Stockholms universitet

STATISTISKA INSTITUTIONEN HT-2021

Written Re-Exam in Probability Theory, 7.5 ECTS credits
Tuesday, 4^{th} of January 2022, 08:00 – 13:00
Examination: On-campus Exam

You are asked to answer below stated questions and to motivate your solutions. Grades are assigned as follows: **A** (91+), **B** (75-90), **C** (66-74), **D** (58-65), **E** (50-57), **Fx** (30-49), and **F** (0-29)

You are **allowed** to use any calculator. Other supplementary material is attached to your exam questions.

The teacher reserves the right to further examine the students on the answers provided.

1. (12 points) Let X and Y be *i.i.d* standard normal random variables, and $U:=X+Y$; $V:=X-Y$
   a) Calculate the joint pdf: $f_{U,V}(u,v)$
   b) Calculate the $Corr(U,V)$
   c) Check if U and V are independent

2. (12 points)

a) Let random variable X have a pdf $f(x) = \begin{cases} \frac{2x-1}{6}, & 1 < x < 3 \\ 0, & otherwise \end{cases}$

Find a monotone function $u(x)$ such that the random variable $Y=u(X)$ has uniform $U(0,1)$ distribution.

b) Check that below stated function is a pdf and calculate its moment generating function $f(x) = \frac{1}{2\beta}\exp\{\frac{-|x-a|}{\beta}\}, -\infty < x, \alpha < \infty, \beta > 0$. Provide detailed explanations.

3. (12 points) Let the joint pdf of (X,Y) be $f(x,y)=1$, $0<x<1$, $x<y<x+1$. Find
   a) marginals of X and Y and their first two moments;
   b) $Corr(X,Y)$

4. (10 points) One tosses two dice: the outcomes are the numbers from 1 to 6. Let X be the "outcome" on the first dice and Y is the minimum of the two. Find joint distribution of (X,Y) and calculate $E[X]$, $E[Y]$, $Var(X)$, $Var(Y)$, and $Cov(X,Y)$

5. (10 points) Let $f(x,y) = \begin{cases} x+y, & 0 < x < 1, \quad 0 < y < 1 \\ 0, & otherwise \end{cases}$
   a) Find the probability density function of $X+Y$
   b) Calculate $P(X+Y \leq 0.5)$

6. (10 points) (Weak Law of Large Numbers) Let $X_1, X_2, \ldots$ be iid random variables with mean μ and finite variance σ^2. Prove, that sample mean of the sample converges in probability: $\overline{X_n} \overset{p}{\Rightarrow}$ μ.

7. (12 points) For the hierarchical model *Y|Λ~Poisson(Λ)* and *Λ~Gamma(α,β),* find the marginal distribution, mean, and variance of *Y.* Show that the marginal distribution of *Y* is a negative binomial if α is an integer.

8. (10 points) Let $X_1, \ldots, X_n$ be $i.i.d\, N(\mu, \sigma^2)$ distributed random sample. Let us further assume that you have already proven that $\bar{X}$ and S are independent. Derive distribution of $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$, where $\bar{X}, S$ denote sample mean and sample variance of the above sample.

9. (12 points) *Let* continuous random vector *(X,Y)* have a joint pdf: $f(x,y) = e^{-y}$, $0 < x < y < \infty$. Compute the following:

   a) $f_X(x)$ and write explicitly *f(x,y)* as one formula that contains both domain and function expression of the pdf
   b) *f(Y | X), (Y | X = x);* $E[Y \mid X = x]$; $Var(Y \mid X = x)$
   c) *P(X+Y>1)*

Good Luck

FORMULA SHEET

CHAPTER 1

	With replacement	Without replacement
Ordered	$\frac{n!}{(n-r)!}$	n^r
Unordered	$\binom{n}{r}$	$\binom{n+r-1}{r}$

Bonferroni's inequality

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$

CHAPTER 2

Univariate transformations:

