

User Manual for the Windows R Version of BACC  
(Bayesian Analysis, Computation, and Communication)

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# Important Information

## About this Manual

This manual describes the software developed in connection with the project Bayesian Communication in the Social Sciences, Siddhartha Chib and John Geweke principle investigators. Acknowledgement in any resulting published work would be appreciated. This project was supported, in part, by Grants SBR-9600040 and SBR-9731037 from the National Science Foundation.

## Help Keep This Software Free

The National Science Foundation supports this software and its continued development. It is important that we document the use of BACC. We respectfully request that all publications and working papers reporting the results of research using BACC software, include the following acknowledgement and reference:

Computations reported in this paper were undertaken [in part] using the Bayesian Analysis, Computation and Communication software (<http://www.econ.umn.edu/~bacc>) described in:

Geweke, J. (1999) "Using Simulation Methods for Bayesian Econometric Models: Inference, Development, and Communication" (with discussion and rejoinder), *Econometric Reviews* 18: 1-126.

## BACC Software and Documentation

BACC software and documentation is available on the web at

<http://www.econ.umn.edu/~bacc>

Please send any comments or questions to

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# Chapter 1

## Getting Started with BACC

### 1.1 Introduction

The BACC software provides the user several commands for doing Bayesian analysis and communications. This document describes the function of these commands and their inputs and outputs. It also outlines some of the theory behind the commands, and provides references to the relevant literature.

The following versions of the BACC software and documentation are available:

- Windows Matlab
- Linux/Unix Matlab
- Windows Splus
- Linux/Unix Splus
- Windows R
- Linux/Unix R
- Windows Gauss
- Linux/Unix Gauss
- Windows Console
- Linux/Unix Console

This particular manual is for the Windows R version of BACC.

### 1.2 Requirements

### 1.3 Installation and Configuration

Follow these steps to install the Windows R version of BACC.

1. Download the zip file `baccWinR.zip` from the software page of the BACC website <http://www.econ.umn.edu/~bacc>.
2. Unzip `baccWinR.zip` to the directory `C:\` using a standard unzipping utility such as PKZIP (available at <http://www.pkware.com>). Other freeware and shareware unzipping utilities are available on the web. This creates the directory `C:\BACC_R` and fills it with all the necessary files. You may also install the software into an alternative directory, in which case the following steps need to be modified accordingly.

The Windows R version of BACC is now installed. Follow these steps to load the BACC library within R.

1. Start R.
2. Change working directory to `BACC_R`, load the R script `baccWin.R` and load the dynamically linked library for BACC.

```
> setwd('C:\BACC_R')
> source('baccWin.R')
> loadBACC()
```

Follow these steps to run the sample program `testBACC.R`.

1. Set the working directory to `C:\BACC_R\Test`.

```
> setwd('C:\BACC_R\Test')
```

2. Run the R script `testBACC.R`.

```
> source('testBACC.R')
```

# Chapter 2

## Models

### 2.1 Introduction

This document specifies the models currently supported by the BACC system. Each section following this one describes one of the supported models. Each model description is organized into subsections, following the pattern of this section. Appendix A gives the probability density and mass functions of the distributions used throughout the document.

#### 2.1.1 Dimension parameters

All the quantities relevant to a model are treated as matrix valued. All matrix sizes are specified in terms of these dimension parameters. Examples of dimension parameters include the number of times a variable is observed, the number of individuals in a cross section, and the number of equations in a linear model. This subsection lists and describes the dimension parameters for a particular model.

#### 2.1.2 Unknown Quantities

Unknown quantities are all the unobserved elements in a model. They include unknown parameters of the model, latent variables, and missing data. Separate sub-sub-sections discuss unknown quantities in each of these categories. Posterior simulation involves drawing these quantities from their posterior distribution; that is, their conditional distribution given known quantities.

#### 2.1.3 Known Quantities

Known quantities are all the observed or user-specified values in a model. They include prior parameters, which index distributions within a family of prior distributions, and observed data. Separate sub-sub-sections discuss known quantities in both of these categories. The user of the BACC software must specify all the known quantities of a model in order to create an instance of the model.

### 2.1.4 Data Generating Process

This section specifies the conditional distribution of the endogenous observed data, given the unknown quantities and any observed data ancillary with respect to the unknown quantities.

### 2.1.5 Prior Distribution

This section specifies the marginal distribution of the unknown quantities, reflecting the user's prior beliefs about these quantities. These unknown quantities may or may not be independent. An example where they are not is a hierarchical prior, in which the prior density is expressed as the product of marginal densities of the “lowest level” unknowns and conditional densities of “higher level” unknowns given “lower level” unknowns.

### 2.1.6 Creating a Model Instance

This section gives all the model specific information a user requires to create a model instance. It specifies a short mnemonic label that identifies the model, the order in which the user gives the names to assign the unknown quantities, and the order in which to supply all the known quantities. To create a model instance, the user issues the `minst` command, with appropriate arguments (see section 3.3.14).

### 2.1.7 Sampling Algorithms

This section has brief descriptions of the algorithm used to generate samples of unknown quantities from their prior and posterior distributions. One subsection each concerns the prior distribution and the posterior distribution. For further details on the algorithms, the user should consult the internal (source code) documentation for the BACC system.

### 2.1.8 Marginal Likelihood

Where there is an analytical expression for the marginal likelihood in a model, this subsection provides that expression.

## 2.2 The Normal Linear Model

### 2.2.1 Dimension parameters

There are  $m$  equations,  $k$  covariate coefficients, and  $T$  observations of each variable.

### 2.2.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\beta$  and a  $m \times m$  precision parameter  $H$ .

### 2.2.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a  $k \times k$  positive definite coefficient precision matrix  $\underline{H}$ , a precision degrees of freedom parameter  $\underline{\nu} > \frac{m-1}{2}$ , and a positive definite precision inverse scale parameter  $\underline{S}$ .

#### Data

There are  $m$  vectors of observations of dependent variables:  $y_1, \dots, y_m$ . Each vector is  $T \times 1$ .

There are  $m$  matrices of observations of ancillary (with respect to unknown quantities) variables:  $Z_1, \dots, Z_m$ . Each matrix is  $T \times k$ .

### 2.2.4 Data Generating Process

$$y \equiv \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} Z_1 \\ \vdots \\ Z_m \end{bmatrix} \beta + \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_m \end{bmatrix} \equiv Z\beta + \epsilon$$

$$\begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_m \end{bmatrix} \mid \begin{bmatrix} Z_1 \\ \vdots \\ Z_m \end{bmatrix} \sim N(0, H^{-1} \otimes I_T)$$

### 2.2.5 Priors

The unknown parameters  $\beta$  and  $H$  are a-priori independent, and have the following marginal distributions.

$$\beta \sim N(\underline{\beta}, \underline{H}_{\beta}^{-1})$$

$$H \sim Wi(\underline{S}^{-1}, \underline{\nu})$$

When  $m = 1$ , the distribution of  $\underline{S}H$  is chi-squared with  $\underline{\nu}$  degrees of freedom.

### 2.2.6 Creating a Model Instance

The mnemonic label identifying the model is `nlm`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\beta$  (“beta” for example) and then the name of  $H$ .

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_\beta$ ,  $\underline{\nu}$ ,  $\underline{S}$ ,  $Z$ ,  $y$ .

### 2.2.7 Sampling Algorithms

#### Generating Prior Samples

Samples from the prior distribution of  $\beta$  and  $H$  are generated independently.

#### Generating Posterior Samples

The algorithm to generate samples from the posterior distribution  $\beta, H|Z, y$  is a Gibbs sampling algorithm with two blocks, based on the following conditional posterior distributions.

$$\beta|H, y, Z \sim N(\bar{\beta}, \bar{H}_\beta^{-1})$$

$$H|\beta, y, Z \sim W(\bar{S}^{-1}, \bar{\nu})$$

where

$$\bar{H}_\beta = \underline{H}_\beta + Z'(H \otimes I_T)Z$$

$$\bar{\beta} = \bar{H}_\beta^{-1}[\underline{H}_\beta \beta + Z'(H \otimes I_T)y]$$

$$\bar{S} = \underline{S} + [s_{ij}], s_{ij} = (y_i - Z_i \beta)'(y_i - Z_i \beta)$$

$$\bar{\nu} = \underline{\nu} + T$$

## 2.3 The Seemingly Unrelated Regressions Model

This is a special case of the Normal Linear Model with  $m > 1$ . Please see section 2.2.

## 2.4 The I.I.D. Finite State Model

### 2.4.1 Dimension parameters

There are  $m$  states,  $N$  individuals and  $T$  observation times.

### 2.4.2 Unknown Quantities

#### Unknown Parameters

There is a  $1 \times m$  state probability vector  $\pi$ .

### 2.4.3 Known Quantities

#### Prior Parameters

There is a  $1 \times m$  parameter  $\underline{\alpha}$  indexing the prior distribution of  $\pi$ .

#### Data

There are state observations  $s_{ti} \in \{1, \dots, m\}$  for each individual  $i$  of  $N$  individuals and each observation period  $t$  of  $T$  periods.

$$S = \begin{bmatrix} s_{1,1} & \cdots & s_{N,1} \\ \vdots & \ddots & \vdots \\ s_{T,1} & \cdots & s_{N,T} \end{bmatrix}$$

### 2.4.4 Data Generating Process

Each observation  $s_{ti}$  is independently and identically distributed as follows.

$$\Pr(s_{ti} = s) = \pi_s \quad s = 1, \dots, m$$

### 2.4.5 Priors

$$\pi \sim \text{Di}(\underline{\alpha})$$

### 2.4.6 Creating a Model Instance

The mnemonic label identifying the model is `iidfs`.

Supply the name you wish to give the unknown quantity  $\pi$  (“pi” for example).

Supply the known quantities in the following order:  $\underline{\alpha}$ ,  $S$ .

### 2.4.7 Sampling Algorithms

#### Generating Prior Samples

Samples from the prior distribution of  $\pi$  are generated independently.

### Generating Posterior Samples

In this model, the posterior distribution for  $\pi$  is the following familiar distribution.

$$\pi|S \sim \text{Di}(\bar{\alpha})$$

where

$$\bar{\alpha} = \underline{\alpha} + n$$

$$n = [n_1 \cdots n_m]$$

and  $n_s$  is the number of observations for which  $s_{ti} = s$ . Posterior samples are drawn independently from this distribution.

### 2.4.8 Marginal Likelihood

The marginal likelihood is given by

$$p(S) = \frac{\Gamma(\sum_{i=1}^m \underline{\alpha}_i)}{\prod_{i=1}^m \Gamma(\underline{\alpha}_i)} \frac{\prod_{i=1}^m \Gamma(\bar{\alpha}_i)}{\Gamma(\sum_{i=1}^m \bar{\alpha}_i)}.$$

## 2.5 The Non-Stationary First Order Markov Finite State Model

### 2.5.1 Dimension parameters

There are  $m$  states,  $N$  individuals and  $T$  observation times.

### 2.5.2 Unknown Quantities

#### Unknown Parameters

There is a  $1 \times m$  initial state probability vector  $\pi$  and an  $m \times m$  Markov transition probability matrix  $P$ .

### 2.5.3 Known Quantities

#### Prior Parameters

The prior parameters are a  $1 \times m$  vector  $\underline{\alpha}_0$  indexing the prior distribution of  $\pi$ , and an  $m \times m$  matrix  $\underline{\alpha}$  indexing the prior distribution of  $P$ .

#### Data

There are state observations  $s_{ti} \in \{1, \dots, m\}$  for each individual  $i$  and each observation time  $t$ .

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{TN} \end{bmatrix}$$

### 2.5.4 Data Generating Process

The  $N$  observation sequences  $\{s_{ti}\}_{t=1}^T$  are i.i.d., with each sequence being first order Markov. The initial distribution is  $\pi$  and the Markov transition matrix is  $P$ .

$$\Pr(s_{1i} = s) = \pi_s \quad s = 1, \dots, m$$

$$\Pr(s_{ti} = s' | s_{t-1,i} = s) = P_{ss'}$$

### 2.5.5 Priors

The  $m$  rows  $P_s$  of  $P$  and  $\pi$  are mutually independent, and have the following marginal distributions.

$$\begin{aligned} \pi &\sim \text{Di}(\underline{\alpha}_0) \\ P_s &\equiv [P_{s1}, \dots, P_{sm}] \sim \text{Di}(\underline{\alpha}_s) \quad s = 1, \dots, m \\ \underline{\alpha}_0 &\equiv [\alpha_{01} \quad \cdots \quad \alpha_{0m}] \end{aligned}$$

$$\underline{\alpha}_s \equiv \begin{bmatrix} \underline{\alpha}_{s1} & \cdots & \underline{\alpha}_{sm} \end{bmatrix} \quad s = 1, \dots, m$$

### 2.5.6 Creating a Model Instance

The mnemonic label identifying the model is **nsfomfs**.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\pi$  (“pi” for example), and then the name of  $P$ .

Supply the known quantities in the following order:  $\underline{\alpha}_0$ ,  $\underline{\alpha}$ ,  $S$ .

### 2.5.7 Sampling Algorithms

#### Generating Prior Samples

Samples from the prior distributions of  $\pi$  and  $P$  are generated independently.

