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OBS! Läs noga igenom anvisningarna i tentamen, t.ex. hur du ska skriva svaren.
Det är ditt ansvar som student att följa de anvisningar som ges.

NOTE! Read the examination instructions carefully, e.g. how to write the answers.
It is your responsibility as a student to follow the given instructions.

Skriv din anonymiseringskod och dagens datum på allt material du lämnar in.
(*Enter your anonymization code and todays date on all submitted materials*)

Anonymiseringskod (*Anonymization code*)	3	1	1	-	0	0	2	2	-	N	F	U

Datum (*Date YYYY-MM-DD*)	2025-01-09	Plats nr. (*Seat No.*)	109

Kurs/Kurskod (*Course/Course code*)	ST721A
Kursmoment (*Course component*)	

Fylls i av tentamensvärd (*To be filled in by invigilator*)

Direkt i skrivning: (kryss)		Svarsblankett: (kryss)		Lösa svarsblad: (antal)	28
Lämnat in blankt: (kryss)		Dator: (kryss)			

Inlämningstid: 19:00 Signatur tentamensvärd: Agnetha Land

Fylls i av lärare/examinator (*To be filled in by teacher/examinator*)

Betyg:	B	Poäng:	73 → 76.7

Signatur rättande lärare/examinator: AA

1	2	3	4	5	6	7	8	9	Σ
-	12	6	8	12	10	9	5	11	73

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Uppg.nr.: (*Task no.*) 6

Lärarens kommentar: (*Teacher's note*)

Exercise: 06

Given, $f(x,y) = \begin{cases} 6xy^2, & 0<x<1, 0<y<1 \\ 0, & \text{otherwise} \end{cases}$

a) Is $f(x,y)$ a pdf?

$$\int_0^1\int_0^1 f(x,y)\,dx\,dy$$

$$= \int_0^1\int_0^1 6xy^2\,dx\,dy$$

$$= \int_0^1 6y^2 \int_0^1 x\,dx\,dy$$

$$= \int_0^1 6y^2 \left[\frac{x^2}{2}\right]_0^1 dy$$

$$= \int_0^1 \left(6\cdot y^2\cdot\frac{1}{2} - 0\right) dy = \int_0^1 3y^2\,dy$$

$$= \frac{3y^3}{3}\Big|_0^1 = 1 - 0$$

$$= 1$$

ok

So, $f(x,y) \in (0,6) \longrightarrow$ non-negative

and $f(x,y)$ is a proper density function

as $\iint f(x,y)\,dx\,dy = 1$.

Poäng: (*Points*)

Tentamensservice/SU/typ_a

Lärarens kommentar: (*Teacher's note*)

Poäng: (*Points*)

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Uppg.nr.: (Task no.) 6

Lärarens kommentar: (Teacher's note)

b) $P(X+Y \geq 0.9)$

$$= \int_0^1 \int_{0.9-y}^1 6xy^2 \, dx \, dy$$

$$= \int_0^1 6y^2 \left[\int_{0.9-y}^1 x \, dx \right] dy$$

$$= \int_0^1 6y^2 \left[\frac{x^2}{2} \right]_{0.9-y}^1 dy$$

$$= \int_0^1 6y^2 \left[\frac{1}{2} - \frac{(0.9-y)^2}{2} \right] dy$$

$$= \int_0^1 6y^2 \left[\frac{1}{2} - \frac{.81 - 1.8y + y^2}{2} \right] dy$$

$$= \int_0^1 6y^2 \left[\frac{1}{2} - \frac{.81}{2} + 0.9y - \frac{y^2}{2} \right] dy$$

$$= \int_0^1 (0.57y^2 + 5.4y^3 - 3y^4) \, dy$$

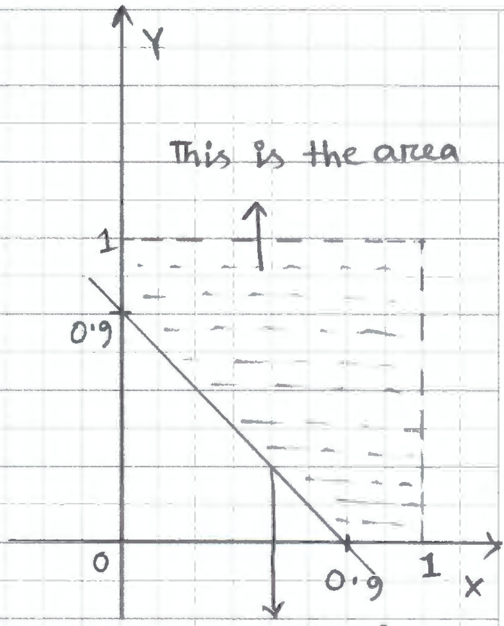
$$= \frac{0.57y^3}{3} + \frac{5.4y^4}{4} - \frac{3y^5}{5} \Big|_0^1$$

$= \cancel{0.19 + 1.35 + 0.6}$ $0.19 + 1.35 - 0.6$

$= 0.94$

So, $P(X+Y \geq 0.9) \approx 0.94$

OK



$x+y=0.9$

$x+y \geq 0.9$

$x \geq 0.9-y$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

c) At first, we have to define marginal distribution of X, integrate away y. Thus, we get,

$$f_X(x) = \int_0^1 6xy^2\,dy$$

$$= 6x\int_0^1 y^2\,dy = 6x \cdot \frac{y^3}{3}\Big|_0^1$$

OK

$$= 2x \cdot 1 - 0 = 2x \quad \text{for, } 0<x<1.$$

using this we can calculate the probabilities such as,

$$P(0.5<X<1) = \int_{0.5}^1 2x\,dx$$

$$= \frac{2x^2}{2}\Big|_{0.5}^1$$

$$= 1 - 0.25$$

$$= 0.75$$

and, $P(0<X<0.5) = \int_0^{0.5} 2x\,dx$

$$= 2\frac{x^2}{2}\Big|_0^{0.5}$$

$$= .25 - 0$$

$$= .25$$

Thus, $\left[P(0.5<X<1) - P(0<X<0.5)\right] = 0.75 - 0.25$

$$= 0.50$$

ok

(Ans).

