Stockholms universitet

OBS! Läs noga igenom anvisningarna i tentamen, t.ex. hur du ska skriva svaren. Det är ditt ansvar som student att följa de anvisningar som ges.

NOTE! Read the examination instructions carefully, e.g. how to write the answers. It is your responsibility as a student to follow the given instructions.

Skriv din anonymiseringskod och dagens datum på allt material du lämnar in.
(*Enter your anonymization code and todays date on all submitted materials*)

Anonymiseringskod (*Anonymization code*)	3	1	1	-	0	0	0	6	-	M	T	A

Datum (*Date YYYY-MM-DD*)	2023-01-03	Plats nr. (*Seat No.*)	91

Kurs/Kurskod (*Course/Course code*)	ST721A
Kursmoment (*Course component*)	

Fylls i av tentamensvärd (*To be filled in by invigilator*)

Direkt i skrivning: (kryss)		Svarsblankett: (kryss)		Lösa svarsblad: (antal)	7
Lämnat in blankt: (kryss)		Dator: (kryss)			

Inlämningstid: 18:56 Signatur tentamensvärd: [signature]

Fylls i av lärare/examinator (*To be filled in by teacher/examinator*)

Betyg:	D	Poäng:	64

Signatur rättande lärare/examinator: AA

1	2	3	4	5	6	7	8	9	Σ
4	12	12	4	0	2	7	12	0	53

× 1,21 → 64,13

Let X and Y be iid normal random variables with pdf N(1, 4)

a) $g_1(x,y) = u = x+y \quad x = h_1(u,v) = \frac{u+v}{2}$

$g_2(x,y) = v = x-y \quad y = h_2(u,v) = \frac{u-v}{2}$

$$J = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = \frac{1}{2}\cdot\left(-\frac{1}{2}\right) - \frac{1}{2}\cdot\frac{1}{2} = -\frac{1}{4} - \frac{1}{4} = -\frac{2}{4} = \left|-\frac{1}{2}\right|$$ OK

$$f_{xy}(x,y) = \frac{1}{\sqrt{2\pi 2}} e^{-(x-1)/(2\cdot 4)} \cdot$$ ok ? ? −

$$f_{uv}(u,v) = \frac{1}{\sqrt{4\pi}} \cdot e^{-\left(\frac{u+v}{2}-1\right)/8} \cdot \frac{1}{\sqrt{4\pi}} \cdot e^{-\left(\frac{u-v}{2}-1\right)/8}$$

$$= \frac{1}{4\pi} \cdot e^{-\left(\frac{u+v}{2}+\frac{u-v}{2}-2\right)/8} = \frac{1}{4\pi} \cdot e^{-(u-2)/8}$$

± Logic (+) Calculation (−)

b) $\text{Corr}(U,V) = \frac{\text{Cov}(U,V)}{\sigma_U \sigma_V}$

$\text{Cov}(U,V) = E[UV] - E[U]\cdot E[V]$

$$E[U\cdot V] = \iint_{-\infty\,-\infty}^{+\infty\,+\infty} u\cdot v \cdot \frac{1}{4\pi} \cdot e^{-(u-2)/8} \, dv\,du = \frac{1}{4\pi}\int_{-\infty}^{+\infty} u\cdot e^{-(u-2)/8} \int_{-\infty}^{+\infty} v\,dv\,du$$

$$= \frac{1}{4\pi}\int_{-\infty}^{+\infty} u\cdot e^{-(u-2)8} \cdot \left.\frac{v^2}{2}\right|_{-\infty}^{+\infty}$$

(−)

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Lärarens
kommentar:
(*Teacher's
note*)

Poäng:
(*Points*)

Datum: (Date YYYY-MM-DD) 2023-01-03
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Lärarens kommentar: (Teacher's note)

$f(x,y) = 6xy^2, \ 0<x<1, \ 0<y<1$
0, otherwise

a) $\int_0^1\int_0^1 6xy^2 dxdy = \int_0^1 6y^2 \int_0^1 x\,dxdy = \int_0^1 6y^2 \left[\frac{x^2}{2}\right]_0^1 dy = \int_0^1 6y^2 \cdot \frac{1}{2} dy$