#### Generating Posterior Samples

In the posterior distribution  $\pi, P|S$ , the parameters  $\pi$  and  $P$  are conditionally independent, and their marginal posterior distributions are the following familiar distributions.

$$\pi|S \sim \text{Di}(\bar{\alpha}_0)$$

$$P_s|S \sim \text{Di}(\bar{\alpha}_s) \quad s = 1, \dots, m$$

where

$$\bar{\alpha}_0 \equiv \underline{\alpha}_0 + n_0$$

$$\bar{\alpha} \equiv \underline{\alpha} + n$$

$$\bar{\alpha}_s \equiv \begin{bmatrix} \bar{\alpha}_{s1} & \cdots & \bar{\alpha}_{sm} \end{bmatrix}$$

$$n_0 \equiv \begin{bmatrix} n_{01} & \cdots & n_{0m} \end{bmatrix}$$

$$n \equiv \begin{bmatrix} n_{11} & \cdots & n_{1m} \\ \vdots & \ddots & \vdots \\ n_{m1} & \cdots & n_{mm} \end{bmatrix}$$

where  $n_{0s}$  is the number of individuals starting in state  $s$ , and  $n_{ss'}$  is the number of transitions from state  $s$  to state  $s'$  in the data.

Posterior samples are drawn independently from this distribution.

### 2.5.8 Marginal Likelihood

The marginal likelihood is available in closed form:

$$p(S) = \frac{\Gamma(\sum_{s=1}^m \underline{\alpha}_{0s})}{\prod_{s=1}^m \Gamma(\underline{\alpha}_{0s})} \frac{\prod_{s=1}^m \Gamma(\bar{\alpha}_{0s})}{\Gamma(\sum_{s=1}^m \bar{\alpha}_{0s})} \cdot \prod_{s=1}^m \left[ \frac{\Gamma(\sum_{s'=1}^m \underline{\alpha}_{ss'})}{\prod_{s'=1}^m \Gamma(\underline{\alpha}_{ss'})} \frac{\prod_{s'=1}^m \Gamma(\bar{\alpha}_{ss'})}{\Gamma(\sum_{s'=1}^m \bar{\alpha}_{ss'})} \right]$$

## 2.6 The Stationary First Order Markov Finite State Model

### 2.6.1 Dimension parameters

There are  $m$  states,  $N$  individuals and  $T$  observation times.

### 2.6.2 Unknown Quantities

#### Unknown Parameters

There is an  $m \times m$  Markov transition probability matrix  $P$ .

### 2.6.3 Known Quantities

#### Prior Parameters

The prior parameter is an  $m \times m$  matrix  $\underline{\alpha}$  indexing the prior distribution of  $P$ .

#### Data

There are state observations  $s_{ti} \in \{1, \dots, m\}$  for each individual  $i$  and each observation time  $t$ .

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{TN} \end{bmatrix}$$

### 2.6.4 Data Generating Process

The  $N$  observation sequences  $\{s_{ti}\}_{t=1}^T$  are i.i.d., with each sequence being first order Markov with transition matrix  $P$ . The initial distribution vector is assumed to be the invariant distribution  $\pi$  for  $P$ .

$$\Pr(s_{1i} = s) = \pi_s \quad s = 1, \dots, m$$

$$\Pr(s_{ti} = s' | s_{t-1,i} = s) = P_{ss'}$$

where  $\pi$  is the left eigenvector of  $P$  corresponding to the eigenvalue  $\lambda = 1$ .

### 2.6.5 Priors

The  $m$  rows  $P_s$  of  $P$  are mutually independent, and have the following marginal distributions.

$$P_s \equiv [P_{s1}, \dots, P_{sm}] \sim \text{Di}(\underline{\alpha}_s) \quad s = 1, \dots, m$$

$$\underline{\alpha}_s \equiv \begin{bmatrix} \alpha_{s1} & \cdots & \alpha_{sm} \end{bmatrix} \quad s = 1, \dots, m$$

### 2.6.6 Creating a Model Instance

The mnemonic label identifying the model is `sfomfs`.

Supply the name you wish to give the unknown quantity  $P$ .

Supply the known quantities in the following order:  $\underline{\alpha}$ ,  $S$ .

### 2.6.7 Sampling Algorithms

#### Generating Prior Draws

Samples from the prior distribution of  $P$  are generated independently.

#### Generating Posterior Draws

In this model, an independence Metropolis-Hastings chain is used to draw from the posterior distribution for  $P$ . The distribution  $P^*|S$  of candidate draws is

$$P_s^*|S \sim \text{Di}(\bar{\alpha}_s) \quad s = 1, \dots, m$$

where

$$\begin{aligned} \bar{\alpha} &\equiv \underline{\alpha} + n \\ \bar{\alpha}_s &\equiv \begin{bmatrix} \bar{\alpha}_{s1} & \cdots & \bar{\alpha}_{sm} \end{bmatrix} \\ n &\equiv \begin{bmatrix} n_{11} & \cdots & n_{1m} \\ \vdots & \ddots & \vdots \\ n_{m1} & \cdots & n_{mm} \end{bmatrix} \end{aligned}$$

where  $n_{ss'}$  is the number of transitions from state  $s$  to state  $s'$  in the data.

The Hastings ratio for this block is given by

$$\prod_{i=1}^N \frac{\pi_{S_{1i}}^*}{\pi_{S_{1i}}}$$

## 2.7 The Poisson Model

### 2.7.1 Dimension parameters

There are  $N$  observations.

### 2.7.2 Unknown Quantities

#### Unknown Parameters

There is a scalar mean parameter  $\lambda$ .

### 2.7.3 Known Quantities

#### Prior Parameters

There is a scalar shape parameter  $\underline{\alpha} > 0$  and a scalar scale parameter  $\underline{\beta} > 0$  indexing the prior distribution of  $\lambda$ .

#### Data

Each observation  $x_i$  is a non-negative integer.

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

### 2.7.4 Data Generating Process

The observations  $x_i$  are independently and identically Poisson distributed.

$$x_i \sim \text{Po}(\lambda)$$

### 2.7.5 Priors

$$\lambda \sim \text{Ga}(\underline{\alpha}, \underline{\beta})$$

### 2.7.6 Creating a Model Instance

The mnemonic label identifying the model is `poisson`

Supply the names you wish to give the unknown quantity  $\lambda$  (“lambda” for example).

Supply the known quantities in the following order:  $\underline{\alpha}$ ,  $\underline{\beta}$ ,  $X$ .

### 2.7.7 Sampling Algorithms

#### Generating Prior Samples

Samples from the prior distribution of  $\lambda$  are generated independently.

### Generating Posterior Samples

In this model, the posterior distribution for  $\lambda$  is the following familiar distribution.

$$\lambda|X \sim \text{Ga}(\bar{\alpha}, \bar{\beta})$$

$$\bar{\alpha} = \underline{\alpha} + \sum_{i=1}^N x_i$$

$$\bar{\beta} = \underline{\beta} + N$$

Posterior samples are drawn independently from this distribution.

### 2.7.8 Marginal Likelihood

The marginal likelihood is given by the following expression.

$$p(X) = \frac{\underline{\beta}^N}{(\underline{\beta} + N)^{\underline{\alpha} + r}} \frac{\Gamma(\underline{\alpha} + r)}{\Gamma(\underline{\alpha})} \frac{1}{\prod_{i=1}^N x_i!},$$

where

$$r = \sum_{i=1}^N x_i$$

## 2.8 The Uniform Model

### 2.8.1 Dimension parameters

There are  $N$  observations.

### 2.8.2 Unknown Quantities

#### Unknown Parameters

There is a scalar support parameter  $\theta$ .

### 2.8.3 Known Quantities

#### Prior Parameters

There is a scalar notional count parameter  $\underline{\alpha} > 0$  and a scalar notional maximal element parameter  $\underline{\beta} \geq 0$  indexing the prior distribution of  $\theta$ .

#### Data

Each observation  $x_i$  is a non-negative real-valued scalar.

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

### 2.8.4 Data Generating Process

Each observation  $x$  is independently and identically distributed with a uniform distribution on  $[0, \theta]$ .

$$x_i \sim \text{i.i.d.} U(0, \theta)$$

### 2.8.5 Priors

$$\theta \sim \text{Pa}(\underline{\alpha}, \underline{\beta})$$

### 2.8.6 Creating a Model Instance

The mnemonic label identifying the model is **uniform**. Supply the name you wish to give the unknown quantity  $\theta$  (“theta” for example).

Supply the known quantities in the following order:  $\underline{\alpha}$ ,  $\underline{\beta}$ ,  $X$ .

### 2.8.7 Sampling Algorithms

#### Generating Prior Samples

Samples from the prior distribution of  $\theta$  are generated independently.

#### Generating Posterior Samples

In this model, the posterior distribution for  $\theta$  is the following familiar distribution.

$$\theta|X \sim Pa(\bar{\alpha}, \bar{\beta})$$

where

$$\begin{aligned}\bar{\alpha} &= \underline{\alpha} + N \\ \bar{\beta} &= \max \left\{ \underline{\beta}, \max_i x_i \right\}\end{aligned}$$

Posterior samples are drawn independently from this distribution.

### 2.8.8 Marginal Likelihood

The marginal likelihood is given by the following expression.

$$p(X) = (\underline{\alpha}/\bar{\alpha}) \prod_{i=1}^N (p_i/\bar{\beta}_i)$$

where

$$\bar{\beta}_i = \max \left\{ \underline{\beta}, \max_{j \leq i} x_j \right\}$$

and

$$p_i = \begin{cases} (\frac{\bar{\beta}_i}{x_i})^{(1+\underline{\alpha}+i)} & \text{if } x_i > \bar{\beta}_i \\ 1 & \text{otherwise.} \end{cases}$$

## 2.9 A Univariate Linear Model with Normal Disturbances

The mnemonic label identifying the model is `n_ult`.

### Dimension Parameters

$T$     number of observations

$K$     number of covariates

### Unknown Quantities

$\beta$      $(K \times 1)$     vector of covariate coefficients

$h$      $(1 \times 1)$     precision of disturbance

### Known Quantities

$\underline{\beta}$      $(K \times 1)$     prior mean of  $\beta$

$\underline{H}_\beta$      $(K \times K)$     prior precision of  $\beta$

$\underline{s}^2$      $(1 \times 1)$     prior inverse scale of  $h$

$\underline{\nu}$      $(1 \times 1)$     prior degrees of freedom of  $h$

$X$      $(T \times K)$     covariates

$y$      $(T \times 1)$     dependant variable

### Data Generating Process

The observables  $y$  are given by

$$y = X\beta + u,$$

where  $u$  is a  $T \times 1$  vector of i.i.d. normal disturbances, with  $u_t|h \sim N(0, h^{-1})$ :

$$p(u|X, \beta, h) = (2\pi)^{-T/2} h^{T/2} \exp(-hu'u/2).$$

### Prior Distribution

The vectors  $\beta$  and  $h$ , together with  $X$ , are mutually independent. The covariate coefficient vector  $\beta$  has distribution  $N(\underline{\beta}, \underline{H}_\beta)$ :

$$p(\beta) = (2\pi)^{-K/2} |\underline{H}_\beta|^{1/2} \exp[-(\beta - \underline{\beta})' \underline{H}_\beta (\beta - \underline{\beta})/2].$$

The precision parameter  $h$  has a scaled chi-squared distribution, with  $\underline{s}^2 h \sim \underline{\nu}$ :

$$p(h) = 2^{-\underline{\nu}/2} \Gamma(\underline{\nu}/2)^{-1} (\underline{s}^2)^{\underline{\nu}/2} h^{(\underline{\nu}-2)/2} \exp(-\underline{s}^2 h/2).$$

**Sampling Algorithm**

See Example 3.4.1 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.10 The Dichotomous Choice Model (with normally distributed disturbances)

### 2.10.1 Dimension parameters

There are  $k$  covariate coefficients and  $T$  observations of each variable.

### 2.10.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\beta$ , a scalar precision parameter  $h$ , and a  $T \times 1$  vector  $\tilde{y}$  of latent outcomes.

### 2.10.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , and a positive definite precision inverse scale parameter  $\underline{S}$ .

#### Data

There is a  $T \times 1$  vector of observations of dependent variables  $y$  taking values in  $\{0, 1\}$ .

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.10.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.10.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.10.6 Creating a Model Instance

The mnemonic label identifying the model is `udcht`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\beta$  (“beta” for example), then the name of  $h$  (“hHomo” for example), and finally the name of the latent variable  $\tilde{y}$  (“yTilde” for example).

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_\beta$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $X$ ,  $y$ .

### 2.10.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> .

## 2.11 The Censored Linear Model (with normally distributed disturbances)

### 2.11.1 Dimension parameters

There are  $k$  covariate coefficients and  $T$  observations of each variable.

### 2.11.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\beta$ , a scalar precision parameter  $h$ , and a  $T \times 1$  vector  $\tilde{y}$  of (possibly) latent outcomes.

### 2.11.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , a positive definite precision inverse scale parameter  $\underline{S}$ , and a censoring parameter  $\underline{c}$ .

#### Data

There is a  $T \times 1$  vector of observations of dependent variables  $y$ .

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.11.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.11.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.11.6 Creating a Model Instance

The mnemonic label identifying the model is **ucensor**.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\beta$  (“beta” for example), then the name of  $h$  (“hHomo” for example), and finally the name of the latent variable  $\tilde{y}$  (“yTilde” for example).