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 7

Lärarens kommentar: (Teacher's note)

Exercise: 07

Let,

X: minimum of two numbers

Y: maximum of two numbers

Now, the joint distribution and marginal distn table is given below:

X \ Y	1	2	3	4	5	6	$f_X(x)$
1	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{11}{36}$
2	—	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{9}{36}$
3	—	—	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{7}{36}$
4	—	—	—	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{5}{36}$
5	—	—	—	—	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$
6	—	—	—	—	—	$\frac{1}{36}$	$\frac{1}{36}$
$f_Y(y)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$	$\Sigma \frac{36}{36}$

good :)

Here, the joint distribution of probability mass function, would be, $f(x,y) = P(X=x, Y=y)$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar: (*Teacher's note*)

Poäng: (*Points*)

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Uppg.nr.: (Task no.) 7

Lärarens kommentar: (Teacher's note)

7. Thus, by definition, $\sum\sum_{(x,y)\in \mathcal{X}^2} f(x,y) = 1$

Now,

$$\boxed{E[X]} = \sum_{i=1}^{6} \text{dice}_{ix}\, P_{ix}$$

$$= \frac{1\cdot 11}{36} + \frac{2\cdot 9}{36} + \frac{3\cdot 7}{36} + \frac{4\cdot 5}{36} + \frac{5\cdot 3}{36} + \frac{6\cdot 1}{36}$$

$$= \frac{11}{36} + \frac{18}{36} + \frac{21}{36} + \frac{20}{36} + \frac{15}{36} + \frac{6}{36}$$

$$= \boxed{\frac{91}{36}}$$ ok

$$E[X^2] = \sum_{i=1}^{6} \text{dice}_{xi}^2\, P_{xi}$$

$$= \frac{1^2\cdot 11}{36} + \frac{2^2\times 9}{36} + \frac{3^2\times 7}{36} + \frac{4^2\times 5}{36} + \frac{5^2\times 3}{36} + \frac{6^2\cdot 1}{36}$$

$$= \frac{11}{36} + \frac{36}{36} + \frac{63}{36} + \frac{80}{36} + \frac{75}{36} + \frac{36}{36}$$

$$= \frac{301}{36}$$ ok

$$\boxed{E[Y]} = \sum_{i=1}^{6} \text{dice}_{iy}\, P_{iy}$$

$$= \frac{1}{36} + \frac{2\cdot 3}{36} + \frac{3\cdot 5}{36} + \frac{4\cdot 7}{36} + \frac{5\times 9}{36} + \frac{6\times 11}{36}$$

$$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36}$$

$$= \boxed{\frac{161}{36}}$$ ok

Thus, $E[Y^2] = \frac{1}{36} + \frac{2^2\cdot 3}{36} + \frac{3^2\cdot 5}{36} + \frac{4^2\cdot 7}{36} + \frac{5^2\times 9}{36} + \frac{6^2\cdot 11}{36}$!

$\longrightarrow$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 7

Lärarens kommentar: (Teacher's note)

so, $E[Y^2] \Rightarrow \frac{791}{36} = \frac{791}{36}$

$$\text{var}[X] = E[X^2] - (E[X])^2$$

$$= \frac{301}{36} - \left(\frac{91}{36}\right)^2$$

$$= 1.971$$

$$\text{var}[Y] = E[Y^2] - (E[Y])^2$$

$$= \frac{791}{36} - \left(\frac{161}{36}\right)^2$$

$$= 1.971$$

$$\text{corr}(X,Y) = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y}$$

we know that, $\text{cov}(X,Y) = E[XY] - E[X] \cdot E[Y]$

Here, $E[XY] = \sum_{i}^{36} g(X,Y)\, f(X,Y)$

$$= 1\cdot 1 \cdot \frac{1}{36} + 1\cdot 2 \cdot \frac{2}{36} + 1 \cdot 3 \cdot \frac{2}{36} + \cdots + 6\cdot 6 \cdot \frac{1}{36}$$

$$= \frac{49}{9} + \frac{65}{36} + \frac{11}{4} + \frac{36}{36} + \frac{85}{36} + \frac{26}{36} = \frac{155}{12}$$

$$\text{cov}(X,Y) = \frac{155}{12} - \left(\frac{91}{36}\right) \cdot \frac{161}{36} = 1.611$$

$$\text{corr}(X,Y) = \frac{1.611}{\sqrt{1.971} \cdot \sqrt{1.971}} = .8177$$

mistake in calculations

$\boxed{\text{corr} \approx 0.48}$

$\longrightarrow$ continue

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar: (*Teacher's note*)

Poäng: (*Points*)

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Uppg.nr.: (Task no.) 7

Lärarens kommentar: (Teacher's note)

Exercise: 07

$corr(X,Y) = 0.8177 \approx 0.82$

Interpretion: In general, ρ_{xy} belongs to -1 to 1. Here, $\rho_{xy} \approx 0.82$ that indecates that there exists a strong and positive correlation between X and Y. Thus, a change of one standard unit of independent variable there it is associated with a change of 0.82 unit in dependent variable.

OK

Poäng: (Points)

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

Uppg.nr.: (Task no.) 5

Lärarens kommentar: (Teacher's note)

Exercise: 05

Given, $f(x,y) = \begin{cases} c * xy(1-x^2) & ; 0<x<1, 0<y<1 \\ 0 & ; \text{otherwise} \end{cases}$

Now, is the function $f(x,y)$ proper density function?

$$\int_0^1\int_0^1 c\,xy(1-x^2)\,dy\,dx$$

$$= \int_0^1\int_0^1 cxy - cx^3y\,dy\,dx$$

$$= \int_0^1 cx \int_0^1 y - x^2y\,dy\,dx$$

$$= \int_0^1 cx \left[\frac{y^2}{2} - \frac{x^2 \cdot y^2}{2}\right]_0^1 dx$$

$$= \int_0^1 cx \left[\frac{1}{2} - \frac{x^2}{2}\right] dx$$

$$= \int_0^1 c\left[\frac{x}{2} - \frac{x^3}{2}\right] dx$$

$$= c\left[\frac{x^2}{4} - \frac{x^4}{2 \cdot 4}\right]_0^1$$

$$= c\left[\frac{1}{4} - \frac{1}{8}\right]$$

$$= c \cdot \frac{1}{8}$$

The function $f(x,y)$ would be a proper density function, if and only if constant $c = 8$.