$= 3\int_0^1 y^2 dy = 3\left[\frac{y^3}{3}\right]_0^1 = 3\left(\frac{1}{3}\right) = 1$ ok

b) $P(X+Y \geq 0{,}9) = \int_0^1\int_{0{,}9-y}^1 6xy^2 dxdy = \int_0^1 6y^2 \int_{0{,}9-y}^1 x\,dxdy = \int_0^1 6y^2 \left[\frac{x^2}{2}\right]_{0{,}9-y}^1 dy$

$= \int_0^1 6y^2 \left[\frac{1}{2} - \frac{(0{,}9-y)^2}{2}\right] dy = 3\int_0^1 y^2 dy - 3\int_0^1 y^2(0{,}9-y)^2 dy$

$= 1 - 3\int_0^1 y^2(0{,}9^2 + y^2 - 2\cdot 0{,}9\cdot y) dy = 1 - 3\int_0^1 \frac{81\cdot y^2}{100} + y^4 - \frac{9}{5}y^3 dy$

$= 1 - \frac{3\cdot 81}{100}\int_0^1 y^2 dy + 3\int_0^1 y^4 dy - 3\cdot\frac{9}{5}\int_0^1 y^3 dy$

$= 1 - \frac{81}{100} - \frac{3}{5} + \frac{27}{20} = \frac{47}{50}$ ok

c) $f_X(x) = \int_0^1 6xy^2 dy = 6x\int_0^1 y^2 dy = 6x\left[\frac{y^3}{3}\right]_0^1 = \frac{6x}{3} = 2x, \ 0<x<1$ ok

$P(0{,}5<X<1) = \int_{\frac{1}{2}}^1 2x\,dx = x^2\Big|_{\frac{1}{2}}^1 = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$P(0<X<0{,}5) = \int_0^{\frac{1}{2}} 2x\,dx = x^2\Big|_0^{\frac{1}{2}} = \frac{1}{4}$

$[P(0{,}5<X<1) - P(0<X<0{,}5)] = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ ok

The outcome is a posetive number. Since $0<x<1$ this is reasonable.

Poäng: (Points)

Tentamensservice/SU/typ_c

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Lärarens
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(*Teacher's*
note)

Poäng:
(*Points*)

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Uppg.nr.: (Task no.) 3

Lärarens kommentar: (Teacher's note)

$$f(x,y) = \begin{cases} x+y, & 0<x<1,\ 0<y<1 \\ 0, & \text{otherwise} \end{cases}$$

$$\int_0^1\int_0^1 x+y\,dx\,dy = \int_0^1 \frac{x^2}{2} + xy \Big|_0^1 dy = \int_0^1 \frac{1}{2} + y\,dy = \frac{y}{2} + \frac{y^2}{2}\Big|_0^1$$

$$= \frac{1}{2} + \frac{1}{2} = 1 //$$ ok

a) $Z = X+Y$

$$f_Z(z) = \int_0^z (z-y+y)\,dy = \int_0^z z\,dy = z\cdot y\Big|_0^z = z^2, \quad 0<z<1$$

$$f_Z(z) = \int_z^2 z\,dy = z\cdot y\Big|_z^2 = 2z - z^2, \quad 1\le z\le 2$$

$$f_Z(z) = \begin{cases} z^2, & 0<z<1 \\ 2z-z^2, & 1\le z\le 2 \\ 0, & \text{otherwise} \end{cases}$$ ok

b) $$P(X+Y>1{,}5) = \int_{1,5}^2 (2z-z^2)\,dz = z^2 - \frac{z^3}{3}\Big|_{1,5}^2 = \left(2^2 - \frac{2^3}{3}\right) - \left((1{,}5)^2 - \frac{(1{,}5)^3}{3}\right)$$

$$= 4 - \frac{8}{3} - \frac{9}{4} + \frac{9}{8} = 4 - \frac{8}{3} - \frac{9}{8} = 4 - \frac{64}{24} - \frac{27}{24} = \frac{96}{24} - \frac{64}{24} - \frac{27}{24} = \frac{5}{24} //$$

ok

Poäng: (Points)

Tentamensservice/SU/typ_c

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(*Points*)

