E\{D(P_2)-Q_1\})P_1/P_0}^{Q_2} \frac{D_2P_2 + (Q_2 - D_2)V_2}{2R(E\{D(P_2) - Q_1\})P_1/P_0}\,\mathrm{d}D_2, \end{aligned} \tag{A.7}$$

where $E\{D(P_1)\}$ and $E\{D(P_2)\}$ are obtained by substituting P_1 and P_2 into Eq. (18), respectively. After integration, Eq. (A.7) gives

$$\begin{aligned} E\{Z\} = {} & \frac{Q_1^2S_1/2 - Q_1^2(S_1 + P_1) + (1+R)(D_0 - bP_1)[Q_1(S_1 + P_1) - S_1(1+R)(D_0 - bP_1)/2]}{2R(D_0 - bP_1)} \\ & + \frac{Q_1^2V_1 + Q_1^2(P_1 - V_1)/2 + (R-1)(D_0 - bP_1)[Q_1V_1 + (R-1)(D_0 - bP_1)(P_1 - V_1)/2]}{2R(D_0 - bP_1)} \\ & - \frac{P_0Q_2^2(S_2 + 2P_2) + (1+R)(Q_1 - D_0 + bP_2)P_1[2Q_2(S_2 + P_2) + S_2P_1(1+R)(Q_1 - D_0 + bP_2)/P_0]}{4RP_1(D_0 - bP_2 - Q_1)} \\ & - \frac{P_0Q_2^2(P_2 + V_2) + (R-1)(Q_1 - D_0 + bP_2)P_1[2Q_2V_2 - (R-1)(Q_1 - D_0 + bP_2)(V_2 - P_2)P_1/P_0]}{4RP_1(D_0 - bP_2 - Q_1)} \\ & - C_1Q_1 - C_2Q_2. \end{aligned} \tag{A.8}$$

Let

$$\begin{aligned} x_0 = {} & [-Q_1h_2^2 + V_2R(Q_1V_2 - bS_2h_2) + RC_2^2(Q_1 - bh_2 - D_0) + (h_2^2 - RV_2^2)D_0 \\ & + C_2[bh_2(h_2 - k_2R) - 2Q_1RV_2 + 2V_2RD_0]]P_1, \end{aligned} \tag{A.9}$$

$$x_1 = -2h_2(Q_1 + b(S_2 - R_2V_2 - bR_1C_2 - D_0)P_1), \tag{A.10}$$

$$x_2 = (D_0 + bC_2R_1 - 4bh_2 - bV_2R - Q_1)P_1, \tag{A.11}$$

$$x_3 = -2bP_1, \tag{A.12}$$

$$x_4 = 9x_1x_2x_3 - 2x_2^3 - 27x_0x_3^2 + \sqrt{(2x_2^3 + 27x_0x_3^2 - 9x_1x_2x_3)^2 - 4(x_2^3 - 3x_1x_3)^2}. \tag{A.13}$$

Setting $\partial E\{Z\}/\partial P_2 = 0$ gives the following necessary condition for P_2 to be optimal:

$$P_2^* = -\frac{2^{4/3}(x_2^2 - 3x_1x_3) + 4x_2x_4^{1/3} + 2^{2/3}x_4^{2/3}}{12x_3x_4^{1/3}} - \frac{\mathrm{i}(6x_1x_3 - 2x_2^2 + 2^{1/3}x_4^{2/3})}{2^{5/3}\sqrt{3}x_3x_4^{1/3}}. \tag{A.14}$$

Let

$$y_0 = [-Q_1^2 P_0 U_2(bh_1 + D_0) + D_0^3(4R^2C_2^2 - R_2^2 P_0 U_2 - 4C_2R(h_2 + k_2R + R_1P_2) + 4R(S_2V_2R + P_2(S_2 - V_2R_2 + P_2))) + D_0^2((2Q_1R_1 + b(S_1R_1^2 - V_1R_2^2))P_0U_2 - 4R^2C_2^2(Q_1 + bP_2) + 4RC_2(Q_1 + bP_2)(h_2 + k_2R + R_1P_2) - 4R(Q_1 + bP_2)(S_2V_2R + P_2(S_2 - V_2R_2 + P_2)))]/(2P_0U_2), \tag{A.15}$$

