Predatory Bidding in Sequential Auctions

Nils-Henrik Morch von der Fehr

Oxford Economic Papers, New Series, Vol. 46, No. 3. (Jul., 1994), pp. 345-356.

Oxford Economic Papers 46 (1994), 345–356

PREDATORY BIDDING IN SEQUENTIAL AUCTIONS

By NILS-HENRIK MØRCH VON DER FEHR

Department of Economics, University of Oslo, P.O. Box 1095 Blindern, 0317 Oslo, Norway

1. Introduction

CONSIDER a standard sequential auction model in which two units of an item are offered in two consecutive English auctions. Buyers want one unit for which they have private and independent valuations. In such a model there exists an equilibrium in which the two units are sold at the same price—equal to the third-highest valuation—in the two auctions. In the second auction optimal bidding will lead to a selling price equal to the valuation of the second-highest-valuation bidder who enters that auction. Foreseeing that, the two buyers with highest valuations can do no better in the first auction than accepting all prices less than or equal to the valuation of the third-highest-valuation bidder: bidding less would mean one of them would not get a unit, while bidding more would mean paying more than what a unit can be obtained for in the second auction (Milgrom and Weber, 1982b; Weber 1983; Milgrom 1985).

In this equilibrium it is implicitly assumed that all buyers will take part in the second auction. Note, however, that when the second auction is reached all remaining buyers, except the one with the highest valuation, do not have any incentive to participate any further; based on the information revealed in the first auction they know that they cannot win. Given this, the assumption that everyone enters the second auction does not seem entirely coherent. Buyers who value their time may prefer to leave. Indeed, casual observation suggests that not even everyone present in an auction house participates in every auction; only a few individuals seem actively engaged in the bidding for any particular object. Some are only observers, and quite a few do not even pay attention to what is going on, preferring to chat to their neighbour, consult the program, or let their thoughts wander. A reasonable assumption for why people do not participate seems to be that they consider it so unlikely that they will win at a price below what they would be willing to pay, that they cannot be bothered to engage in bidding.

It turns out that the 'law-of-one-price' equilibrium is not robust to the assumption that buyers may not participate in the second auction. Consider a variant of the standard model in which some buyers dislike participation and suppose that the above equilibrium is an equilibrium in this 'perturbed' model also. If all buyers bid so long as the price is below their valuation (and below the expected price of the second auction), the first auction provides a ranking of buyer valuations. Thus when the second auction starts, everyone will know whether or not he has the highest valuation amongst the remaining buyers. Consider the player with the second-highest valuation amongst those remaining (i.e. third-highest in the original set). If he finds it costly to participate, he will not enter the second auction, knowing that he cannot win. But then the selling price of the second auction will be below that of the first auction contradicting the law of one price. Furthermore, a rational buyer will realize that if he succeeds in bidding in such a way that he becomes the highest-bidding loser of the first auction, other bidders will indeed consider him as the highest-valuation remaining bidder and may stay out, thus giving him a chance to purchase the object in the second auction at a lower price.

The conclusion is that since bidding behaviour in the first auction reveals information about bidder valuations, and only buyers with non-negative posterior expected payoffs (including any participation cost) will participate in the second auction, there is a strategic incentive for predatory bidding in the first auction. The above argument—which is valid for any positive participation costs, however small—still holds if buyers want more than one unit; equilibrium involves a selling price in the first auction higher than the selling price in the second auction.

It may be noted that the equilibrium price path of our alternative model is consistent with the seemingly common observation that when identical objects are sold in sequential auctions at the same time and place, prices decline for later units. Since this repudiates the law of one price it has been considered an irregularity and termed the 'Price Decline Anomaly' (Ashenfelter 1989). McAfee and Vincent (1991) (see also Milgrom and Weber 1982b) have recently shown that when buyers are risk-averse, non-decreasing absolute risk aversion, which is necessary for pure-strategy equilibrium bidding-functions to exist, induces a declining equilibrium price path in first-price and second-price sealed-bid sequential auction models. While bidding one's valuation remains a dominant strategy in the second auction, risk aversion, by reducing the value of the random second-auction payoff and thus making bidders more reticent to risk foregoing a positive first-period profit, leads to a more aggressive bidding in the first auction. Recently, Black and de Meza (1992) have argued that a decreasing price path may also be explained by assuming that the first-round winner is given an option to purchase further units at the same price. The approach taken in this paper should not be considered rival to those of McAfee and Vincent (1991) and Black and de Maza (1992).[1] The purpose of the present model is to investigate the potential importance of participation costs, and, if anything, the fact that the outcome accommodates the price decline phenomenon should be taken as complementary evidence that auction theory can still be used to account for the 'anomaly'.

