Topic 3: Exploratory Factor Analysis

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Outline

- **Factor Models**
  - 1-factor model
  - 2-factor model
  - m-factor model
  - Terminologies: Loadings, Communalities
  - Model assumptions

- **Exploratory Factor Analysis**
  - Factor rotation problem
  - Estimation of communality problem
  - Model evaluation

- **An illustration: Wechsler Intelligence Scale for Children**
Outline

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  - 2-factor model
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- **Exploratory Factor Analysis**
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- **An illustration: Wechsler Intelligence Scale for Children**
Origin: Tests data

- **Factor analysis** is a technique to develop "scale" to measure unobservable constructs.
- **Spearman (1904):** to explain the correlation among students performances in various courses.
  
  **Observations:** 33 students from the same class.
  **Observed variables:** Test scores for 6 courses.
  **Correlation matrix:**

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<th>Classics</th>
<th>French</th>
<th>English</th>
<th>Math</th>
<th>Pitch</th>
<th>Music</th>
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<td>.40</td>
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</tr>
</tbody>
</table>

*Example:* Classics, English

\[
\frac{0.83}{0.67} \approx \frac{0.70}{0.64} \approx \frac{0.66}{0.54} \approx \frac{0.63}{0.51} \approx 1.2
\]

- Any unobservable construct exist behind observable variables?
One-factor model / Common factor model

- **One-factor model** is set up in the following way:

\[ x_1 = \lambda_1 \xi + \epsilon_1, \]
\[ x_2 = \lambda_2 \xi + \epsilon_2, \]
\[ \vdots \]
\[ x_p = \lambda_p \xi + \epsilon_p. \]

where \( x_1, x_2, \ldots, x_p \) are **standardized data**, i.e.,

\[ E(x_1) = E(x_2) = \cdots = E(x_p) = 0, \]
\[ \text{Var}(x_1) = \text{Var}(x_2) = \cdots = \text{Var}(x_p) = 1. \]

Moreover, we assume the factors satisfy

\[ E(\xi) = 0, \quad \text{Var}(\xi) = 1, \quad E(\epsilon_i) = 0, \]
\[ \text{Cov}(\xi, \epsilon_i) = 0, \quad \text{Cov}(\epsilon_i, \epsilon_j) = 0, \quad i \neq j. \]
Let us use the 1-factor model to explain why the elements in any two rows of the above mentioned matrix are proportional.

The element in the $i$th row and the $j$th column of the correlation matrix is $\text{Corr}(x_i, x_j)$.

Because, in the 1-factor model,

\[
\text{Corr}(x_i, x_j) = \text{Cov}(x_i, x_j) = E(x_i x_j) = E[(\lambda_i \xi + \varepsilon_i) (\lambda_j \xi + \varepsilon_j)]
= E(\lambda_i \lambda_j \xi^2 + \lambda_i \xi \varepsilon_j + \lambda_j \xi \varepsilon_i + \varepsilon_i \varepsilon_j)
= \lambda_i \lambda_j \text{Var}(\xi) + \lambda_i \text{Cov}(\xi, \varepsilon_j) + \lambda_j \text{E}(\xi, \varepsilon_i) + \text{E}(\varepsilon_i, \varepsilon_j) = \lambda_i \lambda_j.
\]

Then, for any column $j$, the ratio between the elements in the $i_1$th row and the $i_2$th row is

\[
\frac{\text{Corr}(x_{i_1}, x_j)}{\text{Corr}(x_{i_2}, x_j)} = \frac{\lambda_i_1 \lambda_j}{\lambda_i_2 \lambda_j} = \frac{\lambda_i_1}{\lambda_i_2},
\]

which is not related to the column index $j$. 