Thus, $\int_0^1\int_0^1 f(x,y)\,dy\,dx$ will be 1.

OK →

Poäng: (Points)

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

exercise : 05

Now, $f(x,y) = \begin{cases} 8xy(1-x^2) \; ; & 0<x<1, 0<y<1 \\ 0 & ; \text{otherwise} \end{cases}$

Here, $8xy(1-x^2) = 8xy - 8x^3y$

Let, $\boxed{U = X * Y}$, $u = g_1(x,y)$, $x = h_1(u,v) = U/Y = U/V$

$V = Y$, $v = g_2(x,y)$, $y = h_2(u,v) = Y$

The jacobian transformation of the distribution is, as follows

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{v} & \frac{-u}{v^2} \\ 0 & 1 \end{vmatrix} = \left|\frac{1}{v}\right| = \frac{1}{v}$$

ok

Now, the joint distribution of U and V is,

$$f_{UV}(u,v) = f_{XY}(x,y)\left(h_1(u,v), h_2(u,v)\right)|J|$$

$$= \left[8\left(\frac{u}{v}\right)\cdot v - 8\left(\frac{u}{v}\right)^3 \cdot v\right]\cdot\frac{1}{v}$$

$$= \left(\frac{8u}{v} - 8\frac{u^3}{v^3}\right)$$

To define $f_U(u)$; we have to integrate away v;
and the limits would be,

$0 < x < 1$ $\qquad$ $0 < y < 1$

$0 < \frac{u}{v} < 1$ $\qquad$ $0 < v < 1$

$0 < u < v \longrightarrow u < v < 1$

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 5

Lärarens kommentar: (Teacher's note)

Ex:05

Now, $f_V^{(u)} = \int_u^1 \left(8u \cdot \frac{1}{v} - 8\frac{u^3}{v^3}\right) dv$

$\int v^{-3} dv = \frac{v^{-3+1}}{-3+1}$

$\int \frac{1}{v} dv = \ln v$

$= 8u \cdot \ln v - 8 \cdot \frac{-u^3}{2v^2} \Big|_u^1$

$= 8u \cdot 0 + 4u^3 - \left(8u \cdot \ln u + \frac{4u^3}{u^2}\right)$

$= \boxed{4u^3 - 8u \cdot \ln u - 4u}$; for $\boxed{0 < u < 1}$

OK

Thus, if $f_V^{(u)}$ is a pdf ? Let, define as,

$\int_0^1 f_V^{(u)} du = \int_0^1 (4u^3 - 8u \cdot \ln u - 4u) du$

$= \frac{4u^4}{4}\Big|_0^1 - \frac{4u^2}{2}\Big|_0^1 - \int_0^1 8u \cdot \ln u \, du$ —— (i)

Here, Let, $x = \ln u$ and $dy = u$

$dx = \frac{1}{u}$ $\quad y = \frac{u^2}{2}$

we know, $xy\Big|_0^1 - \int_0^1 y \, dx dy = \int_0^1 x \, dy \, du$

Implying this, we get,

$-8\left\{\left[\ln u \cdot \frac{u^2}{2}\right]_0^1 - \int_0^1 \frac{u^2}{2} \cdot \frac{1}{u} du\right\}$

$= -8\left\{0 - \frac{u^2}{4}\Big|_0^1\right\}$

$= -8 \cdot \left(-\frac{1}{4}\right)$

$= 2$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 5

ex: 05

putting this value in equation (i) we get

$$\int_0^1 f_V(u)\, du = 1 - 2 + 2$$

$$= 1$$

~~Thus~~ So, that probability density function of $X*Y$ or $f_V(u)$ shows a proper pdf.

ok

Lärarens kommentar: (Teacher's note)

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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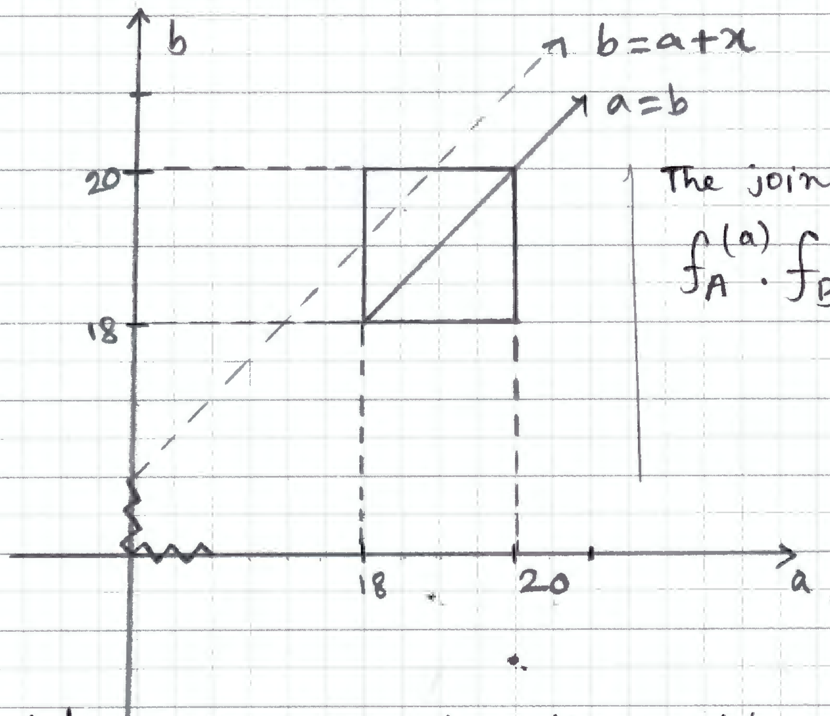
Uppg.nr.: (Task no.) 4

Lärarens kommentar: (Teacher's note)

Exercise : 04 let,

A : A's arrival time   A is r.v $\sim U(18, 20)$

B : B's arrival time   B is r.v $\sim U(18, 20)$



The joint distⁿ is,

$$f_A^{(a)} \cdot f_B^{(b)} = \frac{1}{20-18} \cdot \frac{1}{20-18}$$

$$= \frac{1}{4}$$

ok

let, $X = B - A$; waiting time. We consider waiting time as 0 when $X<0$. i.e B arrives before A.