2. The model

Consider an auction game in which two identical units of an item are sold in two consecutive auctions. The auction procedure is a version of the so-called

[1] Note that since strategies in the law-of-one-price equilibrium of the standard open English sequential auction are (weakly) dominant, the risk-aversion argument of McAfee and Vincent has no bite in the present model.

'English Clock' in which the offer price is raised continuously from some starting value until only one buyer remains (Milgrom and Weber 1982b; McCabe *et al.* 1990).

There are N buyers where N is random and distributed according to the distribution function $F(N)$.[2] Buyers want only one unit of the goods for which they have private and independent valuations. The valuation of buyer i, v_i, is distributed according to the continuous distribution function $H_i(v_i)$, where $h_i(.)$ is the density of H_i and supp $h_i(v_i) = [\underline{v}_i, \bar{v}_i]$, $\underline{v}_i > 0$ and $\bar{v}_i < \infty$, $i = 1, 2, \ldots, N$.

There are four stages in the game. In the first stage buyers simultaneously decide whether or not to enter the first auction. The second stage consists of the first auction in which only those buyers who have decided to enter participate. Correspondingly, in the third and fourth stages, players decide whether or not to enter and play the second auction. The second auction does not take place if no buyer entered the first auction.

The rules of each auction (i.e. stages two and four) are the following:

(i) Bids start at some $\underline{b} < \min\{\underline{v}_i, i = 1, 2, \ldots, N\}$. For simplicity, $\underline{b}$ is normalized to 0.[3]

(ii) The bid is then raised continuously until either
  - (a) only one buyer remains, in which case this buyer receives the good and pays the lowest bid at which he was the only one remaining, or
  - (b) n buyers accept a bid $\hat{b}$ but no buyer accepts any bid strictly greater than $\hat{b}$. In this case the good is allocated by a random device to one of the n buyers where each buyer is given the same probability of obtaining the good.[4] The winning buyer then pays $\hat{b}$.

Entry fee. The intuition that some buyers will not participate in an auction which they know they cannot win is modelled by assuming that there are two types of buyers; a buyer is either an ε-type, in which case entering an auction costs him ε, or he is a 0-type, in which case he does not suffer participation costs and his entry fee is zero. Let the probability that a buyer is an ε-type be independent of the types of the other buyers and equal to α. The game corresponding to a particular α is called G^α.

Information. $F(.)$, $H_1(.), \ldots, H_N(.)$, and α, as well as the rules of the game, are common knowledge. Every participating buyer's acceptances and rejections are publicly observable to other participating buyers, i.e. buyers paying the entry fee, while non-participating buyers cannot observe what other bidders do. This

[2] With appropriate assumptions on $F(.)$ one can ensure that participation yields positive *ex ante* payoff for all buyers; see the discussion of 'participation constraints' below and the proof of result 1.

[3] I discuss the case of positive reserve price bids in Section 3.3.

[4] This tie-breaking rule may be considered the limit of the following stopping-technology: there is a stochastic time lag between deciding to stop bidding and actually doing so ('releasing the button'). If payers are symmetric as far as reaction lags are concerned, n players will be equally likely to win if they decide to stop bidding at the same time, and, furthermore, two bidders leaving simultaneously is a probability zero event. In the limit, as reaction lags go to zero (in probability terms), the outcome described by the tie-breaking rule is reached.

is consistent with the interpretation that the participation cost reflects the opportunity cost of not leaving or the disutility of 'paying attention', in particular, registering the bidding behaviour of rivals. Buyer i's valuation, v_i, is private information to him or her.

Pay-offs. A buyer i who participates in only one auction gets payoff $v_i - b - \tilde{\varepsilon}$ if he wins that auction at bid b, and $-\tilde{\varepsilon}$ if he does not win, where $\tilde{\varepsilon} = \varepsilon$ if the buyer is an ε-type and $\tilde{\varepsilon} = 0$ otherwise. A buyer who participates in both auctions gets payoff $v_i - b - 2\tilde{\varepsilon}$ if he wins the second auction at bid b and $-2\tilde{\varepsilon}$ if he does not win. There is no discounting. Buyers are risk neutral and maximize expected payoffs.