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_\beta$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $\underline{c}$ ,  $X$ ,  $y$ .

### 2.11.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.12 The Univariate Latent Linear Model (with normally distributed disturbances)

### 2.12.1 Dimension parameters

There are  $k$  covariate coefficients and  $T$  observations of each variable.

### 2.12.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\beta$ , a scalar precision parameter  $h$ , and a  $T \times 1$  vector  $\tilde{y}$  of (possibly) latent outcomes.

### 2.12.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , and a positive definite precision inverse scale parameter  $\underline{S}$ .

#### Data

Corresponding to the (possibly) latent outcome  $\tilde{y}$ , there are two  $T \times 1$  vectors  $c$  and  $d$ ,  $c \geq d$ , which describe the observed, set-valued outcome.

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.12.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>.

### 2.12.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>.

### 2.12.6 Creating a Model Instance

The mnemonic label identifying the model is `ullm`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\beta$  (“beta” for example), then the name of  $h$  (“hHomo” for example), and finally the name of the latent variable  $\tilde{y}$  (“yTilde” for example).

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_\beta$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $X$ ,  $c$ ,  $d$ .

### 2.12.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> .

## 2.13 A Univariate Linear Model with Student t Disturbances

The mnemonic label identifying the model is `t_ultm`.

### Dimension Parameters

$T$     number of observations  
 $K$     number of covariates

### Unknown Quantities

$\beta$      $(K \times 1)$     vector of covariate coefficients  
 $h$      $(1 \times 1)$     precision of Student t distribution  
 $\tilde{h}$      $(T \times 1)$     time varying latent precision variable  
 $\lambda$      $(1 \times 1)$     degrees of freedom of Student t distribution

### Known Quantities

$\underline{\beta}$      $(K \times 1)$     prior mean of  $\beta$   
 $\underline{H}_\beta$      $(K \times K)$     prior precision of  $\beta$   
 $\underline{s}^2$      $(1 \times 1)$     prior inverse scale of  $h$   
 $\underline{\nu}$      $(1 \times 1)$     prior degrees of freedom of  $h$   
 $\underline{\lambda}$      $(1 \times 1)$     prior mean of  $\lambda$   
 $X$      $(T \times K)$     covariates  
 $y$      $(T \times 1)$     dependant variable

### Data Generating Process

The observables  $y$  are given by

$$y = X\beta + u,$$

where  $u$  is a  $T \times 1$  vector of independant Student t disturbances, with  $u_t|h, X \sim t(0, h^{-1}, \lambda)$ . Conditioning on the latent  $\tilde{h}$  gives  $u_t|h, \tilde{h} \sim N(0, (h\tilde{h}_t)^{-1})$ :

$$p(u|X, \beta, \tilde{h}, h) = (2\pi)^{-T/2} h^{T/2} \prod_{t=1}^T \tilde{h}_t^{1/2} \exp(-h\tilde{h}_t u_t^2/2).$$

See Section 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> for details.

### Prior Distribution

The vectors  $\beta$ ,  $h$ , and  $(\tilde{h}, \lambda)$ , together with  $X$ , are mutually independent. The covariate coefficient vector  $\beta$  has distribution  $N(\underline{\beta}, \underline{H}_\beta)$ :

$$p(\beta) = (2\pi)^{-K/2} |\underline{H}_\beta|^{1/2} \exp[-(\beta - \underline{\beta})' \underline{H}_\beta (\beta - \underline{\beta})/2].$$

The precision parameter  $h$  has a scaled chi-squared distribution, with  $\underline{s}^2 h \sim \underline{\nu}$ :

$$p(h) = 2^{-\underline{\nu}/2} \Gamma(\underline{\nu}/2)^{-1} (\underline{s}^2)^{\underline{\nu}/2} h^{(\underline{\nu}-2)/2} \exp(-\underline{s}^2 h/2).$$

The time varying latent precision parameters  $\tilde{h}$  are i.i.d. scaled chi-squared variates, with  $\lambda \tilde{h}_t \sim \chi^2(\lambda)$ :

$$p(\tilde{h}|\lambda) = [2^{\lambda/2} \Gamma(\lambda/2)]^{-T} \lambda^{T\lambda/2} \prod_{t=1}^T \tilde{h}_t^{(\lambda-2)/2} \exp(-\lambda \tilde{h}_t/2)$$

The degrees of freedom parameter  $\lambda$  is distributed  $\exp(\underline{\lambda})$ :

$$p(\lambda) = \underline{\lambda}^{-1} \exp(-\lambda/\underline{\lambda}).$$

### Sampling Algorithm

See Section 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.14 The Dichotomous Choice Model (with Student $t$ distributed disturbances)

### 2.14.1 Dimension parameters

There are  $k$  covariate coefficients and  $T$  observations of each variable.

### 2.14.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\beta$ , a scalar precision parameter  $h$ , a  $T \times 1$  vector of precision parameters  $h_t$ , a  $T \times 1$  vector  $\tilde{y}$  of latent outcomes, and a scalar degrees of freedom parameter  $\lambda$ .

### 2.14.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , a positive definite precision inverse scale parameter  $\underline{S}$ , and a degrees of freedom parameter  $\underline{\lambda}$ .

#### Data

There is a  $T \times 1$  vector of observations of dependent variables  $y$  taking values in  $\{0, 1\}$ .

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.14.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.14.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.14.6 Creating a Model Instance

The mnemonic label identifying the model is `t_udcht`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\beta$  (“beta” for example), then the name of  $h$  (“hHomo” for example), then the name of  $h_t$  (“hHetero” for example), then the name of the latent variable  $\tilde{y}$  (“yTilde” for example), and finally the name of the degrees of freedom parameter  $\lambda$  (“lambda” for example).

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_\beta$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $\underline{\lambda}$ ,  $X$ ,  $y$ .

### 2.14.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.15 The Censored Linear Model (with Student $t$ distributed disturbances)

### 2.15.1 Dimension parameters

There are  $k$  covariate coefficients and  $T$  observations of each variable.

### 2.15.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\beta$ , a scalar precision parameter  $h$ , a  $T \times 1$  vector of precision parameters  $h_t$ , a  $T \times 1$  vector  $\tilde{y}$  of (possibly) latent outcomes, and a scalar degrees of freedom parameter  $\lambda$ .

### 2.15.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , a positive definite precision inverse scale parameter  $\underline{S}$ , a degrees of freedom parameter  $\underline{\lambda}$ , and a censoring parameter  $\underline{c}$ .

#### Data

There is a  $T \times 1$  vector of observations of dependent variables  $y$ .

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.15.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.15.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.15.6 Creating a Model Instance

The mnemonic label identifying the model is `t_ucensor`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\beta$  (“beta” for example), then the name of  $h$  (“hHomo” for example), then the name of  $h_t$  (“hHetero” for example), then the name of the latent variable  $\tilde{y}$  (“yTilde” for example), and finally the name of the degrees of freedom parameter  $\lambda$  (“lambda” for example).

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_\beta$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $\underline{\lambda}$ ,  $\underline{c}$ ,  $X$ ,  $y$ .

### 2.15.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.16 The Univariate Latent Linear Model (with Student $t$ distributed disturbances)

### 2.16.1 Dimension parameters

There are  $k$  covariate coefficients and  $T$  observations of each variable.

### 2.16.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\beta$ , a scalar precision parameter  $h$ , a  $T \times 1$  vector of precision parameters  $h_t$ , a  $T \times 1$  vector  $\tilde{y}$  of (possibly) latent outcomes, and a scalar degrees of freedom parameter  $\lambda$ .

### 2.16.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , a positive definite precision inverse scale parameter  $\underline{S}$ , and a degrees of freedom parameter  $\underline{\lambda}$ .

#### Data

Corresponding to the (possibly) latent outcome  $\tilde{y}$ , there are two  $T \times 1$  vectors  $c$  and  $d$ ,  $c \geq d$ , which describe the observed, set-valued outcome.

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.16.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.16.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.16.6 Creating a Model Instance

The mnemonic label identifying the model is `t_ullm`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\beta$  (“beta” for example), then the name of  $h$  (“hHomo” for example), then the name of  $h_t$  (“hHetero” for example), then the name of the latent variable  $\tilde{y}$  (“yTilde” for example), and finally the name of the degrees of freedom parameter  $\lambda$  (“lambda” for example).

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_{\beta}$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $\underline{\lambda}$ ,  $X$ ,  $c$ ,  $d$ .

### 2.16.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.17 A Univariate Linear Model with Finite Mixtures of Normals Disturbances

The mnemonic label identifying the model is `fmn_ulm`.

### Dimension Parameters

|     |                                          |
|-----|------------------------------------------|
| $T$ | number of observations                   |
| $K$ | number of covariates                     |
| $m$ | number of mixture components (or states) |

### Unknown Quantities

|              |                      |                                                    |
|--------------|----------------------|----------------------------------------------------|
| $\gamma$     | $((m + K) \times 1)$ | vector of state means and covariate coefficients   |
| $h$          | $(1 \times 1)$       | constant multiplicative precision component        |
| $\mathbf{h}$ | $(m \times 1)$       | state dependant multiplicative precision component |
| $\tilde{s}$  | $(T \times 1)$       | time varying latent discrete state                 |
| $\pi$        | $(1 \times m)$       | state probabilities                                |

### Known Quantities

|                        |                |                                                   |
|------------------------|----------------|---------------------------------------------------|
| $\underline{h}_\alpha$ | $(1 \times 1)$ | prior precision parameter for state means         |
| $\underline{\beta}$    | $(K \times 1)$ | prior mean of covariate coefficients              |
| $\underline{H}_\beta$  | $(K \times K)$ | prior precision of covariate coefficients         |
| $\underline{s}^2$      | $(1 \times 1)$ | prior inverse scale of $h$                        |
| $\underline{\nu}$      | $(1 \times 1)$ | prior degrees of freedom of $h$                   |
| $m$                    | $(1 \times 1)$ | number of states                                  |
| $\underline{\nu}$      | $(1 \times 1)$ | degrees of freedom parameter for state precisions |
| $r$                    | $(1 \times 1)$ | Dirichlet parameter for state probabilities       |
| $X$                    | $(T \times K)$ | covariates                                        |
| $y$                    | $(T \times 1)$ | dependant variable                                |

### Data Generating Process

The observables  $y$  are given by

$$y = X\beta + u,$$

where  $u$  is a  $T \times 1$  vector of independent discrete normal mixture disturbances, with  $u_t|h, \pi, \alpha, \mathbf{h}, X$  given by:

$$p(u_t|h, \pi, \alpha, \mathbf{h}, X) = (2\pi)^{-1/2} h^{1/2} \sum_{j=1}^m \pi_j h_j^{1/2} \exp[-h \cdot h_j (u_t - \alpha_j)^2 / 2]$$

Conditioning on the latent states gives

$$p(u_t|h, \alpha, \mathbf{h}, X) = (2\pi)^{-1/2} h^{1/2} h_{\tilde{s}_t}^{1/2} \exp[-h \cdot h_{\tilde{s}_t} (u_t - \alpha_{\tilde{s}_t})^2 / 2].$$

See Section 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> for details.

### Prior Distribution

The vectors  $(\gamma, h)$ ,  $\mathbf{h}$ , and  $(\tilde{s}, \pi)$ , together with  $X$ , are mutually independent. The  $\gamma$  parameter vertically stacks the parameters  $\alpha$  and  $\beta$ , where  $\alpha$  is the  $m \times 1$  vector of state dependant means and  $\beta$  is the  $K \times 1$  vector of covariate coefficients. They are independent, with  $\alpha|h \sim N(0, (\underline{h}_\alpha h)^{-1})$  and  $\beta \sim N(\underline{\beta}, \underline{H}_\beta)$ :

$$\begin{aligned} p(\gamma|h) = p(\alpha|h) \cdot p(\beta) &= (2\pi)^{-m/2} (\underline{h}_\alpha h)^{m/2} \exp(-\underline{h}_\alpha h \alpha' \alpha / 2) \\ &\cdot (2\pi)^{-K/2} |\underline{H}_\beta|^{1/2} \exp[-(\beta - \underline{\beta})' \underline{H}_\beta (\beta - \underline{\beta}) / 2]. \end{aligned}$$

The precision parameter  $h$  has a scaled chi-squared distribution, with  $\underline{s}^2 h \sim \underline{\nu}$ :

$$p(h) = 2^{-\underline{\nu}/2} \Gamma(\underline{\nu}/2)^{-1} (\underline{s}^2)^{\underline{\nu}/2} h^{(\underline{\nu}-2)/2} \exp(-\underline{s}^2 h / 2).$$

The state dependant precisions  $h_j$  are i.i.d., with  $\underline{\nu} \cdot h_j \sim \chi^2(\underline{\nu})$ :

$$p(\mathbf{h}) = 2^{m\underline{\nu}} \Gamma(\underline{\nu}/2) (\underline{\nu}^2)^{m\underline{\nu}/2} \prod_{j=1}^m h_j^{(\underline{\nu}-2)/2} \exp(-\underline{\nu} \cdot h_j / 2).$$

The latent states are i.i.d., with the probability  $\Pr[s_t = j]$  given by  $\pi_j$ , for  $j = 1, \dots, m$ :

$$p(\tilde{s}|\pi) = \prod_{t=1}^T \pi_{\tilde{s}_t}$$

The vector  $\pi$  of probabilities is distributed Dirichlet( $r, \dots, r$ ):

$$p(\pi) = \Gamma(mr) \Gamma(r)^{-m} \prod_{j=1}^m \pi_j^{r-1}.$$

### Sampling Algorithm

See section 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.18 The Dichotomous Choice Model (with a scale mixture of normals distribution for the disturbances)

### 2.18.1 Dimension parameters

There are  $k$  covariate coefficients,  $T$  observations of each variable, and  $m$  components for the mixture of normals (i.e.,  $m$  states).