Uppg.nr.: (Task no.) 4

Lärarens kommentar: (Teacher's note)

$$f(x) = \begin{cases} \frac{x+2}{3}, & 1 < x < 4 \\ 0, & \text{otherwise} \end{cases}$$

- is it a pdf ?! (−)

a) Find a monotone function $u(x)$ such that the random variable $Y = u(X) \sim \text{Uniform}(0,1)$

$F_X(X) = u(X)$

$$\int_1^x \frac{t+2}{3}\,dt = \frac{(t+2)^2}{9}\Big|_1^x = \frac{(X+2)^2}{9} - \frac{(1+2)^2}{9}$$

$$= \frac{X^2+2^2+4X}{9} - \frac{1+4+4}{9} = \frac{X^2+4+4X-1-4-4}{9}$$

$$= \frac{X^2+4X-5}{9}$$

$$F_X(X) = \begin{cases} 0, & X \le 1 \\ \frac{X^2+4X-5}{9}, & 1 < X < 4 \\ 1, & X \ge 4 \end{cases}$$

$x = 4$: $\frac{16+16-5}{9} = 3$?? (±) (2p)

b) $f(x) = \frac{1}{2\beta} e^{-\frac{|x-\alpha|}{\beta}}, \quad -\infty < X, \alpha < +\infty, \beta > 0$

This is the double exponential distribution, hence the function is a pdf.

$$M_X(t) = \frac{e^{\alpha t}}{1-(\beta t)^2}, \quad |t| < \frac{1}{\beta}$$

(−) see ex, 2.30 c)

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{tx} \frac{1}{2\beta} e^{-\frac{|x-\alpha|}{\beta}}\,dx\,d\alpha = \frac{1}{2\beta}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-x\left(-t-\frac{1}{\beta}-\frac{\alpha}{x\beta}\right)}\,dx\,d\alpha$$

(−)

$$= \frac{1}{2\beta}\int$$

Poäng: (Points)

⟹

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c) $P(|\bar{X}_1 - \bar{X}_2| < \sigma) \approx 0{,}9$

Since from same population

$\mathbb{E}[\bar{X}_1 - \bar{X}_2] = 0$ ok

$V(\bar{X}_1 - \bar{X}_2) = 2\sqrt{n}$?

$P\left(\frac{-\sigma}{2\sqrt{n}\,\delta} < Z < \frac{\sigma}{2\sqrt{n}\,\delta}\right) \approx 0{,}9$

~~no~~ dependence on "δ"?

$Var(\bar{X}_1) = n\delta^2$

$\Downarrow$

$Var(\bar{\bar{X}}_1 - \bar{\bar{X}}_2) = 2n\delta^2$

From appendix: $\frac{\sigma}{2\sqrt{n}} \approx 1{,}6449$

?

$\bar{n} = ?$

$\Rightarrow \frac{2\sqrt{n}}{\sigma} = \frac{1}{1{,}6449}$

(+) close enough:

(2p)

Lärarens kommentar: (Teacher's note)

Poäng: (Points)

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Uppg.nr.: (Task no.) 6

Lärarens kommentar: (Teacher's note)

A = Time A arrives B = Time b arrives X = Time A waits for B.

$f_a(a) = \frac{1}{24}$, $f_b(b) = \frac{1}{24}$ Because of independence $f(a,b) = \frac{1}{576}$

$$P(B-A \leq 0) = P(B \leq A) = \int_0^{24}\int_0^{a} \frac{1}{576}\, db\, da = \int_0^{24} \frac{b}{576}\Big|_0^a da = \int_0^{24} \frac{a}{576} da$$

$$= \frac{a^2}{2\cdot 576}\Big|_0^{24} = \frac{24^2}{1152} = \frac{1}{2}$$ /ok

$$P(B-A \leq x) = 1 - P(B-A > x) = 1 - \int_0^{24}\int_{x+a}^{x-24} \frac{1}{576}\, db\, da = 1 - \int_0^{24} \frac{b}{576}\Big|_{x+a}^{x-24} da$$

$$= 1 - \int_0^{24} \frac{x-24}{576} - \frac{x+a}{576} da = 1 - \frac{1}{576}\int_0^{24} x-24-(x+a)\, da$$

$$= 1 - \frac{1}{576}\left[xa - 24a - xa - \frac{a^2}{2}\right]_0^{24} = 1 - \frac{1}{576}\left[-24a - \frac{a^2}{2}\right]_0^{24}$$

$$1 - \frac{1}{576}\left[-576 - 288\right] = 1 - \left[-1 + \frac{1}{2}\right]$$ (−)

$$F_X(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 24 \\ \frac{1}{2}, & x = 0 \\ ?, & 0 < x < 24 \end{cases}$$