Participation constraints. Since it is assumed that participation may be costly, particular attention has to be paid to the individual rationality of players' strategies, or the so-called 'participation constraint'; players might not want to participate in the auction game at all. I abstract from this problem by making assumptions which ensure that all N players receive positive (expected) payoff from participation: firstly, it was assumed above that the starting bid is strictly below the lowest possible valuation ($\min\{\underline{v}_i\} > \underline{b} = 0$). Secondly, I assume that the probability that the number of buyers is small, is not too low; in particular, since a buyer's payoff is bounded from below by

$$\Pr(N=1)[v_i - \tilde{\varepsilon}] + \Pr(N>1)[-2\tilde{\varepsilon}]$$

(the event that he participates in both auctions and always loses if other buyers are present), $\Pr(N = 1)/\Pr(N > 1) > 2\varepsilon/[\min\{v_i\} - \varepsilon]$ is a sufficient condition to ensure that all players enter at least one auction. If either of these assumptions were not satisfied, some buyers might prefer staying out of both auctions, and the equilibrium strategies would need to take that into account. By abstracting from such problems, the analysis is considerably simplified, without, it would appear, reducing the generality of the results.[5]

The assumption that buyers want only one unit of the item is not crucial and is relaxed in a later section. In fact, a possible explanation for the 'Price Decline Anomaly' could be that the marginal utility of units is increasing in the number of units owned. In that case, buyers would bid more aggressively in the first than in the second auction in order to obtain both units.

The auction rules are chosen for analytical simplicity, and alternative rules might be considered. What is important for the results is the fact that buyers' behaviour in the first auction reveals information about their valuations, in particular, information about which of the losers of the first auction has the highest valuation. The model is set up so as to maximize such information

[5] An alternative way around the participation constraint problem would be to follow the standard assumption in the literature that strategies are symmetric and increasing in valuations (see e.g. Maskin and Riley 1983). This would imply that only buyers with valuations above some threshold would participate and would yield similar results as the approach chosen here. See also the discussion below regarding reserve bids submitted by the auctioneer.

revelation and thus gives striking results. We conjecture that in settings where bidders' behaviour provides less precise signals of their true valuations, a downward sloping sequence of winning bids would still be an equilibrium outcome, but the result may not be as pronounced as in the present model.

3. Results

3.1. *The standard model—no participation cost*

For the sake of completeness, I first consider the standard model which corresponds to the game G^0 where the probability that people are of the ε-type, α, is zero. In this case it is immediate that the following set of strategies form a perfect Bayesian equilibrium:

Buyer i, $i = 1, 2, \dots, N$:
(i) First auction: enter and, as long as there are two or more buyers left, accept all bids less than or equal to v_i. If there is no more than one other buyer left when the bid equals $\hat{b}$, do not accept any higher bid.
(ii) Second auction: if successful in the first auction, stay out. Otherwise, enter and accept all bids less than or equal to v_i.

Bidding one's valuation is obviously a dominant strategy in the second auction, and thus the equilibrium price in this auction equals the valuation of the second-highest valuation loser of the first auction. Given this, no one will be willing to bid higher in the first auction than the valuation of the third-highest valuation buyer. Thus, prices in the two auctions are identical and equal to the valuation of the third-highest valuation buyer (or zero if $N \leqslant 2$).

Note that except for the buyers with the highest valuations, payoffs to all others are zero. Furthermore, since bidding strategies in the first auction are fully revealing such buyers will be indifferent between entering and not entering the second auction; either way they cannot win.[6] It follows that if there was any cost at all to participation this equilibrium would break down.

3.2. *An example*

Before considering the general model, I consider the extreme case, game G^1, where all buyers incur participation costs ($\alpha = 1$). I do this for two reasons: first, the analysis and intuition are particularly simple in this case, and second, the results are most striking (albeit somewhat extreme and unrealistic).