### 2.18.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\gamma$ , a scalar precision parameter  $h$ , a  $T \times 1$  vector of state indices, a  $1 \times m$  vector of probabilities, an  $m \times 1$  vector of precision parameters  $h_j$ , and a  $T \times 1$  vector  $\tilde{y}$  of latent outcomes.

### 2.18.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a positive scalar precision parameter  $\underline{H}_\alpha$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , a positive definite precision inverse scale parameter  $\underline{S}$ , an  $m \times 1$  vector of precision degrees of freedom parameters  $\underline{\nu}_j$ , an  $m \times 1$  vector of positive definite precision inverse scale parameter  $\underline{S}_j$ , and a  $1 \times m$  vector of hyperparameters  $\underline{r}$ .

#### Data

There is a  $T \times 1$  vector of observations of dependent variables  $y$  taking values in  $\{0, 1\}$ .

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.18.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.18.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.18.6 Creating a Model Instance

The mnemonic label identifying the model is `fmn.udcht`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\gamma$  (“gamma” for example), then the name of  $s$ , then the name of  $p$ , then the name of  $h_j$  (“hState” for example), and finally the name of the latent outcome variable  $\tilde{y}$  (“yTilde” for example).

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_\alpha$ ,  $\underline{H}_\beta$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $\underline{S}_j$ ,  $\underline{\nu}_j$ ,  $\underline{r}$ ,  $X$ ,  $y$ .

### 2.18.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.19 The Censored Linear Model (with a scale mixture of normals distribution for the disturbances)

### 2.19.1 Dimension parameters

There are  $k$  covariate coefficients,  $T$  observations of each variable, and  $m$  components for the mixture of normals (i.e.,  $m$  states).

### 2.19.2 Unknown Quantities

#### Unknown Parameters

There is a  $k \times 1$  coefficient parameter  $\gamma$ , a scalar precision parameter  $h$ , a  $T \times 1$  vector of state indices, a  $1 \times m$  vector of probabilities, an  $m \times 1$  vector of precision parameters  $h_j$ , and a  $T \times 1$  vector  $\tilde{y}$  of (possibly) latent outcomes.

### 2.19.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a positive scalar precision parameter  $\underline{H}_\alpha$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , a positive definite precision inverse scale parameter  $\underline{S}$ , an  $m \times 1$  vector of precision degrees of freedom parameters  $\underline{\nu}_j$ , an  $m \times 1$  vector of positive definite precision inverse scale parameter  $\underline{S}_j$ , a  $1 \times m$  vector of hyperparameters  $\underline{\tau}$ , and a censoring parameter  $\underline{c}$ .

#### Data

There is a  $T \times 1$  vector of observations of dependent variables  $y$ .

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.19.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.19.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.19.6 Creating a Model Instance

The mnemonic label identifying the model is `fmn.ucensor`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\gamma$  (“gamma” for example), then the name of  $s$ , then the name of  $p$ , then the name of  $h_j$  (“hState” for example), and finally the name of the outcome variable  $\tilde{y}$  (“yTilde” for example).

Supply the known quantities in the following order:  $\underline{\beta}$ ,  $\underline{H}_\alpha$ ,  $\underline{H}_\beta$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $\underline{S}_j$ ,  $\underline{\nu}_j$ ,  $\underline{r}$ ,  $\underline{c}$ ,  $X$ ,  $y$ .

### 2.19.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.20 The Univariate Latent Linear Model (with a scale mixture of normals distribution for the disturbances)

### 2.20.1 Dimension parameters

There are  $k$  covariate coefficients,  $T$  observations of each variable, and  $m$  components for the mixture of normals (i.e.,  $m$  states).

### 2.20.2 Unknown Quantities

#### Unknown Parameters

There is a  $(m+k) \times 1$  coefficient parameter  $\gamma$ , a scalar precision parameter  $h$ , a  $T \times 1$  vector of state indices, a  $1 \times m$  vector of probabilities, an  $m \times 1$  vector of precision parameters  $h_j$ , and a  $T \times 1$  vector  $\tilde{y}$  of (possibly) latent outcomes.

### 2.20.3 Known Quantities

#### Prior Parameters

There is a  $k \times 1$  coefficient mean vector  $\underline{\beta}$ , a positive scalar precision parameter  $\underline{H}_\alpha$ , a  $k \times k$  positive definite coefficient matrix  $\underline{H}_\beta$ , a precision degrees of freedom parameter  $\underline{\nu}$ , a positive definite precision inverse scale parameter  $\underline{S}$ , an  $m \times 1$  vector of precision degrees of freedom parameters  $\underline{\nu}_j$ , an  $m \times 1$  vector of positive definite precision inverse scale parameter  $\underline{S}_j$ , and a  $1 \times m$  vector of hyperparameters  $\underline{z}$ .

#### Data

Corresponding to the (possibly) latent outcome  $\tilde{y}$ , there are two  $T \times 1$  vectors  $c$  and  $d$ ,  $c \geq d$ , which describe the observed, set-valued outcome.

There is a  $T \times k$  matrix  $X$  of observations of ancillary (with respect to unknown quantities) variables.

### 2.20.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.20.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

### 2.20.6 Creating a Model Instance

The mnemonic label identifying the model is `fmn.ullm`.

Supply the names you wish to give the unknown quantities in the following order: first the name of  $\gamma$  (“gamma” for example), then the name of  $s$ , then the name of  $p$ , then the name of  $h_j$  (“hState” for example), and finally the name of the outcome variable  $\tilde{y}$  (“yTilde” for example).

Supply the known quantities in the following order:  $\underline{\gamma}$ ,  $\underline{H}_\alpha$ ,  $\underline{H}_\beta$ ,  $\underline{S}$ ,  $\underline{\nu}$ ,  $\underline{S}_j$ ,  $\underline{\nu}_j$ ,  $\underline{r}$ ,  $X$ ,  $c$ ,  $d$ .

### 2.20.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

## 2.21 An Autoregression Model

### 2.21.1 Dimension Parameters

The model features the following dimension parameters.

| Dimension Parameter | Description            |
|---------------------|------------------------|
| $T$                 | number of observations |
| $K$                 | number of covariates   |
| $p$                 | autoregressive order   |

### 2.21.2 Unknown Quantities

The model features the following unknown quantities.

| Unknown Quantity | Dimensions   | Description                           |
|------------------|--------------|---------------------------------------|
| $\beta$          | $K \times 1$ | covariate coefficient vector          |
| $h$              | $1 \times 1$ | residual precision                    |
| $\phi$           | $p \times 1$ | vector of autoregression coefficients |

### 2.21.3 Known Quantities

The model features the following known quantities.

| Known Quantity  | Dimensions   | Description                                 |
|-----------------|--------------|---------------------------------------------|
| $\bar{\beta}$   | $K \times 1$ | prior mean of $\beta$                       |
| $\bar{H}_\beta$ | $K \times K$ | prior precision of $\beta$                  |
| $\bar{\nu}$     | $1 \times 1$ | prior degrees of freedom of $h$             |
| $\bar{s}^2$     | $1 \times 1$ | prior inverse scale of $h$                  |
| $\bar{\phi}$    | $p \times 1$ | prior mean of $\phi$ before truncation      |
| $\bar{H}_\phi$  | $p \times p$ | prior precision of $\phi$ before truncation |
| $X$             | $T \times K$ | covariates                                  |
| $y$             | $T \times 1$ | dependant variable                          |

### 2.21.4 Data Generating Process

The data generating process is given by

$$y_t = \beta' x_t + \epsilon_t$$

where  $x_t$  is the  $t$ 'th row of  $X$ , as a column vector,

$$\epsilon_t = \sum_{i=1}^p \phi_i \epsilon_{t-i} + u_t,$$

and

$$u_t \sim \text{i.i.d. } N(0, h^{-1})$$

### 2.21.5 Prior Distribution

The unknowns are a-priori independent and have the following distributions.

$$\beta \sim N(\bar{\beta}, \bar{H}_{\beta}^{-1})$$

$$\bar{s}^2 h \sim \chi^2(\bar{\nu})$$

The prior for  $\phi$  is obtained by truncating the following density to the region for which  $y$  is stationary.

$$\phi \sim N(\bar{\phi}, \bar{H}_{\phi}^{-1})$$

### 2.21.6 Creating a Model Instance

The mnemonic label identifying the model is **AR**.

Supply the names you wish to give the unknown quantities in the same order as they appear in the table of unknown quantities. Supply the known quantities in the same order as they appear in the table of known quantities.

### 2.21.7 Sampling Algorithm

The sampling algorithm for prior simulation features three blocks, each making independent draws from the prior distribution of one of the unknown quantities. The sampling algorithm for posterior simulation features three blocks, each making draws from the conditional posterior distribution of one of the unknown quantities.

## 2.22 An Autoregression Model with State Dependant Means

### 2.22.1 Dimension Parameters

The model features the following dimension parameters.

| Dimension Parameter | Description            |
|---------------------|------------------------|
| $T$                 | number of observations |
| $K$                 | number of covariates   |
| $p$                 | autoregressive order   |
| $m$                 | number of states       |

### 2.22.2 Unknown Quantities

The model features the following unknown quantities.

| Unknown Quantity | Dimensions         | Description                           |
|------------------|--------------------|---------------------------------------|
| $\gamma$         | $(m + K) \times 1$ | vertical stack of alpha and beta      |
| $h$              | $1 \times 1$       | residual precision                    |
| $\phi$           | $p \times 1$       | vector of autoregression coefficients |
| $P$              | $m \times m$       | state transition probability matrix   |
| $s$              | $T \times 1$       | latent states                         |
| $f$              | $T \times m$       | filter probabilities                  |

### 2.22.3 Known Quantities

The model features the following known quantities.

| Known Quantity   | Dimensions               | Description                                 |
|------------------|--------------------------|---------------------------------------------|
| $\bar{\gamma}$   | $(m + K) \times 1$       | prior mean of $\gamma$                      |
| $\bar{H}_\gamma$ | $(m + K) \times (m + K)$ | prior precision of $\gamma$                 |
| $\bar{\nu}$      | $1 \times 1$             | prior degrees of freedom of $h$             |
| $\bar{s}^2$      | $1 \times 1$             | prior inverse scale of $h$                  |
| $\bar{\phi}$     | $p \times 1$             | prior mean of $\phi$ before truncation      |
| $\bar{H}_\phi$   | $p \times p$             | prior precision of $\phi$ before truncation |
| $\bar{A}$        | $m \times m$             | parameters of prior for $P$                 |
| $X$              | $T \times K$             | covariates                                  |
| $y$              | $T \times 1$             | dependant variable                          |

#### 2.22.4 Data Generating Process

The data generating process is given by

$$y_t = \alpha_{s_t} + \beta' x_t + \epsilon_t$$

where  $x_t$  is the  $t$ 'th row of  $X$ , as a column vector, and  $\alpha$  ( $m \times 1$ ) and  $\beta$  ( $K \times 1$ ) are obtained by partitioning  $\gamma$ ,

$$\epsilon_t = \sum_{i=1}^p \phi_i \epsilon_{t-i} + u_t,$$

and

$$u_t \sim \text{i.i.d. } N(0, h^{-1})$$

#### 2.22.5 Prior Distribution

The unknowns are a-priori independent and have the following distributions.

$$\gamma \sim N(\bar{\gamma}, \bar{H}_\gamma^{-1})$$

$$\bar{s}^2 h \sim \chi^2(\bar{\nu})$$

The prior for  $\phi$  is obtained by truncating the following density to the region for which  $y$  is stationary.

$$\phi \sim N(\bar{\phi}, \bar{H}_\phi^{-1})$$

$$(P_{i1}, \dots, P_{im}) \sim \text{i.i.d. Di}(\bar{A}_{i1}, \dots, \bar{A}_{im})$$

$$\Pr[s_t = j | s_{t-1} = i] = P_{ij}$$

The unknown quantity  $f$  gives, for each observation time  $t$ , the state probabilities at  $t$  given previous states, previous values of the observed variables, and the other unknown quantities. It is not a primitive unknown quantity, and it is included to give the user access to filtered probabilities.

### 2.22.6 Creating a Model Instance

The mnemonic label identifying the model is `Hamilton`.

Supply the names you wish to give the unknown quantities in the same order as they appear in the table of unknown quantities. Supply the known quantities in the same order as they appear in the table of known quantities.

### 2.22.7 Sampling Algorithm

The sampling algorithm for prior simulation features five blocks. Four blocks make independent draws from the prior distributions of  $\gamma$ ,  $h$ ,  $\phi$  and  $P$ . The fifth makes draws from the distribution  $s|P$ . The sampling algorithm for posterior simulation features five blocks, each making draws from the conditional posterior distribution of one of the unknown quantities.