Communalities in 1-factor model

- **Pattern loadings**: $\lambda_1, \lambda_2, etc.$
  Coefficients before the common factor to indicators
- **Communalities** of indicators with the common factor: $\lambda_1^2, \lambda_2^2, etc.$
- **Structure loadings**: $\text{Corr}(x_1, \xi), \text{Corr}(x_2, \xi), etc.$
  Correlation between indicators and common factor.

$$\text{Corr}(x_i, \xi) = \text{Cov}(x_i, \xi) = E[(\lambda_i \xi + \varepsilon_i)\xi] = \lambda_i E(\xi^2) + E(\varepsilon_i \xi) = \lambda_i.$$  

- **Shared variances** between indicators and the common factor:
  $\text{Corr}(x_1, \xi)^2, \text{Corr}(x_2, \xi)^2, etc.$

- Thus, the variances of indicators can be decomposed into two parts:

$$1 = \text{Var}(x_i) = \text{Var}(\lambda_i \xi + \varepsilon_i) = \text{Var}(\lambda_i \xi) + 2 \text{Cov}(\lambda_i \xi, \varepsilon_i) + \text{Var}(\varepsilon_i)$$

\[
= \lambda_i^2 + \text{Var}(\varepsilon_i)
\]

- **Communality**: Variance explained by the common factor.
- **Unique variance**: Variances explained by the unique factor.
2-factor model

- However, in most situations, the number of common factors may be more than 1 and we do not know the number of common factors. Let us present other possible factor models.

- Consider a factor model with two common factors:

\[
\begin{align*}
X_1 &= \lambda_{11} \xi_1 + \lambda_{12} \xi_2 + \varepsilon_1, \\
X_2 &= \lambda_{21} \xi_1 + \lambda_{22} \xi_2 + \varepsilon_2, \\
&\vdots \\
X_p &= \lambda_{p1} \xi_1 + \lambda_{p2} \xi_2 + \varepsilon_p.
\end{align*}
\]

Assume \( \text{cov}(\xi_1, \xi_2) = \phi_{12} \).

- If \( \phi_{12} = 0 \): Orthogonal two-factor model.
- If \( \phi_{12} \neq 0 \): Oblique two-factor model.
• Correlations among the indicators in the two-factor model are

\[
Corr(x_i, x_j) = E(x_i x_j)
\]

\[
= E[(\lambda_{i1}\xi_1 + \lambda_{i2}\xi_2 + \varepsilon_i)(\lambda_{i1}\xi_1 + \lambda_{i2}\xi_2 + \varepsilon_j)]
\]

\[
= \lambda_{i1}\lambda_{j1} + \lambda_{i1}\lambda_{j2}\phi_{12} + \lambda_{i2}\lambda_{j1}\phi_{12} + \lambda_{i2}\lambda_{j2}
\]

\[
= \lambda_{i1}\lambda_{j1} + \lambda_{i2}\lambda_{j2} + (\lambda_{i1}\lambda_{j2} + \lambda_{i2}\lambda_{j1})\phi_{12}.
\]

• It reduces to \(\lambda_{i1}\lambda_{j1} + \lambda_{i2}\lambda_{j2}\) for the orthogonal models.
Communalities in 2-factor model

- **Pattern loadings**: $\lambda_{11}, \lambda_{12}, etc.$
- **Communalities** of indicators with the common factor 1: $\lambda_i^2$.
- **Structure loadings**: $\text{Corr}(x_1, \xi_1), \text{Corr}(x_1, \xi_2), etc.$

$$\text{Corr}(x_i, \xi_1) = E[(\lambda_{i1}\xi_1 + \lambda_{i2}\xi_2 + \varepsilon_i)\xi_1] = \lambda_{i1}E(\xi_1^2) + \lambda_{i2}E(\xi_2\xi_1) + E(\varepsilon_i\xi_1) = \lambda_{i1} + \lambda_{i2}\phi_{12}.$$  

- **Shared variances** between indicators and the common factor 1: $\text{Corr}(x_i, \xi_1)^2$.

$$\text{Corr}(x_i, \xi_1)^2 = (\lambda_{i1} + \lambda_{i2}\phi_{12})^2 = \lambda_{i1}^2 + \lambda_{i2}^2\phi_{12}^2 + 2\lambda_{i1}\lambda_{i2}\phi_{12}.$$  

- The variances of indicators can be decomposed as:

$$1 = \text{Var}(x_i) = \text{Var}(\lambda_{i1}\xi_1 + \lambda_{i2}\xi_2 + \varepsilon_i)$$

$$= \lambda_{i1}^2 + \lambda_{i2}^2 + 2\lambda_{i1}\lambda_{i2}\phi_{12} + \text{Var}(\varepsilon_i)$$