Denote the following set of strategies and beliefs S^1:

Buyer i, $i = 1, 2, \dots, N$:
(i) First auction: always accept a bid less than or equal to ε. So long as two or more other buyers are left, i.e. still accepting bids, also accept any bid

[6] Thus there are other equilibria, with the same equilibrium prices, where some or all of the lower valuation buyers (those not among the top three valuations) enter the second auction with probability less than one (see also footnote 8).

b satisfying

$$b \leqslant \bar{b}_i \equiv 2v_i - \varepsilon \tag{1}$$

If only one other buyer is left when the bid equals $\hat{b}$, do not accept any higher bid.

(ii) Second auction: if successful in the first auction, or any two other buyers accepted higher bids than $\bar{b}_i$, assume that entering the second auction is unprofitable and stay out. Otherwise, enter and accept any bid less than or equal to v_i.

If the bidding strategies in the first auction are fully revealing and the highest-valuation loser of the first auction enters the second, it is a best response for the lower-valuation buyers to stay out since they know they can never win that auction. Then it is indeed optimal for the highest-valuation remaining bidder to enter the second auction since he gets a gross payoff (not including participation costs) equal to his valuation. This, and the fact that if he is amongst the two highest-valuation buyers he wins either the first or the second auction with equal probability, makes it optimal to stay in the first auction as long as the bid plus the entry cost does not exceed twice his valuation. In equilibrium, the price in the first auction equals the third-highest-valuation (less the participation cost), while the price in the second auction is zero.

Result 1 states the existence of a perfect Bayesian equilibrium in which buyers play according to the above strategies. A perfect Bayesian equilibrium is constituted by (i) strategies that are best responses given beliefs—in particular, one has to check that bidding and entry decisions are optimal in both auctions—and (ii) beliefs that are 'consistent', in particular, along the equilibrium path beliefs about rivals' types must be updated according to Bayes' rule, and, furthermore, beliefs have to be specified for out-of-equilibrium moves (i.e. bids and entry decisions that do not correspond to the strategies of any admissible type).

Result 1. S^1 constitutes a perfect Bayesian equilibrium of the game G^1.

Proof. Following the principle of backward induction, I start by considering the second auction. Since the first auction provides a complete ranking of valuations of all unsuccessful buyers, all bidders know their competitors' valuations before the second auction starts. Furthermore, the bidder with the highest valuation among the remaining buyers will win the second auction with probability one. Therefore, the gross gain (i.e. not including the entry fee) for any other buyer is zero and, since participation costs are positive, they stay out.

Turn now to the first auction. I start with the event $N \geqslant 3$. Consider then whether a buyer i might wish to bid above $\bar{b}_i$. If buyer i stays in when the current bid is higher than $\bar{b}_i$, and there are two or more other buyers left, there is a positive probability that he will find himself amongst the two highest-bidding participants when the bid reaches $\hat{b} > \bar{b}_i$. In that event i will receive $v_i - \hat{b} - \varepsilon$ and $v_i - 2\varepsilon$ with equal probability (the former corresponds to the case when he wins the first auction and the latter is i's payoff if he loses the first auction but then enters the second auction as the only buyer). Now, if $\hat{b} > \bar{b}_i$.

$$0.5 \cdot [v_i - \hat{b} - \varepsilon] + 0.5 \cdot [v_i - 2\varepsilon] < -\varepsilon \tag{2}$$

whereas $-\varepsilon$ is the lower bound on a buyer's payoff if he follows his equilibrium strategy. Thus bidding above $\bar{b}_i$ cannot be optimal.

Consider next whether buyer i could benefit from deviating by dropping out of the first auction when the current bid equals some $\hat{b} < \bar{b}_i$ and there are still two or more other buyers left. Let v_k^{-i} be the valuation of the buyer with the kth-highest valuation, not counting buyer i. Then, if i adheres to his equilibrium strategy, his payoff is $\max\{-\varepsilon, v_i - v_2^{-i} - \varepsilon\}$: in the event that $v_2^{-i} > v_i$, he gets $-\varepsilon$ (he loses the first auction and stays out of the second). On the other hand, if $v_2^{-i} < v_i$, since bidding in the first auction stops at $\bar{b}_2^{-i} \equiv 2v_2^{-i} - \varepsilon$, i gets, with equal probability, $v_i - [2v_2^{-i} - \varepsilon] - \varepsilon$ and $v_i - 2\varepsilon$ (depending on whether he wins or loses the first auction), i.e. his expected payoff is $v_i - v_2^{-i} - \varepsilon$. Now we can distinguish between two cases of 'early deviation', depending on whether or not i participates in the first auction at all:

(i) i may stay out of the first auction and only enter the second auction (by assumption, expected payoff is greater than zero, and thus not participating at all is not optimal; see below). Being a non-participant in the first auction, i will not observe his competitors' bids, and thus their valuations, and consequently he will have the same information when the second auction starts as he had *ex ante* (except from the fact that if the second auction takes place, he will know that at least one buyer participated in the first auction). Following i's deviation some other buyer will enter the second auction also, and thus i receives, with equal probability, $\max\{-\varepsilon, v_i - v_1^{-i} - 2\varepsilon\}$ and $\max\{-\varepsilon, v_i - v_2^{-i} - 2\varepsilon\}$, depending on whether he meets the highest-valuation or the second-highest-valuation competitor. Obviously, since $v_1^{-i} \geqslant v_2^{-i}$, the expected mean of this (with respect to all admissible v_1^{-i}'s and v_2^{-i}'s) is strictly less than i's equilibrium payoff $\max\{-\varepsilon, v_i - v_2^{-i} - \varepsilon\}$.

(ii) i may enter the first auction, but then drop out before the bid has reached $\bar{b}_i$ and there are still two or more other buyers left. If, after this deviation, i does not enter the second auction his payoff is $-\varepsilon$ (the cost of participating in the first auction). If i does enter the second auction, he gets with equal probability $\max\{-2\varepsilon, v_i - v_1^{-i} - 2\varepsilon\}$ and $\max\{2\varepsilon, v_i - v_2^{-i} - 2\varepsilon\}$. Again, the expected mean of the payoff from deviation is strictly less than equilibrium profits.

I end the proof by considering the event $N \leqslant 2$ and show that the overall payoff from participation is positive for all buyers. In the event $N \leqslant 2$, buyer i's payoff from adhering to his equilibrium strategy is $\Pr(N = 1)[v_i - \varepsilon] + \Pr(N = 2)[v_i - 2\varepsilon]$ (in the event $N = 2$, the second-auction price will equal 0, thus bidding ε in the first auction is optimal since if winning that auction the buyer saves entry costs in the second auction). If i deviates by not participating in the first auction, but then enters the second, his payoff is $\Pr(N = 1) \cdot 0 + \Pr(N = 2)[v_i - \varepsilon]$ (in the event $N = 1$, the second auction does not take place since no buyer participated in the first). Given the assumption that $\Pr(N = 1)/\Pr(N > 1) > 2\varepsilon/[\min\{\underline{v}_i\} - \varepsilon]$, it follows that $\Pr(N = 1)[v_i - \varepsilon] + \Pr(N = 2) \times [v_i - 2\varepsilon] > \Pr(N = 2)[v_i - \varepsilon]$, i.e. the equilibrium strategy payoff dominates sitting out in the first auction and then re-entering. Lastly, playing the equilibrium strategy payoff dominates not participating at all since a buyer's equilibrium is bounded from below by the positive number $\Pr(N = 1)[v_i - \varepsilon] + \Pr(N > 1)[-2\varepsilon]$ (the latter term is the payoff to buyer i of participating in, and losing, both auctions in the event $N > 1$). □

Remark. I have not been able to prove that S^1 constitutes the unique equilibrium of G^1. It seems clear from the argument in the proof that this equilibrium is the only separating equilibrium, i.e. the only equilibrium in which first-auction strategies, $b_i(v_i)$, are monotone in valuations.[7] However, there may exist pooling equilibria. It can be shown that first-auction strategies cannot contain 'flat' parts, i.e. $b_i(v_i)$ cannot be constant over an interval, but I have not been able to rule out intervals over which $b_i(v_i)$ is decreasing.

3.3. *The general model*

The example presented in the previous section gives striking results; the first-auction price equals twice the valuation of the third-highest valuation buyer while the second-auction price is zero. This result comes from the extreme assumption that all low-valuation buyers leave before the second auction, i.e. $\alpha = 1$. When $\alpha < 1$, this is no longer the case, and even though the price path will still be downward sloping, the outcome is less extreme.