## Chapter 3

# BACC Commands

### 3.1 Overview of BACC Commands

The following is a list of BACC commands with brief descriptions.

|                             |                                                                                                                                                                                               |
|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>dirichletSim</code>   | Generates a sample from a multiple Dirichlet distribution.                                                                                                                                    |
| <code>expect1</code>        | Calculates, for a weighted random sample, the sample mean and standard deviation, estimates of the numerical standard error for the mean, and estimates of the relative numerical efficiency. |
| <code>expectN</code>        | Calculates combined sample means, with numerical standard errors, for a set of different weighted random samples, and tests for the equality of their individual population means.            |
| <code>extract</code>        | Returns simulation matrices for a model instance.                                                                                                                                             |
| <code>gammaSim</code>       | Generates a sample from a gamma distribution.                                                                                                                                                 |
| <code>gaussianSim</code>    | Generates a sample from a Gaussian distribution.                                                                                                                                              |
| <code>listModelSpecs</code> | Lists all available model specifications (e.g. <code>nlm</code> , <code>poisson</code> ).                                                                                                     |
| <code>listModels</code>     | Lists all open model instances.                                                                                                                                                               |
| <code>miDelete</code>       | Closes without saving a (or all) model instances.                                                                                                                                             |
| <code>miLoad</code>         | Loads a model instance stored in a binary file.                                                                                                                                               |
| <code>miLoadAscii</code>    | Loads a model instance stored in a text file.                                                                                                                                                 |
| <code>miSave</code>         | Saves a model instance in a binary file.                                                                                                                                                      |
| <code>miSaveAscii</code>    | Saves a model instance in a text file.                                                                                                                                                        |
| <code>minst</code>          | Creates an instance of a particular model specification.                                                                                                                                      |

|                              |                                                                                                                                                         |
|------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mlike</code>           | Computes various estimates of the marginal likelihood for a model instance, with numerical standard errors.                                             |
| <code>paretoSim</code>       | Generates a sample from a Pareto distribution.                                                                                                          |
| <code>postfilter</code>      | Filters out previously generated draws from the posterior simulation matrix of a given model instance.                                                  |
| <code>postsim</code>         | Generates or appends to the posterior simulation matrix of a given model instance.                                                                      |
| <code>postsimHM</code>       | Generates or appends to the posterior HM simulation matrix of a given model instance.                                                                   |
| <code>priorRobust</code>     | Calculates upper and lower bounds on the mean of a posterior function of interest, as the prior distribution is varied from its original specification. |
| <code>priorfilter</code>     | Filters out previously generated draws from the prior simulation matrix of a given model instance.                                                      |
| <code>priorsim</code>        | Generates or appends to the prior simulation matrix of a given model instance.                                                                          |
| <code>setseedconstant</code> | Sets the seeds of the random number generators to a constant value.                                                                                     |
| <code>setseedtime</code>     | Sets the seeds of the random number generators to the number of seconds since the beginning of 1970.                                                    |
| <code>weightedSmooth</code>  | Estimates a univariate density function for a weighted random sample, using a kernel smoothing algorithm adapted to weighted samples.                   |
| <code>wishartSim</code>      | Generates a sample from a Wishart distribution.                                                                                                         |

## 3.2 Matlab Issues

Help is available within Matlab for BACC commands. Type `help commandName` at the Matlab prompt, or `help BACC` for a list of BACC commands.

## 3.3 Detailed Description of Commands

Each BACC command is described in detail in one of the following sections.

### 3.3.1 The `dirichletSim` Command

#### Description

Generates a sample from a multiple Dirichlet distribution.

#### Usage

```
sample <- dirichletSim(A, n);
```

#### Inputs

|                |                                         |
|----------------|-----------------------------------------|
| <code>A</code> | $m$ by $K$ matrix: Dirichlet parameters |
| <code>n</code> | Integer: number of draws to generate    |

#### Outputs

|                     |                                                                           |
|---------------------|---------------------------------------------------------------------------|
| <code>sample</code> | $n$ by $nK$ matrix: sample generated from multiple Dirichlet distribution |
|---------------------|---------------------------------------------------------------------------|

#### See Also

`paretoSim`, `gaussianSim`, `gammaSim`, `wishartSim`.

#### Example

```
A = array(c(1.0,2.0,3.0,4.0,5.0,6.0),dim=c(2,3))
sample <- dirichletSim(A, 1000);
```

#### Details

The sample consists of  $n$  draws. Each of the  $n$  draws of the sample is an  $m$  by  $K$  matrix with independent rows. Each row has a Dirichlet distribution with parameters given by the corresponding row of  $A$ .

The result is given as a  $n$  by  $mK$  matrix, and each column gives a draw in column major order. See Appendix A for the parameterization of the Dirichlet distribution.

### 3.3.2 The expect1 Command

#### Description

Calculates, for a weighted random sample, the sample mean and standard deviation, estimates of the numerical standard error for the mean, and estimates of the relative numerical efficiency.

#### Usage

```
out <- expect1(logWeight, sample, taper = c(4.0 8.0 15.0));
```

#### Inputs

|                        |                                                     |
|------------------------|-----------------------------------------------------|
| <code>logWeight</code> | Vector of length $M$ : log sample weights           |
| <code>sample</code>    | Vector of length $M$ : sample of scalar draws       |
| <code>taper</code>     | Vector of length $K$ : taper half-widths (optional) |

#### Outputs

|                   |                                                                    |
|-------------------|--------------------------------------------------------------------|
| <code>mean</code> | Real scalar: weighted sample mean                                  |
| <code>std</code>  | Real scalar: weighted sample standard deviation                    |
| <code>nse</code>  | Vector of length $K + 1$ : estimated numerical standard errors     |
| <code>rne</code>  | Vector of length $K + 1$ : estimated relative numerical efficiency |

#### See Also

`expectN`, `priorRobust`.

#### Example

```
# Use default taper values
out <- expect1(lw, z);

# Use alternate taper values
taper = array(c(4.0,8.0),dim=c(1,2))
out <- expectN(lw, z, taper);
```

#### Details

Let  $z = (z_1, \dots, z_M)$  be the sample and  $(\log w_1, \dots, \log w_M)$  be the vector of log weights. Let  $\lambda = (\lambda_1, \dots, \lambda_K)$  be the vector of half-widths. The sample is broken into  $T$  groups of size  $J \equiv M \text{ div } T$  and the last  $M \bmod T$  elements are ignored. Thus  $M_{\text{use}} \equiv JT$  elements are used.

The sample mean and standard deviation are calculated as follows:

$$\bar{z} = \sum_{m=1}^{M_{use}} w_m z_m \bigg/ \sum_{m=1}^{M_{use}} w_m$$

$$\sigma_z = \left[ \sum_{m=1}^{M_{use}} w_m (z_m - \bar{z})^2 \bigg/ \sum_{m=1}^{M_{use}} w_m \right]^{\frac{1}{2}}$$

For the calculation of the first numerical standard error  $\tau_0$ , we assume no serial correlation in  $(z_1, \dots, z_M)$ . This is appropriate for independence or importance sampling. Following Geweke (1989) [3], this leads to

$$\tau \approx \tau_0 \equiv \left[ \sum_{m=1}^{M_{use}} w_m^2 (z_m - \bar{z})^2 \bigg/ \left( \sum_{m=1}^{M_{use}} w_m \right)^2 \right]^{\frac{1}{2}}$$

For the calculation of  $\tau_1$  through  $\tau_K$ , the remaining  $K$  estimates of the numerical standard error, the following method is used. First, **expect1** calculates group and sample means of the numerator quantity  $w_m z_m$  and the denominator quantity  $w_m$ :

$$n(t) = \frac{1}{J} \sum_{m=(t-1)J+1}^{tJ} w_m z_m \quad d(t) = \frac{1}{J} \sum_{m=(t-1)J+1}^{tJ} w_m \quad t = 1, \dots, T$$

$$\bar{n} = \frac{1}{M_{use}} \sum_{m=1}^{M_{use}} w_m z_m \quad \bar{d} = \frac{1}{M_{use}} \sum_{m=1}^{M_{use}} w_m$$

Then it calculates the following sample autocorrelation and autocovariance functions:

$$\gamma_{nn}(t) = \frac{1}{T} \sum_{s=t+1}^T (n(s) - \bar{n})(n(s-t) - \bar{n}) \quad t = 0, \dots, T-1$$

$$\gamma_{dd}(t) = \frac{1}{T} \sum_{s=t+1}^T (d(s) - \bar{d})(d(s-t) - \bar{d}) \quad t = 0, \dots, T-1$$

$$\gamma_{nd}(t) = \frac{1}{T} \sum_{s=t+1}^T (n(s) - \bar{n})(d(s-t) - \bar{d}) \quad t = 0, \dots, T-1$$

$$\gamma_{dn}(t) = \frac{1}{T} \sum_{s=t+1}^T (d(s) - \bar{d})(n(s-t) - \bar{n}) \quad t = 0, \dots, T-1$$

Then it calculates, for each  $k \in \{1, \dots, K\}$ , estimates  $\sigma_{n(k)}^2$ ,  $\sigma_{d(k)}^2$ , and  $\sigma_{nd(k)}$  of  $\sigma_n^2 \equiv \text{Var}[n(t)]$ ,  $\sigma_d^2 \equiv \text{Var}[d(t)]$  and  $\sigma_{nd} \equiv \text{Cov}[n(t), d(t)]$ , based on the taper half-width  $\lambda_k$ :

$$\sigma_{nn(k)}^2 = \gamma_{nn}(0) + 2 \sum_{s=1}^{\lambda_k-1} \frac{\lambda_k - s}{\lambda_k} \gamma_{nn}(s)$$

$$\sigma_{dd(k)}^2 = \gamma_{dd}(0) + 2 \sum_{s=1}^{\lambda_k-1} \frac{\lambda_k - s}{\lambda_k} \gamma_{dd}(s)$$

$$\sigma_{nd(k)}^2 = \gamma_{nd}(0) + \sum_{s=1}^{\lambda_k-1} \frac{\lambda_k - s}{\lambda_k} [\gamma_{nd}(s) + \gamma_{dn}(s)]$$

These calculations are based on conventional time series methods for a wide sense stationary process, described in Geweke (1992) [4].

By the conventional asymptotic expansion, the square of the numerical standard error is approximated by

$$\tau^2 = \text{Var}\left(\frac{n}{d}\right) \approx \begin{bmatrix} \frac{1}{d} & -\frac{1}{d^2} \end{bmatrix} \begin{bmatrix} \sigma_n^2 & \sigma_{nd} \\ \sigma_{nd} & \sigma_d^2 \end{bmatrix} \begin{bmatrix} \frac{1}{d} \\ -\frac{1}{d^2} \end{bmatrix}$$

For each  $k \in \{1, \dots, K\}$  it calculates the approximation  $\tau_k$  using  $\sigma_{nn(k)}^2$ ,  $\sigma_{dd(k)}^2$  and  $\sigma_{nd(k)}^2$  defined above.

Relative numerical efficiencies  $(\nu_0, \dots, \nu_K)$  are calculated using

$$\nu_k \equiv \left(\frac{\tau_0}{\tau_k}\right)^2 \quad k = 0, \dots, K$$

### 3.3.3 The expectN Command

#### Description

Calculates combined sample means, with numerical standard errors, for a set of different weighted random samples, and tests for the equality of their individual population means.

#### Usage

```
out <- expectN(logWeight1, sample1, logWeight2, sample2, taper = c(4.0
8.0 15.0));
```

#### Inputs

|                         |                                                               |
|-------------------------|---------------------------------------------------------------|
| <code>logWeight1</code> | Vector of length $M_1$ : log sample weights for first sample  |
| <code>sample1</code>    | Vector of length $M_1$ : first sample of scalar draws         |
| <code>logWeight2</code> | Vector of length $M_2$ : log sample weights for second sample |
| <code>sample2</code>    | Vector of length $M_2$ : second sample of scalar draws        |
| <code>taper</code>      | Vector of length $K$ : taper half-widths (optional)           |

#### Outputs

|                    |                                                                                                                        |
|--------------------|------------------------------------------------------------------------------------------------------------------------|
| <code>mean</code>  | Vector of length $K + 1$ : estimated combined weighted sample means                                                    |
| <code>nse</code>   | Vector of length $K + 1$ : estimated numerical standard errors                                                         |
| <code>equal</code> | Vector of length $K + 1$ : marginal significance levels for a chi-squared test of the equality of the population means |

#### See Also

`expect1.`

#### Example

```
# Use default taper values
out <- expectN(lw1, z1, lw2, z2);

# Use alternate taper values
taper = array(c(4.0,8.0),dim=c(1,2))
out <- expectN(lw1, z1, lw2, z2, taper);
```

### Details

In general, there are  $N$  pairs of weighted samples, not just two. For each sample  $z^{(i)}$ , **expectN** calculates individual sample moments  $\bar{z}^{(i)}$  and estimates of numerical standard errors  $(\tau_0^{(i)}, \dots, \tau_K^{(i)})$  from the samples  $(z_1^{(i)}, \dots, z_{M_i}^{(i)})$ , the log weights  $(\log z_1^{(i)}, \dots, \log z_{M_i}^{(i)})$ , and the half-taper values  $(\lambda_1, \dots, \lambda_K)$ , in the same way that **expect1** calculates  $\bar{z}$  and  $(\tau_0, \dots, \tau_K)$  from  $(z_1, \dots, z_M)$ ,  $(\log z_1, \dots, \log z_M)$ , and  $(\lambda_1, \dots, \lambda_K)$ .