$$y_1 = [bD_0(-4R^2C_2^2D_0 - (2Q_1R_1 + b(S_1R_1^2 - V_1R_2^2))P_0C_{u2} + 2R_2^2D_0P_0C_{u2} + 4RC_2D_0(h_2 + k_2R + R_1P_2) - 4RD_0(S_2V_2R + P_2(S_2 - V_2R_2 + P_2)) + 4R(Q_1 + bP_2)(S_2V_2R + C_2^2R + P_2(S_2 - V_2R_2 + P_2) - C_2(h_2 + k_2R + R_1P_2))))]/(P_0C_{u2}), \tag{A.16}$$

$$y_2 = [b^2((2Q_1R + b(S_1R_1^2 - V_1R_2^2))P_0C_{u2} - 5R^2P_0D_0C_{u2} + 4RD_0(S_2V_2R + RC_2^2 + P_2(S_2 - V_2R_2 + P_2) - C_2(R_2 + k_2R + R_1P_2)) - 4R(Q_1 + bP_2)(S_2V_2R + RC_2^2 + P_2(S_2 - V_2R_2 + P_2) - C_2(h_2 + k_2R + R_1P_2))))]/(2P_0C_{u2}), \tag{A.17}$$

$$y_3 = b^3R_2^2, \tag{A.18}$$

$$y_4 = 9y_1y_2y_3 - 2y_2^3 - 27y_0y_3^2 + \sqrt{(2y_2^3 + 27y_0y_3^2 - 9y_1y_2y_3)^2 - 4(y_2^3 - 3y_1y_3)^2}. \tag{A.19}$$

Setting $\partial E\{Z\}/\partial P_1 = 0$ gives the following necessary condition for P_1 to be optimal:

$$P_1^* = -\frac{2^{4/3}(y_2^2 - 3y_1y_3) + 4y_2y_4^{1/3} + 2^{2/3}y_4^{2/3}}{12y_3y_4^{1/3}} - \frac{\mathrm{i}(6y_1y_3 - 2y_2^2 + 2^{1/3}y_4^{2/3})}{2^{5/3}\sqrt{3}y_3y_4^{1/3}}. \tag{A.20}$$

The model requires an iterative solution procedure. The optimal prices (i.e. P_1^* and P_2^* in Eqs. (8) and (9)) from the deterministic linear model are used in Eqs. (30) and (31) to obtain initial values for Q_1 and Q_2 which are in turn used in Eqs. (A.9)–(A.20) to obtain new values for P_1 and P_2 and this iterative process is continued until convergence.

References

Burleson, F.J., 2002. Interview: Manager of Little Professor Bookstore, Charlotte, NC, May 8.

Dana Jr., J.D., 2001. Monopoly price dispersion under demand uncertainty. International Economic Review 42, 649–670.

Feng, Y., Gallego, G., 1995. Optimal stopping times for end of season sales and optimal stopping times for promotional fares. Management Science 41, 1371–1391.

Feng, Y., Xiao, B., 2000. Optimal policies of yield management with multiple predetermined prices. Operations Research 48, 332–343.

Feng, Y., Gallego, G., 2000. Perishable asset revenue management with markovian time dependent demand intensities. Management Science 46, 941–958.

Gallego, G., van Ryzin, G.J., 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons. Management Science 40, 999–1020.

Harris, M., Raviv, A., 1981. A theory of monopoly pricing schemes with demand uncertainty. The American Economic Review 71, 347–365.

Khouja, M., 1999. The single-period (news-vendor) inventory problem: A literature review and suggestions for future research. Omega 27, 537–553.

Noble, P.M., Gruca, T.S., 1999. Industrial pricing: Theory and managerial practice. Marketing Science 18, 435–454.

Platt, J., Gwiazdowski, A., 2002. Book sales total $25 billion in 2001. Available from <http://www.publishers.org/press/pr030102.htm>.

Raman, K., Chatterjee, R., 1995. Optimal monopolist pricing under demand uncertainty in dynamic markets. Management Science 41, 144–163.

Simon, H., 1992. Pricing opportunities—and how to exploit them. Sloan Management Review 33, 55–66.

Weatherford, L.R., Bodily, S.E., 1992. A taxonomy and research overview of perishable-asset revenue management: Yield management, overbooking and pricing. Operations Research 40, 831–844.