Consider the first auction in G^α and let n be the number of buyers that have dropped out before the bid at which buyer i drops out. Let $v^1, v^2, \ldots, v^n$ be the valuations of these buyers ordered such that $v^1 \leqslant v^2 \leqslant \cdots \leqslant v^n$. Define

$$Ev_s^i = \sum_{k=1}^{n} [1 - \alpha]\cdot\alpha^{N-2-k}\cdot v^k + [1 - \alpha^{N-2-n}]\cdot v_i \tag{3}$$

Before commenting on this expression, I present the following set of strategies and beliefs, denoted S^α.

Buyer i, $i = 1, 2, \ldots, N$:

(i) First auction: always accept bids less than or equal to $\tilde{\varepsilon}$. As long as there are two or more other buyers left, accept also any bid b satisfying

$$b \leqslant \bar{b}_i^\alpha \equiv 2v_i - Ev_s^i - \tilde{\varepsilon} \tag{4}$$

$$\tilde{\varepsilon} = \begin{cases} 0 & \text{if 0-type} \\ \varepsilon & \text{otherwise} \end{cases}$$

[7] First-auction strategies that are monotonously decreasing in valuations can be ruled out by observing that low-valuation types would always want to deviate by placing a low first-auction bid to mimic the behaviour of the highest-possible-valuation type; thereby they can enter the second auction and buy a unit for the price of zero.

If there is only one other buyer left when the bid equals $\hat{b}$, do not accept any higher bid.

(ii) Second auction: if successful in the first auction, or if being an ε-type and any two other buyers accepted higher bids than $\bar{b}_i^{\alpha}$, assume that entering the second auction is unprofitable and stay out. Otherwise, enter and accept any bid less than or equal to v_i.

Note that these strategies have a form similar to those in S^1. The difference is that here a buyer has to take into account the fact that in the second auction he may have to compete against some lower-valuation buyers with zero entry costs. Assume that in the first auction the bid has reached some value $\hat{b}$ and that n buyers with valuations $v^1 \leqslant v^2 \leqslant \cdots \leqslant v^n$ have already dropped out. If the bidding stops 'in the next instant', i.e. of the remaining $N-n$ buyers only two accept $\hat{b}$, $N-n-2$ bidders have valuations equal to some $\hat{v} = v(\hat{b}) \geqslant v^n$. Consider the $N-2$ buyers with valuations less than or equal to $\hat{v}$. The event that the highest-valuation 0-type among these buyers has valuation equal to v^k, $k = 1, \ldots, n$, has probability $[1-\alpha]\cdot\alpha^{N-2-k}$ ($1-\alpha$ is the probability that the person with valuation v^k is zero-type and α^{N-2-k} is the probability that none of those with higher valuations have zero entry costs). The corresponding probability that his valuation is equal to $\hat{v}$, is $1-\alpha^{N-2-n}$. If all 0-types enter the second auction, the highest-valuation loser of the first auction has to pay an expected second-auction price equal to

$$EP_2 = \sum_{k=1}^{n} [1-\alpha]\cdot\alpha^{N-2-k}\cdot v^k + [1-\alpha^{N-2-n}]\cdot\hat{v} \tag{5}$$

Thus in this game the second-auction price is positive (in expected terms), whereas in the example analyzed in Section 3.2 it was zero. It follows that when $\alpha < 1$, the willingness to bid in the first auction is correspondingly lower. In particular, a buyer i who has entered the first auction accepts (does not accept) all bids b less (greater) than $\bar{b}_i^{\alpha}$ since if he were to win at such a bid, his gain would be $0.5\{[v_i - b] + [v_i - EP_2 - \bar{\varepsilon}]\} > (<) 0.5\{[v_i - \bar{b}_i^{\alpha}] + [v_i - Ev_s^i - \bar{\varepsilon}]\} = 0$.

By an analogous proof to that of result 1, we have:

Proposition 2. S^{α} constitutes a perfect Bayesian equilibrium of the game G^{α}.

As argued above, when $\alpha < 1$, the price in the second auction will typically not equal zero (i.e. the starting bid) since in addition to the highest-valuation loser of the first auction, 0-types will enter the second auction as well. However, the price in the first auction will be at least as high as that in the second. In particular, since

$$\lim_{\alpha \to 1} Ev_i^s = 0 \quad \text{and} \quad \lim_{\alpha \to 0} Ev_i^s = v_i \tag{6}$$

one gets the outcome of G^1 as the probability that buyers are of the ε-type approaches 1, while, in the opposite extreme, the law of one price (approximately) holds (as $\varepsilon \to 0$ the prices in the two auctions become arbitrarily close).[8] In a sense, a downward-sloping price path is the generic outcome in this model, since for any $\alpha > 0$ and $\varepsilon > 0$ the winning bid in the second auction is less than that in the first.