The estimated sample means  $\bar{z}_k$  are given by

$$\bar{z}_k = \sum_{i=1}^N \frac{\bar{z}^{(i)}}{\tau_k^{2(i)}} \bigg/ \sum_{i=1}^N \frac{1}{\tau_k^{2(i)}} \quad k = 1, \dots, K$$

The estimated numerical standard errors  $\tau_k$  are given by

$$\frac{1}{\tau_k^2} = \sum_{i=1}^N \frac{1}{\tau_k^{2(i)}}$$

For each  $k$ , the marginal significance level is the value of  $p_k$  such that

$$\begin{bmatrix} \bar{z}_k^{(2)} - \bar{z}_k^{(1)} & \dots & \bar{z}_k^{(N)} - \bar{z}_k^{(N-1)} \end{bmatrix} \cdot \Sigma^{-1} \cdot \begin{bmatrix} \bar{z}_k^{(2)} - \bar{z}_k^{(1)} \\ \bar{z}_k^{(3)} - \bar{z}_k^{(2)} \\ \vdots \\ \bar{z}_k^{(N)} - \bar{z}_k^{(N-1)} \end{bmatrix} = \chi_{1-p_k}^2(N-1)$$

where  $\Sigma$  is the following matrix

$$\begin{bmatrix} \tau_k^{2(1)} + \tau_k^{2(2)} & -\tau_k^{2(2)} & 0 & \dots & 0 & 0 \\ -\tau_k^{2(2)} & \tau_k^{2(2)} + \tau_k^{2(3)} & -\tau_k^{2(3)} & \dots & 0 & 0 \\ 0 & -\tau_k^{2(3)} & \tau_k^{2(3)} + \tau_k^{2(4)} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \tau_k^{2(N-2)} + \tau_k^{2(N-1)} & -\tau_k^{2(N-1)} \\ 0 & 0 & 0 & \dots & -\tau_k^{2(N-1)} & \tau_k^{2(N-2)} + \tau_k^{2(N-1)} \end{bmatrix}$$

### 3.3.4 The extract Command

#### Description

Returns simulation matrices for a model instance.

#### Usage

```
sim <- extract(modelInst);
```

#### Inputs

**modelInst** Integer: model instance identifier.

#### Outputs

**sim** Structure: simulation matrices

#### Example

```
sim <- extract(mi);
```

#### Details

The return value is a structure (named list in S-PLUS and R) with the following fields (components in S-PLUS and R):

**id** Model instance identifier.

**logWeightPost** Log weights for posterior draws.

**logPrior** Value of log prior for posterior draws.

**logXPrior** Value of transformed log prior values for posterior draws

**logLike** Value of log likelihood for posterior draws.

**logPriorHM** Value of log prior for posterior HM draws.

**logLikeHM** Value of log likelihood for posterior HM draws.

**logWeightPrior** Log weights for prior draws.

**logPriorPrior** Value of log prior for prior draws.

**logLikePrior** Value of log likelihood for prior draws.

**\*** Posterior simulation matrix of unknown quantity named \*.

**\*Prior** Prior simulation matrix of unknown quantity named \*.

**\*HM** Posterior HM simulation matrix of unknown quantity named \*.

All simulation matrices have three dimensions. The first two dimensions give the row and column of the unknown quantity. The third dimension is the simulation dimension. Each value of the third index gives a different draw of the unknown quantity.

### 3.3.5 The gammaSim Command

#### Description

Generates a sample from a gamma distribution.

#### Usage

```
sample <- gammaSim(alpha, beta, n);
```

#### Inputs

|                    |                                                            |
|--------------------|------------------------------------------------------------|
| <code>alpha</code> | Real scalar: shape parameter of gamma distribution         |
| <code>beta</code>  | Real scalar: inverse scale parameter of gamma distribution |
| <code>n</code>     | Integer: number of draws to generate                       |

#### Outputs

|                     |                                                           |
|---------------------|-----------------------------------------------------------|
| <code>sample</code> | $n$ by 1 matrix: sample generated from gamma distribution |
|---------------------|-----------------------------------------------------------|

#### See Also

`paretoSim`, `gaussianSim`, `dirichletSim`, `wishartSim`.

#### Example

```
alpha <- 3.0
beta <- 5.0
sample <- gammaSim(alpha, beta, 1000);
```

#### Details

Each of the  $n$  draws of the sample is a scalar with a gamma distribution. The result is given as a  $n$  by 1 matrix.

See Appendix A for the parametrization of the gamma distribution.

### 3.3.6 The gaussianSim Command

#### Description

Generates a sample from a Gaussian distribution.

#### Usage

```
sample <- gaussianSim(mean, precision, n);
```

#### Inputs

|                        |                                                       |
|------------------------|-------------------------------------------------------|
| <code>mean</code>      | Vector of length $K$ : mean of Gaussian distribution  |
| <code>precision</code> | $K$ by $K$ matrix: precision of Gaussian distribution |
| <code>n</code>         | Integer: number of draws to generate                  |

#### Outputs

|                     |                                                                |
|---------------------|----------------------------------------------------------------|
| <code>sample</code> | $n$ by $K$ matrix: sample generated from Gaussian distribution |
|---------------------|----------------------------------------------------------------|

#### See Also

`paretoSim`, `dirichletSim`, `gammaSim`, `wishartSim`.

#### Example

```
mean = array(c(1.0,2.0),dim=c(2,1))
precision = array(c(1.0,0.0,0.0,1.0),dim=c(2,2))
sample <- gaussianSim(mean, precision, 1000);
```

#### Details

Each of the  $n$  draws of the sample is a vector of length  $K$  with a Gaussian distribution. The result is given as a  $n$  by  $K$  matrix.

See Appendix A for the parametrization of the Gaussian distribution.

### 3.3.7 The `listModelSpecs` Command

#### Description

Lists all available model specifications (e.g. `nlm`, `poisson`).

#### Usage

```
listModelSpecs();
```

#### Inputs

None.

#### Outputs

None.

#### See Also

```
minst, listModels.
```

#### Example

```
listModelSpecs();
```

#### Details

A printed message gives a list of model specifications.

### 3.3.8 The `listModels` Command

#### Description

Lists all open model instances.

#### Usage

```
listModels();
```

#### Inputs

None.

#### Outputs

None.

#### See Also

`minst`, `miDelete`, `listModelSpecs`.

#### Example

```
listModels();
```

#### Details

A printed message gives the model instance identification number, the name of the model specification (e.g. `nlm`, `poisson`), and the number of prior, posterior, and posterior HM draws.

### 3.3.9 The miDelete Command

#### Description

Closes without saving a (or all) model instances.

#### Usage

```
miDelete(modelInst);
```

#### Inputs

|                        |                                     |
|------------------------|-------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier. |
|------------------------|-------------------------------------|

#### Outputs

None.

#### See Also

`minst`, `listModels`.

#### Example

```
miDelete(mi);
```

### 3.3.10 The miLoad Command

#### Description

Loads a model instance stored in a binary file.

#### Usage

```
modelInst <- miLoad(filename);
```

#### Inputs

|                       |                                                        |
|-----------------------|--------------------------------------------------------|
| <code>filename</code> | String: name of binary file storing the model instance |
|-----------------------|--------------------------------------------------------|

#### Outputs

|                        |                                     |
|------------------------|-------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier. |
|------------------------|-------------------------------------|

#### See Also

`miSave`, `minst`, `miLoadAscii`.

#### Example

```
mi <- miLoad("miFile");
```

### 3.3.11 The miLoadAscii Command

#### Description

Loads a model instance stored in a text file.

#### Usage

```
modelInst <- miLoadAscii(filename);
```

#### Inputs

|                       |                                                      |
|-----------------------|------------------------------------------------------|
| <code>filename</code> | String: name of text file storing the model instance |
|-----------------------|------------------------------------------------------|

#### Outputs

|                        |                                     |
|------------------------|-------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier. |
|------------------------|-------------------------------------|

#### See Also

`miSaveAscii`, `minst`, `miLoad`.

#### Example

```
mi <- miLoadAscii("miFile.txt");
```

### 3.3.12 The miSave Command

#### Description

Saves a model instance in a binary file.

#### Usage

```
miSave(modelInst, filename);
```

#### Inputs

|                        |                                                                  |
|------------------------|------------------------------------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier.                              |
| <code>filename</code>  | String: name of binary file in which to store the model instance |

#### Outputs

None.

#### See Also

`miLoad`, `minst`, `miSaveAscii`.

#### Example

```
miSave(mi, "miFile");
```

#### Details

If the file already exists, it is written over.

### 3.3.13 The miSaveAscii Command

#### Description

Saves a model instance in a text file.

#### Usage

```
miSaveAscii(modelInst, filename);
```

#### Inputs

`modelInst`     Integer: model instance identifier.

`filename`     String: name of text file in which to store the model instance

#### Outputs

None.

#### See Also

`miLoadAscii`, `minst`, `miSave`.

#### Example

```
miSaveAscii(mi, "miFile.txt");
```

#### Details

If the file already exists, it is written over. The ascii version of a model instance is platform independent and human readable, but long and inefficient.

### 3.3.14 The `minst` Command

#### Description

Creates an instance of a particular model specification.

#### Usage

```
modelInst <- minst(modelSpecName, unknownNames, knowns);
```

#### Inputs

`modelSpecName` String: name of model specification

`unknownNames` List of strings: user provided names for unknown quantities

`knowns` List of matrices: user provided matrices of known quantities

#### Outputs

`modelInst` Integer: model instance identifier.

#### Example

```
a = array(c(1,1,2),dim=c(1,3))
s = array(c(1,1,2,3,3,3),dim=c(3,2))
myMI <- minst('iidfs', 'pi', a, s);
```

#### Details

The available model specifications are described in Chapter 2. For each model specification, there is a section “Creating a Model Instance” with the relevant information, namely

- The name of the model specification.
- The order in which the user specifies the names for the unknown quantities of the model.
- The order in which the user provides the matrices giving the values of the known quantities of the model.

### 3.3.15 The `mlike` Command

#### Description

Computes various estimates of the marginal likelihood for a model instance, with numerical standard errors.

#### Usage

```
out <- mlike(modelInst, p = c(0.1 0.5 0.9), taper = c(4.0 8.0 15.0));
```

#### Inputs

|                        |                                                         |
|------------------------|---------------------------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier.                     |
| <code>p</code>         | Vector of length $L$ : truncation parameters (optional) |
| <code>taper</code>     | Vector of length $K$ : taper half-widths (optional)     |

#### Outputs

|                    |                                                           |
|--------------------|-----------------------------------------------------------|
| <code>ml</code>    | Vector of length $L$ : marginal likelihood estimates      |
| <code>mlnse</code> | $L$ by $K + 1$ matrix: numerical standard error estimates |

#### See Also

`postsim`, `postsimHM`.

#### Example

```
# Use default truncation and taper values
out <- mlike(mi);
# Use alternate truncation values
p = array(c(0.1,0.3,0.5,0.7,0.9),dim=c(1,5))
out <- mlike(mi, p);
# Use alternate truncation and taper values
taper = array(c(4.0,8.0),dim=c(1,2))
out <- mlike(mi, p, taper);
```

#### Details

The method used is a modification described in Geweke [5] of the method proposed in Gelfand and Dey [2].

The truncation parameters  $p_l \in [0, 1]$  index the truncated multivariate normal distribution  $f(\cdot)$  discussed in Geweke [5]. For each  $p_l$ , `mlike` generates (internally) an unweighted vector  $(z_1^l, \dots, z_M^l)$ , where  $M$  is the number of posterior samples in the given model instance.

For each  $l$ , **mlike** calculates the sample mean  $\bar{z}_l$  and numerical standard errors  $(\tilde{\tau}_1^l, \dots, \tilde{\tau}_M^l)$  from  $(z_1^l, \dots, z_M^l)$  and  $(\lambda_1, \dots, \lambda_K)$  in the same way that **expect1** calculates  $(\tau_0, \dots, \tau_K)$  from  $(z_1, \dots, z_M)$ , a vector of equal log weights, and  $(\lambda_1, \dots, \lambda_K)$ . Then for all  $l$ , the estimate of the log marginal likelihood is given by

$$\mu_l = -\log \bar{z}_l$$

and for all  $l$  and  $k$ , the estimate of the numerical standard error for the log marginal likelihood is given by

$$\tau_{kl} = \frac{\tilde{\tau}_{kl}}{\bar{z}_l}$$

When numerical standard error is small, results are not sensitive to the choice of  $p$ . In these cases  $L = 1$  and  $p_1 = 0.5$  will suffice. However the additional computational burden of increasing  $L$  is negligible. If you are concerned about standard errors, it is best to use several values of  $p_l$ , for example,  $p = (0.1, 0.2, \dots, 0.9)$ .

### 3.3.16 The `paretoSim` Command

#### Description

Generates a sample from a Pareto distribution.