Communality wrt $\xi_1$  Communality wrt $\xi_2$  Unique variance

**Total communality of $x_i$;**

Variance explained by both common factors
m-factor model

- Consider a factor model with $m$ common factors ($m$ is usually much smaller than $p$):

\[
\begin{align*}
\mathbf{x}_1 &= \lambda_{11}\xi_1 + \lambda_{12}\xi_2 + \cdots + \lambda_{1m}\xi_m + \epsilon_1, \\
\mathbf{x}_2 &= \lambda_{21}\xi_1 + \lambda_{22}\xi_2 + \cdots + \lambda_{2m}\xi_m + \epsilon_2, \\
&\quad \vdots \\
\mathbf{x}_p &= \lambda_{p1}\xi_1 + \lambda_{p2}\xi_2 + \cdots + \lambda_{pm}\xi_m + \epsilon_p,
\end{align*}
\]

with assumptions

\[
\begin{align*}
E(\xi_l) &= 0, \quad Var(\xi_l) = 1, \\
E(\epsilon_i) &= 0, \\
Cov(\xi_l, \epsilon_i) &= 0, \quad l = 1, \ldots, m; \quad i = 1, \ldots, p. \\
Cov(\epsilon_i, \epsilon_j) &= 0, \quad i \neq j, \\
Cov(\xi_l, \xi_k) &= \phi_{lm}, \quad l \neq k.
\end{align*}
\]
Why we need the communalities

• In the exploratory factor analysis (EFA), we want to explore a factor model with the smallest number of common factors that explain the maximum amount of correlation among the indicators.

1. Communalities are defined to measure how many percentages of variances among the indicators can be explained by the common factors.

2. Estimating variances shared by factors is easier than estimating the loadings directly. In order to estimated the loadings, we need an estimate of communality of each indicator.
Exercise: Question 5.2 in P 130

5.2 Consider the two-indicator two-factor model represented by the following equations:

\[ A = 0.85F_1 + 0.12F_2 + U_A \]
\[ B = 0.74F_1 + 0.07F_2 + U_B \]
\[ C = 0.67F_1 + 0.18F_2 + U_C \]
\[ D = 0.21F_1 + 0.93F_2 + U_D \]
\[ E = 0.05F_1 + 0.77F_2 + U_E \]
\[ F = 0.08F_1 + 0.62F_2 + U_F. \]

The usual assumptions hold for the above model. Also, assume that the common factors \( F_1 \) and \( F_2 \) are uncorrelated.

(a) What are the pattern loadings of indicators \( A, C, \) and \( E \) on the factors \( F_1 \) and \( F_2 \)?

(b) What are the structure loadings of indicators \( A, C, \) and \( E \) on the factors \( F_1 \) and \( F_2 \)?

(c) Compute the correlations between the following sets of indicators: (i) \( A, B \); (ii) \( C, D \); (iii) \( E, F \).

(d) What percentage of the variance of indicators \( A, C, \) and \( F \) is not accounted for by the common factors \( F_1 \) and \( F_2 \)?

(e) Identify sets of indicators that share more than 90% of the total shared variance with each common factor. Which indicators should therefore be used to interpret each common factor?
Exercise: Question 5.2 in P 130

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The usual assumptions hold for the above model. Also, assume that the common factors \( F_1 \) and \( F_2 \) are uncorrelated. ORTHOGONAL MODEL

(a) What are the pattern loadings of indicators \( A, C, \) and \( E \) on the factors \( F_1 \) and \( F_2 \)?
(b) What are the structure loadings of indicators \( A, C, \) and \( E \) on the factors \( F_1 \) and \( F_2 \)?
(c) Compute the correlations between the following sets of indicators: (i) \( A, B \); (ii) \( C, D \); (iii) \( E, F \).
\[ \text{CORR}(A, B) = 0.85 \times 0.74 + 0.12 \times 0.07 \]
(d) What percentage of the variance of indicators \( A, C, \) and \( F \) is not accounted for by the common factors \( F_1 \) and \( F_2 \)?
\[ \text{VAR}(U_A) = 1 - 0.85^2 - 0.12^2 = 0.26 \]
(e) Identify sets of indicators that share more than 90% of the total shared variance with each common factor. Which indicators should therefore be used to interpret each common factor?
\[ F_1: \{A, B, C\}, \quad F_2: \{D, E, F\} \]
Review of Part I