The limiting result also suggests what is a necessary condition for a downward-sloping price path: bidding behaviour in the first auction must convey information which induces lower-valuation participants—in particular, the third-highest valuation buyer—to stay out of the second auction with positive probability.[9] If so, predatory bidding, i.e. accepting bids above the third-highest valuation (the 'equal-price' outcome), leads to higher payoff since it deters lower-valuation buyers from entering the subsequent auction.

The equilibrium payoff to the auctioneer is twice the third-highest valuation minus ε. This is (approximately) the same as in the $\alpha = 0$ equilibrium in which prices in the two auctions both equal the third-highest valuation. The bidders' *ex ante* expected payoffs are also (approximately) the same whether there is an entry fee or not. In both cases the two highest-valuation participants get one unit each and in expected terms they pay a price equal to the third-highest valuation. Assume for a moment that ε is a fee charged by the auction house. It follows that the auction house will be indifferent between a 'per auction fee' and an 'entrance fee'. Furthermore, buyer and seller (expected) payoffs are independent of whether fees are fixed ('per auction fee') or sunk ('entrance fee'), even though the actual outcomes are different.

If we were to allow for strategic behaviour by the auctioneer, we might also consider whether he could benefit from introducing a reserve price bid in the second auction to take advantage of the information revealed in the first. Consider the example when $\alpha = 1$ and assume the auctioneer submits a second-auction reserve price equal to the winning bid of the first auction minus ε.[10] Then bidding in the first auction would stop at the third-highest valuation, and prices in the two auctions would be (approximately) equal. Thus, the auctioneer's revenues are the same with and without such a reserve price bid, i.e. equal to twice the third-highest valuation minus ε. More generally, if (i) the

[8] It should be noted that $\varepsilon > 0$ is in fact not a necessary condition for the existence of the equilibrium described in proposition 2. Since, as observed in Section 3.1, when $\varepsilon = 0$ all lower-valuation losers of the first auction are indifferent between entering and not entering the second auction, S^α will constitute an equilibrium (in mixed strategies) when $\varepsilon = 0$ also, where α should be interpreted as the probability with which lower valuation buyers enter the second auction. Thus the equilibrium considered may alternatively be interpreted as an equilibrium in behavioral strategies of a standard auction game with no participation costs.

[9] Since an ε-type will stay out if two or more other buyers bid higher than him in the first auction, it is only necessary for the result to hold that the identities of the two highest-bidding buyers are conveyed (in the case considered below, when buyers want more than one unit, only the identity of the winner has to be revealed for the result to hold). Thus, the introduction of entry costs may lead to a downward-sloping price path in sealed-bid auctions as well.

[10] If he were to submit a second-auction reserve bid equal to the first-auction winning bid, this would lead to a first-auction price equal to the second-highest valuation (since it is a dominant strategy to buy in the first auction in order to avoid paying the entry costs twice) and no one would enter the second auction.

second-auction reserve price bid is a (known) function of information revealed in the first auction, and (ii) the introduction of the reserve price bid does not affect entry decisions (as in the example alluded to in footnote 10), a reserve price bid will lead to correspondingly less aggressive first-auction bidding, and, thus, revenues will not be affected.

3.4. *When buyers want more than one unit*

A variant of the model above is to introduce buyers who want more than one unit. In particular, I consider a model where buyers' utility is proportional to the number of units they buy, i.e. the gross valuation of buyer i net of participation costs and what he has to pay for each unit, is given by $M \cdot v_i$, where $M = 0, 1, 2,$ is the number of units he obtains. Call this game G_M^α. Denote by S_M^α the following set of strategies and beliefs:

Buyer i, $i = 1, 2, \ldots, N$:

(i) First auction: accept any bid b satisfying

$$b \leqslant \hat{b}_i^\alpha \equiv 2v_i - E\hat{v}_s^i - \tilde{\varepsilon} \tag{7}$$

$$E\hat{v}_s^i = \sum_{k=1}^{n} [1 - \alpha] \cdot \alpha^{N-1-k} \cdot v^k + [1 - \alpha^{N-1-n}] \cdot v_i$$

$$\tilde{\varepsilon} = \begin{cases} 0 & \text{if 0-type} \\ \varepsilon & \text{otherwise} \end{cases}$$

(ii) Second auction: If unsuccessful in the first auction and if an ε-type, assume that entering the second auction is unprofitable and stay out. Otherwise, enter and accept any bid less than or equal to v_i.