We want to define the probability of time, when they arrive simultaneously. $P(X=0)$; but due to continuety of time $P(X=0)$ is expressed as

$P(X \leq 0)$ or $P(B-A \leq 0)$ or $P(B \leq A)$.

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 4

Lärarens kommentar: (Teacher's note)

Thus, $\int_{18}^{20}\int_{18}^{a} \frac{1}{4}\, db\, da = \int_{18}^{20} \left[\frac{b}{4} \Big|_{18}^{a} \right] da$

$$= \int_{18}^{20} \left(\frac{a}{4} - \frac{18}{4}\right) da$$

$$= \left(\frac{a^2}{4\cdot 2} - \frac{18a}{4}\right)_{18}^{20}$$

$$= \left(\frac{400}{8} - \frac{360}{4}\right) - \left(\frac{18^2}{8} - \frac{18\cdot 18}{4}\right)$$

$= \frac{1}{2}$; of course! this calculation is redundant. if or we considere only the fact that the probability of arriving before one to another it will be always $\frac{1}{2}$.

Now, we also want to cacklate the prob^y of arriving A before B. ~~Not~~ Thus,

$$P(X \leq x) = 1 - P(X \geq x)$$

$$= 1 - P(B - A > x) \text{ or}$$

$$1 - P(B > x + A) \Longleftrightarrow$$

$$1 - \int_{18}^{20-x}\int_{a+x}^{20} \frac{1}{4}\, db\, da$$

$$= 1 - \int_{18}^{20-x} \left[\frac{b}{4}\right]_{a+x}^{20} da$$

→

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 4

Lärarens kommentar: (Teacher's note)

Ex: 04

$$P[X \leq x] = 1 - \int_{18}^{20-x} \left(\frac{20}{4} - \frac{a+x}{4}\right) da$$

$$= 1 - \left[\frac{20a}{4} - \left(\frac{a^2}{4} + \frac{xa}{4}\right)\right]_{18}^{20-x}$$

$$= 1 - \left[5a - \frac{a^2}{4} - \frac{xa}{4}\right]_{18}^{20-x}$$

$$= 1 - \left\{\left[5(20-x) - \frac{(20-x)^2}{4} - \frac{x(20-x)}{4}\right] - \left(5 \cdot 18 - \frac{18^2}{4} - \frac{x \cdot 18}{4}\right)\right\}$$

$$= 1 - \left\{\left(100 - 5x - \frac{400 - 40x + x^2}{4} - \frac{20x - x^2}{4}\right) - \left(90 - \frac{324}{4} - \frac{18x}{4}\right)\right\}$$

$$= 1 - \left\{\cancel{100} - \cancel{5x} - \cancel{100} + \cancel{10x} - \cancel{\frac{x^2}{4}} - \cancel{5x} + \cancel{\frac{x^2}{4}} - 90 + 81 + \frac{18x}{4}\right\}$$

$$= 1 - \frac{9x}{2} + 9$$

$$= 10 - \frac{9x}{2}$$

so that,

$$F_X(x) = \begin{cases} 0 & ; \text{ if } X < 0 \\ 10 - \frac{9X}{2} & ; \text{ if } 0 < X < 2 \\ \frac{1}{2} & ; \text{ if } x = 0 \\ 1 & ; \text{ otherercwise.} \end{cases}$$

ok.

(Ans).

can not be correct!

say, $x \approx 0$ but $x > 0$ $10 - \frac{9 \cdot \text{"}0\text{"}}{2} \approx 10$

+

"x" should be [0,2]

$x = B - A$

ok

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Ark nr.: (Sheet no.) 15

Anonymiseringskod (Anonymization code) 311-0022-NFU

Uppg.nr.: (Task no.) 2

Lärarens kommentar: (Teacher's note)

Exercise : 02)

Let, $-\infty < \mu_x < \infty$ $\quad -\infty < \mu_y < \infty$

$\mu \qquad \gamma$

$0 < \sigma_x$ and $0 < \sigma_y$ as τ and $-1 < \rho < 1$

σ

Given,

$$f(x,y) = \frac{1}{2\pi\sigma\tau(\sqrt{1-\rho^2})} \cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu}{\sigma}\right)^2 - 2\rho\left(\frac{x-\mu}{\sigma}\right)\left(\frac{y-\gamma}{\tau}\right) + \left(\frac{y-\gamma}{\tau}\right)^2\right]}$$

; for $-\infty < x < \infty$ and $-\infty < y < \infty$.

a) The marginal distribution of X can be defined as,

$$f_X^{(x)} = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma\tau(\sqrt{1-\rho^2})} \cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu}{\sigma}\right)^2 - 2\rho\left(\frac{x-\mu}{\sigma}\right)\cdot\left(\frac{y-\gamma}{\tau}\right) + \left(\frac{y-\gamma}{\tau}\right)^2\right]} dy$$

$$= \frac{1 \cdot e^{-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2(1-\rho^2)}}}{2\pi\sigma\tau(\sqrt{1-\rho^2})} \int_{-\infty}^{\infty} e^{-\frac{1}{2(1-\rho^2)}\left[\rho^2\left(\frac{x-\mu}{\sigma}\right)^2 - 2\rho\left(\frac{x-\mu}{\sigma}\right)\left(\frac{y-\gamma}{\tau}\right) + \left(\frac{y-\gamma}{\tau}\right)^2 - \rho^2\left(\frac{x-\mu}{\sigma}\right)^2\right]}$$