4 possible outcomes

① $x < 0$

② $x \leq 24$ (−)

③ $x = 0$

④ $0 < x < 24$

Poäng: (Points)

Tentamensservice/SU/typ_c

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Uppg.nr.: (Task no.) 7

Lärarens kommentar: (Teacher's note)

$$f(x_1,x_2,x_3,x_4)=\begin{cases}\frac{3}{4}(x_1^2+x_2^2+x_3^2+x_4^2), & 0<x_i<1,\ i=1,2,3,4\\ 0, & \text{otherwise}\end{cases}$$

$$\frac{3}{4}\int_0^1\int_0^1\int_0^1\int_0^1 x_1^2\,dx_1dx_2dx_3dx_4=\frac{3}{4}\int_0^1\int_0^1\int_0^1 \frac{x_1^3}{3}\Big|_0^1 dx_2dx_3dx_4$$

$$=\frac{3}{4}\int_0^1\int_0^1\int_0^1 \frac{1}{3}dx_2dx_3dx_4=\frac{3}{4}\int_0^1\int_0^1 \frac{x_2}{3}\Big|_0^1 dx_3dx_4=\frac{3}{4}\int_0^1\int_0^1\frac{1}{3}dx_3dx_4$$

$$=\frac{3}{4}\int_0^1\frac{x_3}{3}\Big|_0^1 dx_4=\frac{3}{4}\int_0^1\frac{1}{3}dx_4=\frac{3}{4}\left[\frac{x_4}{3}\right]_0^1=\frac{3}{4}\cdot\frac{1}{3}=\frac{1}{4}$$

By symmetry: $\frac{3}{4}\int_0^1\int_0^1\int_0^1\int_0^1 x_2^2\,dx_1dx_2dx_3dx_4=\frac{1}{4}$

$$\frac{3}{4}\int_0^1\int_0^1\int_0^1\int_0^1 x_3^2\,dx_1dx_2dx_3dx_4=\frac{1}{4}$$

$$\frac{3}{4}\int_0^1\int_0^1\int_0^1\int_0^1 x_4^2\,dx_1dx_2dx_3dx_4=\frac{1}{4}$$

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{4}{4}=1$$

ok

a) $$\frac{3}{4}\int_{1/2}^1\int_0^1\int_0^1\int_0^{3/4} x_1^2\,dx_1dx_2dx_3dx_4=\frac{3}{4}\int_{1/2}^1\int_0^1\int_0^{3/4}\frac{x_1^3}{3}\Big|_0^1 dx_2dx_3dx_4=\frac{3}{4}\int_{1/2}^1\int_0^{3/4}\frac{x_2}{3}\Big|_0 dx_3dx_4$$

$$=\frac{3}{4}\int_{1/2}^1\int_0^1\frac{1}{4}dx_3dx_4=\frac{3}{4}\int_{1/2}^1\frac{x_3}{4}\Big|_0^1 dx_4=\frac{3}{4}\left[\frac{x_4}{4}\right]_{1/2}^1=\frac{3}{4}\left[\frac{1}{4}-\frac{\frac{1}{2}}{4}\right]=\frac{3}{32}$$

$$\frac{3}{4}\int_{1/2}^1\int_0^1\int_0^1\int_0^{3/4} x_2^2\,dx_2dx_1dx_3dx_4=\frac{3}{4}\int_{1/2}^1\int_0^1\int_0^1\frac{x^3}{3}\Big|_0^{3/4}dx_1dx_3dx_4=\frac{3}{4}\int_{1/2}^1\int_0^1\frac{9x_1}{64}\Big|_0^1 dx_3dx_4$$

$$=\frac{3}{4}\int_{1/2}^1\int_0^1\frac{9}{64}dx_3dx_4=\frac{3}{4}\int_{1/2}^1\frac{9}{64}dx_4=\frac{3}{4}\left[\frac{9x_4}{64}\right]_{1/2}^1=\frac{3}{4}\left[\frac{9}{64}-\frac{9}{128}\right]=\frac{27}{512}$$