#### Usage

```
sample <- paretoSim(alpha, beta, n);
```

#### Inputs

|                    |                                                        |
|--------------------|--------------------------------------------------------|
| <code>alpha</code> | Real scalar: tail parameter of Pareto distribution     |
| <code>beta</code>  | Real scalar: location parameter of Pareto distribution |
| <code>n</code>     | Integer: number of draws to generate                   |

#### Outputs

|                     |                                                            |
|---------------------|------------------------------------------------------------|
| <code>sample</code> | $n$ by 1 matrix: sample generated from Pareto distribution |
|---------------------|------------------------------------------------------------|

#### See Also

`dirichletSim`, `gaussianSim`, `gammaSim`, `wishartSim`.

#### Example

```
alpha <- 1.0
beta <- 4.0
sample <- gammaSim(alpha, beta, 1000);
```

#### Details

Each of the  $n$  draws of the sample is a scalar with a pareto distribution. The result is given as a  $n$  by 1 matrix.

See Appendix A for the parametrization of the Pareto distribution.

### 3.3.17 The `postfilter` Command

#### Description

Filters out previously generated draws from the posterior simulation matrix of a given model instance.

#### Usage

```
postfilter(modelInst, filter);
```

#### Inputs

|                        |                                                                      |
|------------------------|----------------------------------------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier.                                  |
| <code>filter</code>    | Vector of integers of length $n$ : indices of existing draws to keep |

#### Outputs

None.

#### Example

```
filter = seq(101,1000)

postfilter(mi, filter);
```

#### Details

The  $i$ th draw of the posterior simulation matrix is kept if and only if  $i = f_j$  for some  $j$  from 1 to  $n$ .

### 3.3.18 The `postsim` Command

#### Description

Generates or appends to the posterior simulation matrix of a given model instance.

#### Usage

```
postsim(modelInst, m, n);
```

#### Inputs

|                        |                                                                      |
|------------------------|----------------------------------------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier.                                  |
| <code>m</code>         | Integer: number of posterior draws to record                         |
| <code>n</code>         | Integer: number of posterior draws to generate for each one recorded |

#### Outputs

None.

#### See Also

`minst`, `postfilter`, `mlike`, `priorsim`, `postsimHM`, `extract`.

#### Example

```
postsim(mi, 1000, 1);
```

#### Details

Generates draws of unknown quantities from their posterior distribution. Generates  $mn$  new posterior draws, and appends every  $n$ th draw to the posterior simulation matrix. If there are any draws from a previous invocation of `postsim`, the first new draw comes from the transition kernel of the Markov chain used for posterior simulation. Otherwise, it comes from the initial distribution of the Markov chain.

Use the `extract` command to obtain the posterior draws.

### 3.3.19 The `postsimHM` Command

#### Description

Generates or appends to the posterior HM simulation matrix of a given model instance.

#### Usage

```
postsimHM(modelInst, m, n, scalePrecision);
```

#### Inputs

|                             |                                                                                        |
|-----------------------------|----------------------------------------------------------------------------------------|
| <code>modelInst</code>      | Integer: model instance identifier.                                                    |
| <code>m</code>              | Integer: number of posterior draws to record                                           |
| <code>n</code>              | Integer: number of posterior draws to generate for each one recorded                   |
| <code>scalePrecision</code> | Real scalar: factor used to rescale the precision matrix of the random walk innovation |

#### Outputs

None.

#### See Also

`minst`, `mlike`, `postsim`, `extract`.

#### Example

```
postsimHM(mi, 1000, 1, 10.0);
```

#### Details

Generates draws of unknown quantities from their posterior distribution using a Gaussian random walk Metropolis chain with proposal covariance proportional to the sample covariance of draws from the posterior simulation matrix. Generates  $mn$  new posterior draws, and appends every  $n$ th draw to the posterior simulation matrix. If there are any draws from a previous invocation of `postsimHM`, the first new draw comes from the transition kernel of the Markov chain used for posterior simulation. Otherwise, it comes from the initial distribution of the Markov chain.

Use the `extract` command to obtain the posterior HM draws.

### 3.3.20 The priorRobust Command

#### Description

Calculates upper and lower bounds on the mean of a posterior function of interest, as the prior distribution is varied from its original specification.

#### Usage

```
out <- priorRobust(logWeight, sample, factors);
```

#### Inputs

|                  |                                                                             |
|------------------|-----------------------------------------------------------------------------|
| <b>logWeight</b> | Vector of length $m$ : log weights                                          |
| <b>sample</b>    | Vector of length $m$ : posterior sample of some scalar function of interest |
| <b>factors</b>   | Vector of length $n$ : bound factors for robustness analysis                |

#### Outputs

|             |                                                                                   |
|-------------|-----------------------------------------------------------------------------------|
| <b>mean</b> | Real scalar: posterior sample mean for original prior specification               |
| <b>std</b>  | Real scalar: posterior sample standard deviation for original prior specification |
| <b>U</b>    | Vector of length $n$ : exact upper bounds                                         |
| <b>L</b>    | Vector of length $n$ : exact lower bounds                                         |
| <b>Ut</b>   | Vector of length $n$ : asymptotic upper bounds                                    |
| <b>Lt</b>   | Vector of length $n$ : asymptotic lower bounds                                    |

#### Example

```
K = array(c(5.0,10.0,20.0),dim=c(1,3))
out <- priorRobust(lw, beta, K);
```

#### Details

For each bound factor, calculates exact lower and upper bounds and asymptotic lower and upper bounds for the posterior mean. For each bound parameter  $k_i$ , **priorRobust** calculates exact lower and upper bounds  $L_i$  and  $U_i$  for the posterior mean of the function of interest  $g$ , for the following set of prior density kernels.

$$\left\{ p^*(\cdot) : \frac{1}{k_i} p(\theta) \leq p^*(\theta) \leq k_i p(\theta) \quad \forall \theta \in \Theta \right\}$$

where  $p(\cdot)$  is the actual prior density. It uses the algorithm described in Geweke and Petrella [6]. Also for each  $k_i$ , **priorRobust** calculates asymptotically valid

lower and upper bounds  $\tilde{L}_i$  and  $\tilde{U}_i$ , using the results of DeRobertis and Hartigan [1].

### 3.3.21 The priorfilter Command

#### Description

Filters out previously generated draws from the prior simulation matrix of a given model instance.

#### Usage

```
priorfilter(modelInst, filter);
```

#### Inputs

|                        |                                                                      |
|------------------------|----------------------------------------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier.                                  |
| <code>filter</code>    | Vector of integers of length $n$ : indices of existing draws to keep |

#### Outputs

None.

#### Example

```
filter = seq(101,1000)

priorfilter(mi, filter);
```

#### Details

The  $i$ th draw of the prior simulation matrix is kept if and only if  $i = f_j$  for some  $j$  from 1 to  $n$ .

### 3.3.22 The priorsim Command

#### Description

Generates or appends to the prior simulation matrix of a given model instance.

#### Usage

```
priorsim(modelInst, m, n);
```

#### Inputs

|                        |                                                                  |
|------------------------|------------------------------------------------------------------|
| <code>modelInst</code> | Integer: model instance identifier.                              |
| <code>m</code>         | Integer: number of prior draws to record                         |
| <code>n</code>         | Integer: number of prior draws to generate for each one recorded |

#### Outputs

None.

#### See Also

`minst`, `priorfilter`, `postsim`, `extract`.

#### Example

```
priorsim(mi, 1000, 1);
```

#### Details

Generates draws of unknown quantities from their prior distribution. Generates  $mn$  new prior draws, and appends every  $n$ th draw to the prior simulation matrix. If there are any draws from a previous invocation of `priorsim`, the first new draw comes from the transition kernel of the Markov chain used for prior simulation. Otherwise, it comes from the initial distribution of the Markov chain.

Use the `extract` command to obtain the prior draws.

### 3.3.23 The `setseedconstant` Command

#### Description

Sets the seeds of the random number generators to a constant value.

#### Usage

```
setseedconstant();
```

#### Inputs

None.

#### Outputs

None.

#### See Also

`setseedtime`.

#### Example

```
setseedconstant();
```

#### Details

This is useful for ensuring that repeated invocations of a command generating random values lead to the same results.

### 3.3.24 The `setseedtime` Command

#### Description

Sets the seeds of the random number generators to the number of seconds since the beginning of 1970.

#### Usage

```
setseedtime();
```

#### Inputs

None.

#### Outputs

None.

#### See Also

`setseedconstant`.

#### Example

```
setseedtime();
```

#### Details

This is useful for ensuring that repeated invocations of a command generating random values lead to different results.

### 3.3.25 The weightedSmooth Command

#### Description

Estimates a univariate density function for a weighted random sample, using a kernel smoothing algorithm adapted to weighted samples.

#### Usage

```
out <- weightedSmooth(logWeight, sample, ktype = uniform, krange = quantile,
  wwf = 0.5, nplot = 1000, range_a1 = 0.001, range_a2 = 0.009);
```

#### Inputs

|                        |                                                                        |
|------------------------|------------------------------------------------------------------------|
| <code>logWeight</code> | Vector of length $M$ : log weights                                     |
| <code>sample</code>    | Vector of length $M$ : a posterior sample of some function of interest |
| <code>ktype</code>     | String: kernel type (optional)                                         |
| <code>krange</code>    | String: kernel range type (optional)                                   |
| <code>wwf</code>       | Real scalar: window width fraction (optional)                          |
| <code>nplot</code>     | Integer: number of ordered pairs to generate (optional)                |
| <code>range_a1</code>  | Real scalar: left bound range parameter (optional)                     |
| <code>range_a2</code>  | Real scalar: right bound range parameter (optional)                    |

#### Outputs

|                |                                        |
|----------------|----------------------------------------|
| <code>x</code> | Vector of length $N$ : ordinate values |
| <code>y</code> | Vector of length $N$ : abscissa values |

#### Example

```
out <- weightedSmooth(lw, z);
nplot <- 2000
ktype <- triangular
out <- weightedSmooth(lw, z);
```

#### Details

The estimated density at a point  $z$  is

$$f(z) = \frac{\sum_{m=1}^M w_m K\left(\frac{z-z_m}{h}\right)}{h \sum_{m=1}^M w_m}$$

The functional form of the kernel function  $K$  depends on the value of `ktype` according to Table 3.1.

Table 3.1: Values of **Ktype**

| <b>Ktype</b>    | $K$                                               |
|-----------------|---------------------------------------------------|
| <b>uniform</b>  | $K(t) = \frac{1}{2}\chi_{(-1,1)}(t)$              |
| <b>triangle</b> | $K(t) = (1 -  t )\chi_{(-1,1)}(t)$                |
| <b>biweight</b> | $K(t) = \frac{15}{16}(1 - z^2)^2\chi_{(-1,1)}(t)$ |

For any set  $S$ , the function  $\chi_S(\cdot)$  is a set membership indicator function.

The value  $h$  is given by

$$h = \lambda(q_{\frac{3}{4}} - q_{\frac{1}{4}})$$

where  $q_\alpha$  denotes the  $\alpha$ 'th sample quantile of  $z$ .

The **weightedSmooth** command generates  $N$  ordered pairs  $(x_i, y_i)$ . The values  $x_i$  are evenly spaced between  $x_{min}$  and  $x_{max}$ , determined by **Krange** according to Table 3.2. The values  $y_i$  satisfy  $y_i = f(x_i)$ .

Table 3.2: Values of **Krange**

| <b>Krange</b>   | $x_{min}$ | $x_{max}$ |
|-----------------|-----------|-----------|
| <b>quantile</b> | $q_{a_1}$ | $q_{a_2}$ |
| <b>absolute</b> | $a_1$     | $a_2$     |

For most plotting routines,  $N$  should be in the range of 200 to 400. The choice of  $\lambda$  depends on how smooth the resulting plot is desired to be. As with all kernel smoothing methods, some experimentation will probably be necessary. The greater the number of simulations available, the smaller  $\lambda$  can be and still retain visual smoothness. It is generally easier to use the **Krange=quantile** option and specify  $a_1$  in the range .001 to .01 and  $a_2$  in the range .99 to .999; this will include the important part of the estimated density while not wasting space on the plot for points where the density is small.

### 3.3.26 The wishartSim Command

#### Description

Generates a sample from a Wishart distribution.

#### Usage

```
sample <- wishartSim(A, nu, n);
```

#### Inputs

|           |                                                                    |
|-----------|--------------------------------------------------------------------|
| <b>A</b>  | $m$ by $m$ matrix: inverse scale parameter of Wishart distribution |
| <b>nu</b> | Real scalar: degrees of freedom parameter of Wishart distribution  |
| <b>n</b>  | Integer: number of draws to generate                               |

#### Outputs

|               |                                                                 |
|---------------|-----------------------------------------------------------------|
| <b>sample</b> | $n$ by $m^2$ matrix: sample generated from Wishart distribution |
|---------------|-----------------------------------------------------------------|

#### See Also

paretoSim, gaussianSim, gammaSim, dirichletSim.

#### Example

```
A = array(c(1.0,0.0,0.0,1.0),dim=c(2,2))
nu <- 100
sample <- wishartSim(A, nu, 1000);
```

#### Details

Each of the  $n$  draws of the sample is an  $m$  by  $m$  matrix with a Wishart distribution. The result is given as a  $m$  by  $m^2$  matrix.

See Appendix A for the parametrization of the Wishart distribution.