$\longrightarrow$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 2

Lärarens kommentar: (Teacher's note)

Ex:02

$$f_X^{(x)} = \frac{e^{-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2(1-\rho^2)}}}{2\pi\sigma\tau(\sqrt{1-\rho^2})}\int_{-\infty}^{\infty} e^{-\frac{1}{2(1-\rho^2)}\left\{\left[\left(\left(\frac{y-\gamma}{\tau}\right)^2-\left(\rho\left(\frac{x-\mu}{\sigma}\right)\right)^2\right)\right]^2 \cdot \left[-\rho^2\left(\frac{x-\mu}{\sigma}\right)^2\right]\right\}} dy$$

$$= \frac{e^{-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2(1-\rho^2)} \cdot \frac{\rho^2\left(\frac{x-\mu}{\sigma}\right)^2}{2(1-\rho^2)}}}{2\pi\sigma\tau\sqrt{1-\rho^2}}\int_{-\infty}^{\infty} e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{y-\gamma}{\tau}\right)-\rho\left(\frac{x-\mu}{\sigma}\right)\right]^2} dy$$

$$= \frac{e^{-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2\cancel{(1-\rho^2)}}\left[1-\cancel{\rho^2}\right]}}{2\pi\sigma\tau\cdot\sqrt{1-\rho^2}} \cdot \int_{-\infty}^{\infty} e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{y-\gamma}{\tau}\right)-\left(\rho\left(\frac{x-\mu}{\sigma}\right)\right)\right]^2} dy$$

let, $u = \left(\frac{y-\gamma}{\tau}\right)-\rho\left(\frac{x-\mu}{\sigma}\right)\cdot\frac{1}{\sqrt{(1-\rho^2)}}$

$du = \frac{1}{\tau}\, dy \cdot \frac{1}{\sqrt{(1-\rho^2)}}$

$dy = \tau\cdot\sqrt{(1-\rho^2)}\cdot du \quad ; \quad -\infty < u < \infty$

Implying this we can rewrite this $f_X^{(x)}$ as,

$$f_X^{(x)} = \frac{1\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{2\pi\sigma\cancel{\tau}\cdot\cancel{\sqrt{1-\rho^2}}}\int_{-\infty}^{\infty} e^{-\frac{u^2}{2}}\, du\, \cancel{\tau}\cdot\cancel{\sqrt{1-\rho^2}}$$

$\longrightarrow$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar: (*Teacher's note*)

Poäng: (*Points*)

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Uppg.nr.: (Task no.) 02

Lärarens kommentar: (Teacher's note)

Ex:02

$\int_{-\infty}^{\infty} e^{-\frac{u^2}{2}} du$ shows the Kernel of normal distribution; which is equal to $\sqrt{2\pi}$.

Now, $f_X(x) = \dfrac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{2\pi\sigma} \cdot \sqrt{2\pi}$

$= \dfrac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

So that, $f_X(x) \sim N(\mu_X, \sigma_X^2)$ ok

b) By Symmetry; we can also write

$f_Y(y) \sim N(\mu_Y, \sigma_Y^2)$

Here, $f_Y(y) = \dfrac{e^{-\frac{(y-\gamma)^2}{2\tau^2}}}{\sqrt{2\pi}\,\tau} \sim n(\mu_Y, \sigma_Y^2)$

ok

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
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Poäng:
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Uppg.nr.: (Task no.) 02

Lärarens kommentar: (Teacher's note)

c) $$\rho_{xy} = \frac{\mathrm{cov}(X,Y)}{\sigma_x \sigma_y} = \frac{E[(X-\mu)(Y-\gamma)]}{\sigma \cdot \tau}.$$

$$= E\left[\left(\frac{X-\mu}{\sigma}\right)\cdot\left(\frac{Y-\gamma}{\tau}\right)\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{x-\mu}{\sigma}\right)\left(\frac{y-\gamma}{\tau}\right) f(x,y)\, dx\, dy$$

Let, $S = \left(\frac{x-\mu}{\sigma}\right)\left(\frac{y-\gamma}{\tau}\right)$ and, $t = \left(\frac{x-\mu}{\sigma}\right)$

$x = t\sigma + \mu$

$$S = t\left(\frac{y-\gamma}{\tau}\right)$$

$$\frac{S}{t} = \left(\frac{y-\gamma}{\tau}\right) \quad \text{and} \quad y = \gamma + \frac{S\tau}{t}$$

Now, the jacobian transformation is,

$$\begin{vmatrix} \frac{\partial x}{\partial s} = 0 & \frac{\partial x}{\partial t} = \sigma \\ \frac{\partial y}{\partial s} = \frac{\tau}{t} & \frac{\partial y}{\partial t} = \frac{-s\tau}{t^2} \end{vmatrix} = \left|\frac{-\sigma\tau}{t}\right| = \frac{\sigma\tau}{t}$$

$$\rho_{xy} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s \cdot f\left((t\sigma,\mu),\left(\gamma + \frac{s\tau}{t}\right)\right) ds\, dt \left|\frac{\sigma\tau}{t}\right|$$

$$= \frac{1}{2\pi\sigma\tau\sqrt{1-\rho^2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s \cdot e^{-\frac{1}{2(1-\rho^2)}\left[t^2 - 2\rho s + \left(\frac{s}{t}\right)^2\right]} \left|\frac{\sigma\tau}{t}\right| ds\, dt$$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
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Uppg.nr.: (Task no.) 02

Lärarens kommentar: (Teacher's note)

$$\rho_{xy} = \frac{1}{2\pi\sigma\tau\sqrt{1-\rho^2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s \cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{s-\rho t^2}{t}\right)^2+t^2(1-\rho^2)\right]} \frac{\sigma\tau}{\sqrt{t^2}}\, ds\, dt$$