Note that, with this set of strategies, a buyer's maximum acceptable bid in the first auction is slightly different from that in the previous section. Here a buyer wants to squeeze out even the second-highest valuation competitor in order to obtain the second unit at the lowest possible price, whereas when he wanted only one unit he was mainly concerned with the third-highest-valuation buyer whom he might encounter if he lost the first auction. As a result, in this model bidding in the first auction goes on until only one buyer remains. Again by an analogous proof to that of Result 1, we have:

Proposition 3. S_M^α constitutes a perfect Bayesian equilibrium of the game G_M^α.

4. Conclusion

This paper has been devoted to an exploration of the implications of allowing for participation costs in sequential auctions. In particular, a perfect Bayesian equilibrium in a two-unit auction game has been investigated and the following results have been demonstrated: in the first auction buyer strategies are fully revealing; in the second auction only some buyers remain and the price is lower than the first-auction price. Thus, the fact that a declining price path in sequential auctions is consistent with risk-neutral bidders, provides additional support for the conclusion of McAfee and Vincent (1991) and Black and de Meza (1992) that auction theory can be adapted to account for 'the Price Decline Anomaly'.

The present analysis has been based on a model of open English auctions, but the main result of a downward-sloping price path would appear to generalize to other types of auctions. Since buyers that find participation costly will stay out of an auction if they know that someone else is willing to bid higher than themselves, it is necessary for the result to hold only that the identities of the highest-bidding participants in the first auction are revealed. Thus, participation costs may lead to a downward-sloping price path in closed, or sealed-bid, auctions as well. Consequently, for settings in which entry costs would seem particularly important, such as in procurement (where participants often sink substantial investment before bidding on contracts), the argument in this paper might be of some relevance.

ACKNOWLEDGEMENTS

I am grateful to Geir Asheim, Friedel Bolle, David Harbord, Steiner Holden, Meg Meyer, David de Meza, Paul Klemperer, workshop participants at Nuffield College, and two anonymous referees for helpful comments and suggestions on earlier versions of this paper.

REFERENCES

Ashenfelter, O. (1989). 'How Auctions Work for Wine and Art', *Journal of Economic Perspectives*, **3**, 23–33.

Black, J. and de Meza, D. (1992). 'Systematic Price Divergences Between Successive Auctions Are No Anomaly', *Journal of Economics & Management Strategy*, **1**, 607–28.

McAfee, R. and Vincent, D. (1991). 'The Afternoon Effect', CMSEMS Discussion Paper No. 961, Northwestern University, Evanston, IL.

McCabe, K. A., Rassenti, S. J. and Smith, V. L. (1990). 'Auction Institutional Design: Theory and Behaviour of Simultaneous Multiple-Unit Generalizations of the Dutch and English Auctions', *American Economic Review*, **80**, 1276–83.

Maskin, E. and Riley, J. G. (1983). 'The Gains to Making Losers Pay in High-Bid Austions', in R. Engelbrecht-Wiggans, M. Shubik and R. M. Stark (eds), *Auctions, Bidding and Contracting: Uses and Theory*, New York University Press, New York.

Milgrom, P. R. (1985). 'The Economics of Competitive Bidding: A Selective Study', in L. Hurwicz, D. Schmeidler and H. Sonnenschein (eds), *Social Goals and Social Organization*, Cambridge University Press, Cambridge.

Milgrom, P. R. and Weber, R. J. (1982a). 'A Theory of Auctions and Competitive Bidding', *Econometrica*, **50**, 1089–22.

Milgrom, P. R. and Weber, R. J. (1982b). 'A Theory of Auctions and Competitive Bidding, II', unpublished manuscript.

Weber, R. J. (1983). 'Multiple-Object Auctions', in R. Engelbrecht-Wiggans, M. Shubik and R. M. Stark (eds), *Auctions, Bidding and Contracting: Uses and Theory*, New York University Press, New York.