## Chapter 4

# A BACC Tutorial

In order to answer commonly asked questions, this chapter contains a step-by-step tutorial with explanations of what each step is doing and what each term means.

## 4.1 Working through a model instance

Before creating a model instance, you need to load all the known quantities. These quantities can be either vector or matrix data objects.

The following code demonstrates how a normal linear model called `sim` is created and manipulated.

```
# Specify the names for the unknown quantities
unknownNames <- c("beta","h")

# Specify the known quantities of the model instance.
# Only the name of the data objects are accepted. These data
# objects must be preloaded.

knowns <- list(betahd,Hhd,nuhd,shd,Xhd,yhd)

# Create an instance of the normal linear model
modelInst <- minst("nlm",unknownNames,knowns)

# Simulate 5000 prior samples
setseedconstant()
priorsim(modelInst,5000,1)

# Simulate 1000 posterior samples
setseedconstant()
postsim(modelInst,1000,1)

# Filter out the first 100 posterior samples
f<-101:1000
postfilter(model,f)

# Add 4100 new posterior samples
setseedconstant()
postsim(modelInst,4100,1)

# Simulate 10000 new HM posterior samples
setseedconstant()
postsimhm(modelInst,10000,1,1)

# extract all the quantities related to the model created
out<-extract(modelInst)

# Get the vector of posterior samples of the first element
# of beta
beta1<-out$beta[1,1,]
```

```

# Get the vector of log weight evaluations
lw<-out$logweightPost

# Get the vector of prior samples of the first element
# of beta
betap1<-out$betaPrior[1,1,]

# Find the posterior mean and standard deviation of beta1
# using the default value of taper
exp1<-expect1(lw,beta1)
# The following four operations are only to demonstrate
# what are included in exp1. exp1$... is good enough for use
postmean<-exp1$mean
poststd<-exp1$std
postnse<-exp1$nse
postrne<-exp1$rne

# Specify truncation parameters for marginal likelihood computation
p <- 1:9*0.1      # A short way to write p<-c(0.1,0.2,...,0.9)
mlike.out <- mlike(modelInst,p)
# As exp1, it is sufficient to use mlike.out$... to get the
# components. Replace ... by ml, mlNSE, mlHM, mlNSEHM as desired

# Specify the logweight and function of interest variables
# for expectN. These variables must be preloaded
mlist<-list(lw,beta1,lw,betap1)
expN<-expectN(mlist)
# Again, it is sufficient to use expN$... to get the components
# mean, nse, p of expN

# Find the minimum and maximum values of the posterior mean
# of beta1 as the prior is changed from its original specification
robust.out <- priorRobust(lw,beta1)
# The components of robust.out would be mean, std, exactUP, exactDown,
# DeRHUp, and DeRHDown

# Generate (x,y) paris tracing an estimated posterior marginal
# density of beta1
smooth.out <- weightedSmooth(lw,beta1)
# plot the estimated posterior marginal density of beta1
plot(smooth.out) # Or equivalently, plot(smooth.out$y~smooth.out$x)

# Save the current model instance in the test file "baccSim"
# under the current working directory
miSaveAscii(modelInst,"baccSim")

```

## 4.2 Simulating from various distributions

### 1. Dirichlet

```
a<-matrix(1:6,2,3,byrow=T)
sample<-dirichletSim(a,1000)
cat("dirichlet mean:\n",mean(sample), "\n")
```

### 2. Gamma

```
sample<-gammaSim(3,5,1000)
cat("gamma mean:\n",mean(sample), "\n")
```

### 3. Gaussian(Multivariate Normal)

```
mean<-c(1,2)
precision<-matrix(c(1,0,0,1),2,2)
sample<-gaussianSim(mean,precision,1000)
cat("gaussian mean:\n",apply(sample,2,mean), "\n")
```

### 4. Pareto

```
sample<-paretoSim(3,5,1000)
cat("pareto mean:\n", mean(sample), "\n")
```

### 5. Wishart

```
scale<-matrix(c(1,0,0,1),2,2)
sample<-wishartSim(scale,10,1000)
cat("wishart mean:\n",apply(sample,2,mean), "\n")
```

# Appendix A

## Distributions

This appendix gives the density and mass functions for the distributions used in this document.

### A.1 The Dirichlet Distribution

A random vector  $\pi$  of length  $n$  has the Dirichlet distribution with parameter vector  $\alpha \in \mathbb{R}_+^n$ , denoted  $\pi \sim \text{Di}(\alpha)$ , if its probability density function is

$$p(\pi|\alpha) = \begin{cases} m^{-1/2} \frac{\Gamma(\sum_{i=1}^m \alpha_i)}{\prod_{i=1}^m \Gamma(\alpha_i)} \prod_{i=1}^m \pi_i^{\alpha_i-1} & \pi \in \Delta_n \equiv \{p \in \mathbb{R}_+^n : \sum_{i=1}^n p_i = 1\} \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[\pi_i|\alpha] &= \frac{\alpha_i}{\sum_{j=1}^n \alpha_j} \\ \text{Var}[\pi_i|\alpha] &= \frac{\mathbb{E}[\pi_i|\alpha](1 - \mathbb{E}[\pi_i|\alpha])}{1 + \sum_{j=1}^n \alpha_j} \\ \text{Cov}[\pi_i, \pi_j|\alpha] &= \frac{-\mathbb{E}[\pi_i|\alpha]\mathbb{E}[\pi_j|\alpha]}{1 + \sum_{k=1}^n \alpha_k} \end{aligned}$$

### A.2 The Gamma Distribution

A random scalar  $\lambda$  has the Gamma distribution with shape parameter  $\alpha > 0$  and scale parameter  $\beta > 0$ , denoted  $\lambda \sim \text{Ga}(\underline{\alpha}, \underline{\beta})$ , if its probability density function is

$$p(\lambda|\alpha, \beta) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} & \lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[\lambda|\alpha, \beta] &= \frac{\alpha}{\beta} \\ \text{Var}[\lambda|\alpha, \beta] &= \frac{\alpha}{\beta^2} \end{aligned}$$

### A.3 The Normal Distribution

A random vector  $x$  has the Normal Distribution with mean parameter vector  $\mu \in \mathbb{R}^k$  and positive definite  $k \times k$  variance parameter matrix  $\Sigma$ , denoted  $x \sim \mathcal{N}(\mu, \Sigma)$ , if its probability density function is

$$p(x|\mu, \Sigma) = |\Sigma|^{-\frac{1}{2}} (2\pi)^{-\frac{k}{2}} \exp \left[ -\frac{1}{2} (x - \mu)' \Sigma^{-1} (x - \mu) \right] \quad \forall x \in \mathbb{R}^n$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[x|\mu, \Sigma] &= \mu \\ \text{Var}[x|\mu, \Sigma] &= \Sigma \end{aligned}$$

### A.4 The Pareto Distribution

A random scalar  $x$  has the Pareto Distribution with parameters  $\alpha > 0$  and  $\beta \geq 0$ , denoted  $\theta \sim \text{Pa}(\alpha, \beta)$ , if its probability density function is

$$p(\theta|\alpha, \beta) = \begin{cases} \alpha \beta^\alpha \theta^{-(\alpha+1)} & \theta \geq \beta \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[\theta|\alpha, \beta] &= \frac{\alpha \beta}{\alpha - 1} \\ \text{Var}[\theta|\alpha, \beta] &= \frac{\alpha \beta^2}{(\alpha - 1)^2 (\alpha - 2)} \end{aligned}$$

### A.5 The Poisson Distribution

A discrete random variable  $x$  has the Poisson distribution with mean parameter  $\lambda > 0$ , denoted  $x \sim \text{Po}(\lambda)$ , if its probability mass function is

$$p(x) = \begin{cases} e^{-\lambda} \frac{\lambda^x}{x!} & x \in \{0, 1, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[x|\lambda] &= \lambda \\ \text{Var}[x|\lambda] &= \lambda \end{aligned}$$

## A.6 The Wishart Distribution

An  $m \times m$  random matrix  $H$  has the Wishart distribution with positive definite  $m \times m$  scale parameter matrix  $A$  and degrees of freedom parameter  $\nu > m$ , denoted  $H \sim \text{Wi}(A, \nu)$ , if its probability density function is

$$p(H|A, \nu) = \begin{cases} \frac{\pi^{-m(m-1)/4} |A|^{-\nu/2}}{2^m \nu^{m/2} \prod_{i=1}^m \Gamma(\frac{\nu-i+1}{2})} \cdot |H|^{(\nu-m-1)/2} \exp \left[ -\frac{1}{2} \text{tr}(A^{-1}H) \right] & H \text{ p.d.} \\ 0 & \text{otherwise} \end{cases}$$

The mean, and mean of the matrix  $H^{-1}$  are given by

$$\begin{aligned} \mathbb{E}[H|A, \nu] &= \nu A \\ \mathbb{E}[H^{-1}|A, \nu] &= \frac{1}{\nu - m - 1} A^{-1} \end{aligned}$$



## Appendix B

# A Brief S-PLUS/R Tutorial

This section gives a brief overview of S-PLUS and R commands, and their data types. It is not intended to be a complete tutorial. Consult the S-PLUS and R manuals for further information. You can find S-PLUS manuals at URL:

`http://www.splus.mathsoft.com/splus/resources/doc`  
and R manuals at URL:

`http://lib.stat.cmu.edu/R/CRAN/contents.html#doc`

### B.1 Basic syntax of expressions

Variable names in S-PLUS and R are case sensitive, so that `x` and `X` are different. A function call consists of a function name followed by an argument list (which may be empty) in parentheses:

```
setseedconstant()  
plot(smooth.out)  
gaussianSim(mean,precision,1000)
```

One of the most frequently used operators is the *assignment* operator `<-` (or `_`). If the value of a function is not assigned to an object using `<-` or `_`, it is automatically printed and stored as `.Last.value`. When values returned by BACC functions are complicated objects, it is a good idea to use an assignment statement to store them.

### B.2 Data objects

Data in S-PLUS and R are organized into data objects. Each data object has a name, consisting of alphanumeric characters and periods (`.`). Names cannot start with a number.

Four basic S-PLUS/R data objects are used in BACC commands:

- Vectors:

1. Creating a vector:

The following examples show some useful functions for creating vectors

```
# A vector with elements 1, 2, 3, 4, 5
x1<-1:5

# A vector with elements 1, 3, 4, 6, 10, 5
x2<-c(1,3,4,6,10,5)

# A vector of strings
y<-c("Obs", "Age")

# A vector with elements from 0.1 to 0.6 with step size 0.05
z<-seq(0.1,0.6,0.05)

# A zero vector of length 6
z<-rep(0,6)
```

2. Attributes of a vector

- `length` The length of the vector.

```
length(x1)
# Returns 5
```

```
length(y)
# Returns 2
```

- `mode` One of numeric, character, logical, or complex.

```
mode(x1)
# Returns "numeric"
```

```
mode(y)
# Returns "character"
```

- `names` Label associated with values.

3. Concatenating vectors

```
z<-c(x1,x2)
# z is the vector obtained by vertically stacking x1 and x2
```

- Matrices:

1. Creating a matrix

```

# Create a 6 by 2 matrix using the values of z. The first
# column of z1 is equal to x1 and the second column is
# equal to x2.
z1<-matrix(z,6,2)
z1[,1] # First column of z1, equal to x1

# Create a 2 by 6 matrix using the values of z. The first
# row of z2 is equal to x1 and the second row is equal to x2
z2<-matrix(z,2,6,byrow=T)
z2[1,] # First row of z2, equal to x1

```

## 2. Attributes of a matrix

- `length` The total number of element
 

```

length(z1)
# Returns 12

length(z2)
# Returns 12

```
- `mode` As above.
- `dim` The number of rows and columns of a matrix
 

```

dim(z1)
# Return the vector (6, 2)

nrow(z1)
# Returns the number of rows of z1, i.e. 6

ncol(z1)
# Returns the number of columns of z1, i.e. 2

```
- `dimnames` The row and column names

## 3. some other manipulations of matrices

```

# z3 is z2, reshaped to have dimensions 3 by 4
z3<-matrix(z2,3,4)

# z4 is the transpose of z2
z4<-t(z2)

# Indexing the elements of a matrix
z1[,1] # First column of z1
z2[1,] # First row of z2

```

- arrays

Arrays are like matrices, but with an arbitrary number of dimensions. The examples for matrices above can be generalized for arrays.

- **lists** Unlike vectors, matrices, and arrays, lists may contain data objects with different data types (or modes).

1. Create a list

```
# Create a list with components x1, x2 (numeric vectors),
# y (character vector), and z1 (matrix)
mylist<-list(x1,x2,y,z1)
```

2. Attributes of a list

- **length** The number of components in the list
 

```
length(mylist)
# Returns 4
```
- **mode** The mode of a list is always "list"
- **names** The names of the components
 

```
# Check names of the components in the list 'mylist'
names(mylist)
# Returns a vector with elements "x1", "x2", "y", "z1"
```

3. To access list components

- (a) To access list components by name

```
# Show the value of component z1 of mylist
mylist$z1

# Show the dimensions of component z1 of mylist
dim(mylist$z1)
```

- (b) To access list components by index

```
Indexes must be enclosed in double brackets([[ ]])
# Display the value of component z1 of mylist
mylist[[4]]
```

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