Nothing ; just write $|t| = \sqrt{t^2}$ and $\left(t^2 - 2\rho s + \left(\frac{s}{t}\right)^2\right)$ by simplifying $\left(\frac{s-\rho t}{t}\right)^2 + t^2(1-\rho^2)$:

$$\rho_{xy} = \int_{-\infty}^{\infty} \frac{1 \cdot e^{-\frac{t^2(1-\rho^2)}{2(1-\rho^2)}}}{\sqrt{2\pi}} \cdot \left[\int_{-\infty}^{\infty} \frac{s \cdot e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(s-\rho t)^2}{t^2}\right]}}{\sqrt{2\pi}\sqrt{(1-\rho^2)t^2}}\, ds\right] dt$$

$$= \int_{-\infty}^{\infty} \frac{1 \cdot e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \frac{s \cdot e^{-\frac{(s-\rho t)^2}{2(1-\rho^2)t^2}}}{\sqrt{2\pi}\sqrt{(1-\rho^2)t^2}}\, ds\right] dt$$

The inner integrand is $E[S]$; with

$E[S] = \rho t^2$ and $var[S] = (1-\rho^2)t^2$

So, $$\rho_{xy} = \int_{-\infty}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \cdot \rho t^2\, dt$$

$$= \rho \int_{-\infty}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \cdot t^2\, dt$$

Here, $E(t^2) \sim N(0,1)$ $E[t] = 1$

$$\rho_{xy} = \rho \cdot 1$$

ok, good

(proved)

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
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Uppg.nr.: (Task no.) 03

Lärarens kommentar: (Teacher's note)

Exercice: 03, we know the mgf can be defined as,

a) $M_X(t) = E[e^{tX}] \wedge X \sim \text{Gamma}(\alpha, \beta)$

where, $0 \leq x < \infty$; $\alpha, \beta > 0$

Thus, $$E[e^{tX}] = \int_0^\infty e^{tX} \cdot \frac{1}{\Gamma\alpha \cdot \beta^\alpha} x^{\alpha-1} \cdot e^{-x/\beta} dx$$

$$= \frac{1}{\Gamma\alpha \cdot \beta^\alpha} \int_0^\infty e^{tX - \frac{X}{\beta}} x^{\alpha-1} dx$$

$$= \frac{1}{\Gamma\alpha \cdot \beta^\alpha} \int_0^\infty x^{\alpha-1} e^{-x(\frac{1}{\beta} - t)} dx$$

Kernel of gamma $\left(\alpha, \frac{1}{(\frac{1}{\beta}-t)}\right)$; must be integrate to, $\Gamma\alpha$ and $\frac{1}{(\frac{1}{\beta}-t)}$. Here,

$\beta = \frac{1}{\frac{1}{\beta}-t}$; for some reason we simplify it as, $\frac{1}{\frac{1-\beta t}{\beta}} = \frac{\beta}{(1-\beta t)}$. Now, the mgf would be,

$$E[e^{tX}] = \frac{1}{\Gamma\alpha \cdot \beta^\alpha} \cdot \Gamma\alpha \left(\frac{\beta}{1-\beta t}\right)^\alpha$$

$$= \frac{1}{(1-\beta t)^\alpha} \text{ ; where, } t < \frac{1}{\beta}.$$

Thus, the denumanitore will be zerco and ~~mgf~~ mf will be negative.

$\longrightarrow$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 03

Lärarens kommentar: (Teacher's note)

Ex:03

We know that,

$$E[X^n] = M_X^n(0) = \frac{d^n}{dt^n} M_X(t)\Big|_{t=0}$$

So, $E[X] = \frac{d}{dt}\left(\frac{1}{(1-\beta t)^\alpha}\right)\Big|_{t=0}$

$$= -\alpha \cdot -\beta \cdot \frac{1}{(1-\beta t)^{\alpha+1}}\Big|_{t=0}$$

$$= \alpha\beta \cdot 1 = \alpha\beta$$

$$E[X^2] = M_X''(t) = \frac{d}{dt}\left[\frac{\alpha\beta}{(1-\beta t)^{\alpha+1}}\right]\Big|_{t=0}$$

$$= \alpha\beta \cdot (-\alpha-1) \cdot (1-\beta t)^{-\alpha-2} \cdot -\beta \Big|_{t=0}$$

$$= \alpha\beta(\alpha\beta + \beta) \cdot 1$$

$$= \alpha^2\beta^2 + \alpha\beta^2$$

Now, $Var[X] = E[X^2] - [E(X)]^2$

$$= \alpha^2\beta^2 + \alpha\beta^2 - (\alpha\beta)^2$$

$$= \alpha^2\beta^2 + \alpha\beta^2 - \alpha^2\beta^2$$

$$= \alpha\beta^2$$

α

(Ans).

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar: (*Teacher's note*)

Poäng: (*Points*)

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Uppg.nr.: (Task no.) 09

Lärarens kommentar: (Teacher's note)

Exercise: 09

Given, $f(x,y)=1$, $0<y<1$, $y<x<y+1$

$$\int_0^1\int_y^{y+1} 1\,dx\,dy = \int_0^1 x\Big|_y^{y+1} dy$$

$$= \int_0^1 \cancel{y}+1-\cancel{y}\,dy$$

$$= y\Big|_0^1$$

ok

$$= 1-0$$

$$= 1$$

$f(x,y) \in 1.$

a) To find the marginal dist^n of X, have to integrate away y and the limits are,

$y<x<y+1$

$y<x \qquad x<y+1$

$0<y<x \qquad x-1<y \longrightarrow x-1<y<1$

$$\int_0^x 1\,dy = x\Big|_0^1 = x \quad \text{and} \quad \int_{x-1}^1 1\,dy = x\Big|_{x-1}^1 = 2-x$$

$$f_X(x) = \begin{cases} x & ;\text{ if } 0<x<1 \\ 2-x & ;\text{ if } 1<x<2 \end{cases}$$

$$f_Y(y) = \int_y^{y+1} 1\,dx = 1 \qquad \text{for, } 0<y<1$$

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 09

Ex: 09

let, $\boxed{U=2X}$; $X=\frac{U}{2}$ $\frac{dX}{dU}=1\cdot\frac{1}{2}$

$f_U^{(u)}=\frac{1}{2}\rightarrow\begin{cases}0<\frac{u}{2}<1 \text{ or } 0<U<2\\ 1<\frac{u}{2}<2 \text{ or } 2<u<4\end{cases}$