Question
What is the model assumptions for the general factor model?
Review of Part I

Question

What is the model assumptions for the general factor model?

\[ x_i = \lambda_{i1}\xi_i + \lambda_{i2}\xi_2 + \cdots + \lambda_{im}\xi_m + \varepsilon_i, \ i = 1, \ldots, p. \]

- Observed: \( x_1, \ldots, x_p \).
- Unobserved: \( \xi_1, \ldots, \xi_m \) \ and \( \varepsilon_1, \ldots, \varepsilon_p \).
  - assumed: \( E(\xi_l) = E(\varepsilon_i) = 0, \ Var(\xi_l) = 1, \ Cov(\varepsilon_i, \varepsilon_j) = Cov(\xi_l, \varepsilon_i) = 0. \)
  - unknown: \( m, \lambda_{il}, \ Cov(\xi_l, \xi_k) \) \ and \( Var(\varepsilon_i) \).
Review of Part I
Review of Part I

Question

For any specific factor model:

- how the variances explained by intelligence factor \( l \) and the variances explained by all intelligence factors are related to the total communalities: \( 1 - \text{var}(\epsilon_i) \)?

- how to derive its theoretical correlations among test scores?

\[
\text{corr}(x_i, \xi_l)^2 \quad \text{corr}(x_i, x_j)
\]

\[
\lambda_{il} \quad \lambda^2_{il}
\]
Review of Part I

Question

What is the task of EFA?
Question

What is the task of EFA?

• Answer 1: To interpret the correlations among indicators by using the factor models.
Question

What is the task of EFA?

• Answer 1: To interpret the correlations among indicators by using the factor models.

• Answer 2: To find a factor model of which:

  the theoretical correlations $\rightarrow$ the sample correlations observed

  (but still $m << p$ and interpretable)
Reformulating the task of EFA

- Let the reproduced correlation matrix be a matrix of which
  - diagonals: the total communalities;
  - off-diagonals: the theoretical correlations.

- Let the residual correlation matrix be the difference between the sample correlation matrix and the reproduced correlation matrix, i.e.
  - diagonals: the unique variances;
  - off-diagonals: residuals of the correlations.

- Root mean square residual:
  \[
  RMSR = \sqrt{\frac{\sum_{i=1}^{p} \sum_{j=i}^{p} res_{ij}^2}{p(p-1)/2}}.
  \]

- We seek for a factor model where
  \[ RMSR \approx 0. \]
Problems in the task of EFA

Problem 1: Factor rotation problem

The Factor analysis solution is not unique.

Different factor models may have the same reproduced correlation matrix.
Example: The common factors $\xi_1$ and $\xi_2$ are independent.

- **MODEL I**

\[
\begin{align*}
X_1 &= 0.8 \xi_1 + \varepsilon_1, \\
X_2 &= 0.5 \xi_1 + \varepsilon_2, \\
X_3 &= 0.6 \xi_2 + \varepsilon_3
\end{align*}
\]

- **MODEL II**

\[
\begin{align*}
X_1 &= (0.8 \cos \theta) \xi_1 + (0.8 \sin \theta) \xi_2 + \varepsilon_1, \\
X_2 &= (0.5 \cos \theta) \xi_1 + (0.5 \sin \theta) \xi_2 + \varepsilon_2, \\
X_3 &= (-0.6 \sin \theta) \xi_1 + (0.6 \cos \theta) \xi_2 + \varepsilon_3
\end{align*}
\]

Q: Reproduced correlation matrix = ?
"Factor indeterminacy"
Factor rotation techniques

Solution to Problem 1 – Factor rotation techniques

We choose the solution with most theoretically plausible interpretation.