→

Poäng: (Points)

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Lärarens kommentar: (Teacher's note)

$$\frac{3}{4}\int_{\frac{1}{2}}^{1}\int_{0}^{3/4}\int_{0}^{1}\int_{0}^{1} x_3^2 \, dx_3 dx_1 dx_2 dx_4 = \frac{3}{4}\int_{\frac{1}{2}}^{1}\int_{0}^{3/4}\int_{0}^{1}\int \frac{x_3^3}{3}\Big|_0^1 dx_1 dx_2 dx_4 = \frac{3}{4}\int_{\frac{1}{2}}^{1}\int_{0}^{3/4} \frac{x_1}{3}\Big|_0^1 dx_2 dx_4$$

$$= \frac{3}{4}\int_{\frac{1}{2}} \frac{x_2}{3}\Big|_0^{\frac{3}{4}} = \frac{3}{4}\int_{\frac{1}{2}}^{1} \frac{1}{4} dx_4 = \frac{3}{4}\left[\frac{1}{4} - \frac{1}{8}\right] = \frac{3}{32}$$

$$\frac{3}{4}\int_0^1\int_0^{\frac{3}{4}}\int_0^1\int_{\frac{1}{2}}^1 x_4^2 \, dx_4 dx_3 dx_2 dx_1 = \frac{3}{4}\int_0^1\int_0^{\frac{3}{4}}\int_0^1 \frac{x_4^3}{3}\Big|_{\frac{1}{2}}^1 dx_3 dx_2 dx_1 = \frac{3}{4}\int_0^1\int_0^{\frac{3}{4}} \frac{x_3 7}{24}\Big|_0^1 dx_2 dx_1$$

$$= \frac{3}{4}\int_0^1 \frac{7x_2}{24}\Big|_0^{\frac{3}{4}} dx_1 = \frac{3}{4}\left[\frac{7}{32}\right] = \frac{21}{127}$$

Answer:

$$\frac{21}{127} + \frac{3}{32} + \frac{27}{512} + \frac{3}{32} = \frac{3249}{8128}$$

5p

b) $$f(x_2, x_3) = \int_0^1\int_0^1 \frac{3}{4}(x_1^2 + x_2^2 + x_3^2 + x_4^2) \, dx_1 dx_4 = \frac{3}{4}\int_0^1 \frac{x_1^3}{3} + x_2^2 \cdot x_1 + x_3^2 \cdot x_1 + x_4^2 \cdot x_1 \Big|_0^1 dx_4$$

$$= \frac{3}{4}\int_0^1 \frac{1}{3} + x_2^2 + x_3^2 + x_4^2 \, dx_4 = \frac{3}{4}\left[\frac{x_4}{3} + x_2^2 \cdot x_4 + x_3^2 \cdot x_4 + \frac{x_4^3}{3}\right]_0^1$$

$$= \frac{3}{4}\left[\frac{1}{3} + x_2^2 + x_3^2 + \frac{1}{3}\right] = \frac{1}{2} + \frac{x_2^2 + x_2^2 \cdot 3}{4}$$

$$E[x_2 x_3] = \int_0^1\int_0^1\int_0^1\int_0^1 x_2 \cdot x_3 \cdot \frac{3}{4}(x_1^2 + x_2^2 + x_3^2 + x_4^2) \, dx_1 dx_2 dx_3 dx_4$$

$$x_2 x_3 \frac{3}{4}\int_0^1\int_0^1\int_0^1 \frac{x_1^3}{3} + x_2^2 + x_3^2 + x_4^2 \Big|_0^1 dx_2 dx_3 dx_4 = x_3 \frac{3}{4}\int_0^1\int_0^1 \frac{x_2}{} + \frac{x_2^3}{3} + x_3 \cdot x_2 + x_4 \cdot x_2 \Big|_0^1 dx_3 dx_4$$

(+)

$=$

Poäng: (Points)