?? should be:

$f_{2X}(u)=\begin{cases}\frac{u}{4}, & 0<u<2\\ \frac{4-u}{4}, & 2<u<4\\ 0, & \text{otherwise}\end{cases}$

again, let, ~~$f(v)$~~ $V=3Y$ $Y=\frac{V}{3}=1\cdot\frac{1}{3}$

$f_V^{(v)}=\frac{1}{3}\rightarrow\{0<\frac{v}{3}<1 \text{ or } 0<V<3$ OK

Now, $\boxed{E[2X]}=2E[X]$

$=2\left[\int_0^1 x\cdot x\,dx+\int_1^2 x\cdot(2-x)\,dx\right]$

$=2\left[\left(\frac{x^2}{2}\right)_0^1+\left(x^2-\frac{x^3}{3}\right)_1^2\right]$

$=2\left[\frac{1}{2}+\left(4-\frac{8}{3}-1+\frac{1}{3}\right)\right]$

$=2\cdot1$

$=\boxed{2}$ ⟶ This is the first moment

OK

? $E[(2X)^2]=2\,E[X^2]$

$=2\left[\int_0^1 x^2\cdot x\,dx+\int_1^2 x^2(2-x)\,dx\right]$

$=2\left\{\left[\frac{x^4}{4}\right]_0^1+\left[\frac{2x^3}{3}-\frac{x^4}{4}\right]_1^2\right\}$

$=2\left\{\left[\frac{1}{4}\right]+\left(\frac{16}{3}-\frac{16}{4}-\frac{2}{3}+\frac{1}{4}\right)\right\}$

$=2\cdot\frac{7}{6}=\frac{7}{3}$ ⟶ second moment ⟶

2

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar: (*Teacher's note*)

Poäng: (*Points*)

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Lärarens kommentar: (Teacher's note)

$$Var[2X] = \left[E[X^2] - (E(x))^2\right] \cdot 2^2$$

$$= 4\left[\frac{\cancel{7}\,2}{\cancel{6}\,3} - 1^2\right]$$

$$= 4 \cdot \frac{1}{\cancel{6}\,3} = \frac{4}{\cancel{6}\,3} \Rightarrow var[X] = \frac{1}{6}$$ slight mistake

$$E[3Y] = 3 \cdot E[Y]$$

$$= 3 \cdot \int_0^1 y \cdot 1 \cdot dy = \frac{y^2}{2}\Big|_0^1 = \left[\frac{1}{2}\right] 3$$

$$= \frac{3}{2} \longrightarrow \text{First moment}$$

or

$$E[(3Y)^2] = 3\int_0^1 y^2 \cdot 1\, dy = 3\left[\frac{y^3}{3}\right]_0^1 = 3 \cdot \frac{1}{3}$$

$$= \frac{3}{3} = 1 \longrightarrow \text{second moment}$$

$$var[3Y] = 3^2\left[\frac{1}{3} - \left(\frac{1}{2}\right)^2\right]$$

$$= 9 \cdot \left[\frac{1}{3} - \frac{1}{4}\right] = 9 \cdot \frac{1}{12}$$ Same mistake

$$= \frac{9}{12} = \frac{3}{4}$$

c) we know, $Corr(2X, 3Y) = \dfrac{cov(2X, 3Y)}{\sigma_{2X}\, \sigma_{3Y}}$

$$= \frac{2 \cdot 3\left[cov(X, Y)\right]}{2 \cdot 3\left[\sigma_X, \sigma_Y\right]}$$

$$= \frac{cov(X, Y)}{\sigma_X\, \sigma_Y}$$

Poäng: (Points)

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Lärarens kommentar:
(*Teacher's note*)

Poäng:
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Uppg.nr.: (Task no.) 09

Lärarens kommentar: (Teacher's note)

ex:09

$$\operatorname{cov}(X,Y) = E[X,Y] - E[X]\cdot E[Y]$$

$$E[X,Y] = \int_0^1\int_y^{y+1} xy \cdot 1 \, dx\, dy$$

$$= \int_0^1 y \left[\frac{x^2}{2}\right]_y^{y+1} dy$$

$$= \int_0^1 y \left[\frac{(y+1)^2}{2} - \frac{y^2}{2}\right] dy$$

$$= \int_0^1 y \left[\left(\frac{y^2+2y+1}{2}\right) - \frac{y^2}{2}\right] dy$$

$$= \int_0^1 y\left(\cancel{\frac{y^2}{2}} + y + \frac{1}{2} - \cancel{\frac{y^2}{2}}\right) dy$$

$$= \int_0^1 y^2 + \frac{y}{2} \, dy = \frac{y^3}{3} + \frac{y^2}{4}\Big|_0^1$$

$$= \frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = 7/12$$

$$\operatorname{cov}(X,Y) = \frac{7}{12} - \left(1\cdot\frac{1}{2}\right) = \frac{1}{12}.$$

$$\operatorname{corr}(2X, 3Y) = \frac{1/12}{\sqrt{\frac{1}{6}}\sqrt{1/12}} = \sqrt{\frac{\frac{1}{144}}{\sqrt{1/72}}}$$

$$= \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

ok

(Ans).