Some factor rotation techniques (self-study):
- Orthogonal rotations: Varimax, Quartimax, etc.
- Oblique rotations: ...
Problem 2: Estimation of communality problem

- The estimates of the pattern loading can be obtained from computing the eigenstructure of the matrix:

\[
R - \Phi \\begin{pmatrix}
\text{var}(e_1) & 0 \\
0 & \text{var}(e_2)
\end{pmatrix}
\]

- The estimates of the communalities relies on the estimates of the pattern loadings.

See any problem? Circularity
Problem 2: Estimation of communality problem

- The estimates of the pattern loading can be obtained from computing the eigenstructure of the matrix:

\[ R - \Lambda \Phi \Lambda' \]

where \( R \) is the correlation matrix of the observed variables, \( \Lambda \) is the matrix of pattern loadings, and \( \Phi \) is the diagonal matrix of communalities.

- The estimates of the communalities rely on the estimates of the pattern loadings.

See any problem? **Circularity**
Factor extraction techniques

Solution 1 – Principal Component Factoring (PCF)

(i) The 1st estimate of the total communalities for each indicator is 1;

(ii) PCA is performed on the sample correlation matrix with the first estimate of the communalities (1s) on diagonals;

• Determine the number of factors;
• Obtain the estimate of the loadings;
• Use the estimate of the loadings to compute a new estimate of the communalities;
Factor extraction techniques

Solution 2 – Principal Axes Factoring (PAF)

(i) The 1st estimate of the total communalities for each indicator is 1;

(ii) PCA is performed on the sample correlation matrix with the \( i \)th estimate of the communalities on diagonals:

- Determine the number of factors;
- Obtain the estimate of the loadings;
- Use the estimate of the loadings to compute the \((i + 1)\)th estimate of the communalities;

(iii) Repeat step 2 until “small” changes in the communalities.
Factor extraction techniques

Comments on PCF and PAF

- Orthogonal factor model
- Results are very similar.
- PAF is more popular.
- Disadvantage: unique factors are correlated.
Model evaluation

**Question**

Should the data be analyzed by factor analysis?

**KMO**

- Kaisers Measure of overall sampling adequacy
- the higher the better. 0.8?

**Question**

How well can the factors explain the data?

**RMSR**

- Root mean square residual
- the smaller the better. 0.01?
Model evaluation

**Question**

Should the data be analyzed by factor analysis?

**KMO**

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**Question**

How well can the factors explain the data?

**RMSR**

- Root mean square residual
- the smaller the better. 0.01?
Objectives of Factor Analysis

Analysing a correlation matrix to:

- identify the smallest number of factors that best explain the correlations;
- identify the most plausible solution (via rotation);
- estimate loadings, communalities, etc.;
- interpret the factors;
- if possible, estimate factor scores.
Example – Wechsler Intelligence Scales for Children

175 children

The verbal tests were:
X1 Information
X2 Similarities
X3 Arithmetic
X4 Digit Span
X5 Comprehension
X6 Vocabulary

The performance tests were:
X7 Picture Arrangement
X8 Picture Completion
X9 Block Design
X10 Matrix Reasoning
X11 Coding Digit Symbol
Information

Information measures general cultural knowledge, long-term memory, and acquired facts. Children are asked questions about different topics like geography, science and historical figures. The questions shouldn't be difficult for a child with a well rounded education but they do encompass a wide range of knowledge.

Information is a supplemental Verbal Comprehension subtest.

Example: Who was Queen Elizabeth I
Example: What do your lungs do?
Example: What is photosynthesis?

Suggestions: The focus of our store is thinking skills material rather than general knowledge. We suggest you encourage your child to read quality non-fiction and help them make links between what they're studying at school now and what they've studied in the past so that they retain information. The titles below can be used to check that your child has the expected grade level knowledge and to build general knowledge:

- Spectrum Test Prep
- Daily Mind Builders
Similarities

**Similarities** measures logical thinking, verbal concept formation and verbal abstract reasoning. Two similar but different objects or concepts are presented, and the student is asked to tell how they are alike or different.

Similarities is an untimed core Verbal Comprehension subtest.

Examples:
- How are whales and lions similar?
- How are anger and delight similar?
- How are boys and girls similar?

**Suggestions:**
- Building Thinking Skills
- Analogies
Arithmetic

Arithmetic measures numerical accuracy, reasoning and mental arithmetic ability. Mental arithmetic and story problems play an important part in the student’s success.