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Lärarens kommentar: (Teacher's note)

$$f(x,y) = \begin{cases} c \cdot xy(1-y^2), & 0<x<1,\ 0<y<1 \\ 0, & \text{otherwise} \end{cases}$$

① $\int_0^1\int_0^1 c \cdot xy(1-y^2)\,dx\,dy = 1$

$= \int_0^1\int_0^1 cxy - cxy^3\,dx\,dy = 1$

$= \int_0^1\int_0^1 cxy\,dx\,dy - \int_0^1\int_0^1 cxy^3\,dx\,dy = 1$

$= c\int_0^1 y \cdot \int_0^1 x\,dx\,dy - c\int_0^1 y^3 \int_0^1 x\,dx\,dy = 1$

$= c\int_0^1 y\left[\frac{x^2}{2}\right]_0^1 dy - c\int_0^1 y^3\left[\frac{x^2}{2}\right] dy = 1$

$= \frac{c}{2}\left[\frac{y^2}{2}\right]_0^1 - \frac{c}{2}\left[\frac{y^4}{4}\right]_0^1 = 1$

$= \frac{c}{4} - \frac{c}{8} = 1 \longrightarrow \frac{2c}{8} - \frac{c}{8} = 1$

$c = 8$ // OK

$u = g_1(x,y) = x \cdot y$ $\quad x = h_1(u,v) = \frac{u}{v}$

$v = g_2(x,y) = y$ $\quad y = h_2(u,v) = v$

② $J = \begin{vmatrix} \frac{1}{v} & -\frac{u}{v^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v} - \left(-\frac{u}{v^2} \cdot 0\right) = \left|\frac{1}{v}\right|$

③ $f_{UV}(u,v) = \left(8 \cdot \frac{u}{v} \cdot v - 8\frac{u}{v} \cdot v^3\right) \cdot \frac{1}{v} = (8u - 8u \cdot v^2)\frac{1}{v}$

$= 8\frac{u}{v} - 8u \cdot v$

④ $f_U(u) = \int_1^u 8\frac{u}{v} - 8uv\,dv = 8u\int_1^u \frac{1}{v}dv - 8u\int_1^u v\,dv = 8u[\ln(v)]_1^u - 8u\left[\frac{v^2}{2}\right]_1^u$

$= 8u\left(\ln(u) - \ln(1) - \left(\frac{u^2}{2} - \frac{1}{2}\right)\right) = 8u\left(\ln(u) - \frac{u^2}{2} + \frac{1}{2}\right)$ // (TG) (+)

Poäng: (Points)

Tentamensservice/SU/typ_c

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Regler i skrivsalen

- Följ tentamensvärds anvisningar.
- Väskor och ytterkläder ska placeras på anvisad plats.
- Placera ID-handling väl synlig på bordet framför dig.
- Ingen student får lämna skrivsalen under de första 30 minuterna.
- Endast en student i taget får besöka toaletten. Vid toalettbesök skriv ditt namn och klockslag på avsedd lista. Efter toalettbesöket ska du åter ange klockslag på listan.
- Elektronisk utrustning som mobiltelefon eller Smartwatch ska vara avstängd och placerad på anvisad plats.
- Under tentamen gäller tystnad – det är förbjudet att prata, eller på annat sätt kommunicera, med andra studenter under pågående tentamen.
- Innan tentamenshandlingarna lämnas in; skriv sidnummer, anonymiseringskod och datum på alla inlämnade papper.

Om något är oklart – fråga gärna tentamensvärden. Lycka till!

Rules in the examination hall

- Follow the invigilator's instructions.
- Bags and outerwear must be placed at the designated place.
- Place your ID document clearly visible on the table in front of you.
- No student may leave the examination hall for the first 30 minutes.
- Only one student at a time may visit the toilet. Before visiting the toilet, write your name and time on the intended list. After the toilet visit, enter the time on the list again.
- Electronic equipment such as a mobile phone or Smartwatch must be switched off and placed at the designated place.
- During the exam, silence applies – you are not allowed to talk, or otherwise communicate, with other students during the exam.
- Before submitting the examination documents; remember to write the page number, anonymization code, and date on all papers.

Please do not hesitate to ask the invigilator if anything is unclear. Good luck!

Tentamensservice
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