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Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar:
(*Teacher's note*)

Poäng:
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Uppg.nr.: (Task no.) 08

Lärarens kommentar: (Teacher's note)

Exercise: 08

Given, $f(x,y) = \begin{cases} x+y, & 0<x<1, \ 0<y<1 \\ 0, & \text{otherwise} \end{cases}$

a) Let,

$U = -(X+Y)$ $\quad u = g_1(x,y) \quad x = h_1(u,v) = -(u+v)$

$V = Y$ $\quad v = g_2(x,y) \quad y = h_2(u,v) = v$

To define $f_U(u)$; need to integrate away V and the limits are as follows,

$0<x<1$ $\qquad$ $0<y<1$

$0<-(u+v)<1$ $\qquad$ $0<v<1$

$0<-u-v<1 \longrightarrow -u-v<1$ $\qquad$ $0<v<-u$

$0<-u-v$ $\qquad -u<1+v$

$v<-u$ $\qquad -1-u<v \longrightarrow -1-u<v<1$

$$\int_0^{-u} x+y \, d\cancel{x} = \int_0^{-u} -u-\cancel{y}+\cancel{y} \, dv = -uv\Big|_0^{-u}$$

$$= u^2$$

and $\int_{-1-u}^{1} -u \, dv = -u\cdot v\Big|_{-1-u}^{1} = -u-[-u(-1-u)]$

$= -u-[u+u^2]$

$= -u-u+u^2$

$= u^2-2u$

So that, $f_U(u) = \begin{cases} u^2 & ; 1>u>0 \\ u^2-2u & ; 1<u<2 \\ 0 & ; \text{otherwise} \end{cases}$

"u" should be defined on interval $[-2, 0)$, right?

$u = -x-y \quad 0<x<1, \ 0<y<1$

Poäng: (Points)

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Lärarens kommentar:
(*Teacher's note*)

Poäng:
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Lärarens kommentar: (Teacher's note)

if $f_U^{(u)}$ a pdf ?

$$\int_0^1 u^2\, du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}$$

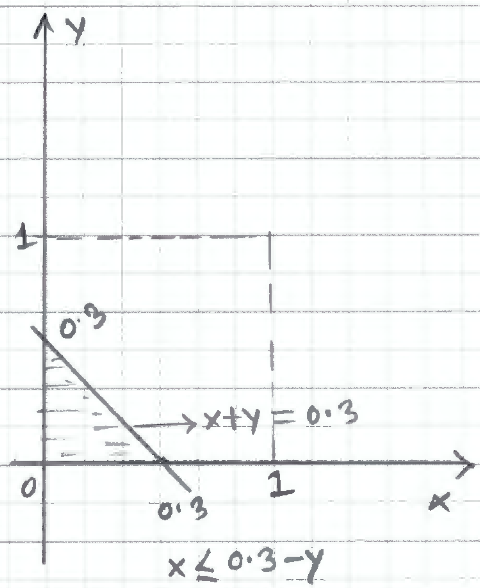
$$\int_1^2 2u^2 - 2u\, du = \frac{u^3}{3} - 2\frac{u^2}{2}\Big|_1^2 = \frac{8}{3} - 4 - \frac{1}{3} + 1$$

$$= \frac{2}{3}$$

Both domains $\wedge$ functions shows,

$\int f_U^{(u)}\, du = 1$; so that, it's a pdf.

b) $P[X+Y \leq 0.3]$



$P(X+Y \leq 0.3)$

$$\Rightarrow \int_0^1 \int_0^{0.3-y} x+y\; dx\, dy$$

$$= \int_0^1 y \left[\frac{x^2}{2}\right]_0^{0.3-y} dy$$

$$= \int_0^1 y \left[\frac{(0.3-y)^2}{2}\right] dy$$

$$= \int_0^1 y \left(\frac{0.09}{2} - \frac{.6y + y^2}{2}\right) dy$$

$$= \int_0^1 y \left(0.045 - 0.3y + \frac{y^2}{2}\right) dy$$

$$= \left(\frac{y^2 \cdot 0.045}{2} - \frac{0.3y^3}{3} + \frac{y^4}{2 \cdot 4}\right)_0^1 \longrightarrow$$

Poäng: (Points)

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Lärarens kommentar:
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Poäng:
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Stockholms universitet

Uppg.nr.: (Task no.) 08

Lärarens kommentar: (Teacher's note)

~~$= 0.225 - 0.15 - \frac{1}{3}$~~

$$P(X+Y \leq 0.3)$$

$$= \frac{9}{400} - \frac{1}{10} + \frac{1}{8}$$

$$= 0.0475$$

ok.

(Ans).

mean

?

median

Poäng: (Points)

Tentamensservice/SU/typ_a

Lärarens kommentar: (*Teacher's note*)

Poäng: (*Points*)

Regler i skrivsalen

- Följ tentamensvärds anvisningar.
- Väskor och ytterkläder ska placeras på anvisad plats.
- Placera ID-handling väl synlig på bordet framför dig.
- Ingen student får lämna skrivsalen under de första 30 minuterna.
- Endast en student i taget får besöka toaletten. Vid toalettbesök skriv ditt namn och klockslag på avsedd lista. Efter toalettbesöket ska du åter ange klockslag på listan.
- Elektronisk utrustning som mobiltelefon eller Smartwatch ska vara avstängd och placerad på anvisad plats.
- Under tentamen gäller tystnad – det är förbjudet att prata, eller på annat sätt kommunicera, med andra studenter under pågående tentamen.
- Innan tentamenshandlingarna lämnas in; skriv sidnummer, anonymiseringskod och datum på alla inlämnade papper.

Om något är oklart – fråga gärna tentamensvärden. Lycka till!

Rules in the examination hall

- Follow the invigilator's instructions.
- Bags and outerwear must be placed at the designated place.
- Place your ID document clearly visible on the table in front of you.
- No student may leave the examination hall for the first 30 minutes.
- Only one student at a time may visit the toilet. Before visiting the toilet, write your name and time on the intended list. After the toilet visit, enter the time on the list again.
- Electronic equipment such as a mobile phone or Smartwatch must be switched off and placed at the designated place.
- During the exam, silence applies – you are not allowed to talk, or otherwise communicate, with other students during the exam.
- Before submitting the examination documents; remember to write the page number, anonymization code, and date on all papers.

Please do not hesitate to ask the invigilator if anything is unclear. Good luck!

Tentamensservice
tentamensservice@su.se | www.su.se/tentamensservice

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