Arithmetic is a supplemental Working Memory subtest.

Example: How many carrots are there in this picture?

Example: Michelle is 2 years younger than Peter and 5 years older than Sam. If Sam is 6 how old is Michelle?

Example: Kathy’s lunch bill was $22.50. If she leaves a 15% tip how much money does she need to leave?

Suggestions:

• Primary Grade Challenge Math
Digit Span

Digit Span measures short-term auditory memory and attention. The digits have no logical relationship to each other and are presented in random order by the examiner. The student must then recite the digits correctly by recalling them in the same order. On the second part of this subtest the student must remember the order in which digits are presented, but recite them in reverse order.

Digit Span is an untimed core Working Memory subtest.

Examples:
For Digit Span forward tester would read numbers like "2, 3, 9, 1"
and child would respond with the same numbers

For Digit Span backward the tester would read numbers like "24, 3, 7, 12"
and child would respond "12, 7, 3, 24"

Suggestions:

- Look! Listen! Think!
- Memory Challenge (visualization of numbers as chunks can help memory of auditory information)
- Listening Skills titles (to build attention to the information as it is provided)
Comprehension

Comprehension measures common-sense social knowledge, practical judgment in social situations, and level of social maturation, along with the extent of development of their moral conscience. Children are asked to explain situations, actions, or activities that they'd be expected to be familiar with.

Comprehension is a core Verbal Comprehension subtest.

Example: Why do we turn out lights when we leave a room?

Suggestions:

- What Would You Do?
- Nifty Fifty
Vocabulary

Vocabulary measures the students’ verbal fluency and concept formation, word knowledge, and word usage.

Vocabulary is an untimed core Verbal subtest

Example:

Children are shown pictures or a word is said aloud. They are asked to provide the name of the object or to define the word.

What is this?

What does simple mean?

Suggestions:

- Building Thinking Skills
- Analogies
- Nifty Fifty
- Word Roots
- Vocabulary Cartoons
Picture Completion

**Picture Completion** measures a student’s ability to recognize familiar items and to identify missing parts. The student’s task is to separate essential and nonessential parts from the whole. It is necessary to observe each item closely and concentrate on picture detail. Students must name or indicate the missing part by saying the name of the part or by pointing to it.

Picture Completion is a timed supplemental Perceptual Reasoning subtest

**Suggestions:**

- Thinker Doodles
- Dot-to-Dots
- Visual Discrimination
- Look! Listen! Hear!
Block Design

Block Design measures an individual’s ability to analyze and synthesize an abstract design and reproduce that design from colored plastic blocks. Spatial visualization and analysis, simultaneous processing, visual-motor coordination, dexterity, and nonverbal concept formation are involved. The students use logic and reasoning to successfully complete the items.

Block Design subtest is a timed core Perceptual Reasoning subtest.

Children are given bi-colored blocks and must arrange them to duplicate a printed image or modelled design.

Suggestions:
- Pattern Block Activity Pack
- Tangoes
- Building Thinking Skills Figural Activities
- Architecto Games
Matrix Reasoning

Matrix Reasoning measures visual processing and abstract, spatial perception and may be influenced by concentration, attention, and persistence.

Matrix Reasoning is an untimed core Perceptual Reasoning subtest.

Children are shown colored matrices or visual patterns with something missing. The child is asked to select the missing piece from a range of options.

Suggestions:

- Building Thinking Skills
- Math Analogies
- Visual Discrimination
- Look! Listen! Hear!
- Number Patterns
- MiniLUK
**Coding**

*Coding* measures visual-motor dexterity, associative nonverbal learning, and nonverbal short-term memory. Fine-motor dexterity, speed, accuracy and ability to manipulate a pencil contribute to task success; perceptual organization is also important.

Coding is a timed core Processing Speed subtest.

For children aged 6-7 the test is picture based. Children are given a worksheet like the example below. The first line contains the key. They must place a mark within all the other figures so that they match the key.

For children aged 8-16 the key consists of boxes containing a numeral in the top line and a symbol in the bottom line. They must write the symbol corresponding to each numeral in the worksheet provided.