X is a discrete r.v. Let $Y = g(X)$, then Y has the following pmf:

$$f_y(y) = \sum_{x \in g^{-1}(y)} f_X(x)$$

X is a continuous r.v with domain χ. Let $Y = g(X)$

	Probability density function of Y
If $g(x)$ is monotone for $x \in \chi$	$f_Y(y) = f_X(g^{-1}(y)) \left\lvert \frac{d}{dy} g^{-1}(y) \right\rvert$
If $g(x)$ is not monotone for $x \in \chi$. But $g_i(x) = g(x)$ for $x \in A_i$ is monotone on A_i.	$f_Y(y) = \sum_{i=1}^{k} f_X\left(g_i^{-1}(y)\right) \left\lvert \frac{d}{dy} g_i^{-1}(y) \right\rvert$

	Cumulative density function of Y
If $g(x)$ ↑↑ for all $x \in \chi$	$F_Y(y) = F_X(g^{-1}(y))$
If $g(x)$ ↓↓ for all $x \in \chi$	$F_Y(y) = 1 - F_X(g^{-1}(y))$

The expected value of a r.v $g(X)$:

$$E[g(X)] \begin{cases} \int_{\infty}^{\infty} g(x) f_X(x) dx & \text{, if } X \text{ is continuous} \\ \sum_{x \in \chi} g(x) f_X(x) & \text{, if } X \text{ is discrete} \end{cases}$$

The moment generating function for a r.v X is given by:

$$M_X(t) = E(e^{tX})$$

The $n{:}th$ moment of X:

$$E(X^n) = M_X^n(0) = \frac{d^n}{dt^n} M_X(t)|_{t=0}$$

Leibnitz Formula (Change order of $\int$ and $\frac{d}{dx}$)

$$\frac{d}{d\theta} \int_{a(\theta)}^{b(\theta)} f(x,\theta) dx = f(b(\theta),\theta) \cdot \frac{d}{d\theta} b(\theta) - f(a(\theta),\theta) \frac{d}{d\theta} a(\theta) + \int_{a(\theta)}^{b(\theta)} \frac{d}{d\theta} f(x,\theta) dx$$

CHAPTER 3

Chebyshev's Inequality: Let X be a random variable, $g(x) \geq 0, \forall x \geq 0$ and $\forall r > 0$

$$P(g(X) \geq r) \leq \frac{E[g(X)]}{r}$$

Exponential families:

A family of pdf's or pmf's is called an exponential family if it can be expressed as:

$$f(x|\boldsymbol{\theta}) = h(x) c(\boldsymbol{\theta}) \exp\left\{\sum_{i=1}^{k} w_i(\boldsymbol{\theta}) t_i(x)\right\}$$

Binomial Theorem:

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

Gamma function:

$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$; $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$; $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$; If α is an integer: $\Gamma(n) = (n-1)!$

CHAPTER 4

	Discrete	Continuous
Joint pdf/pmf	$P((X,Y) \in A) = \sum_{(x,y)\in A} f(x,y)$	$P((X,Y) \in A) = \int_A\int f_{X,Y}(x,y)dxdy$
Marginal pdf/pmf	$f_X(x) = \sum_y f_{X,Y}(x,y)$ $f_y(y) = \sum_x f_{X,Y}(x,y)$	$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$ $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx$
Conditional pdf/pmf	$f(y\|x) = \frac{f(x,y)}{f_X(x)}$; $f(x\|y) = \frac{f(x,y)}{f_y(y)}$	

Bivariate transformations:
Let $(X,Y) \sim f(x,y)$. Suppose the functions $u = g_1(x,y)$ and $v = g_2(x,y)$ have the inverse functions $x = h_1(u,v)$ and $y = h_2(u,v)$.

The joint pdf for (U,V) is given by:

$$f_{U,V}(u,v) = f_{X,Y}\big(h_1(u,v), h_2(u,v)\big)|J|$$

Where:

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

The expected value of $g(X,Y)$

$$E\big(g(X,Y)\big) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f(x,y)dxdy$$

Conditional expected value of $g(Y)$ given $X = x$

$$E[g(Y)|x] = \begin{cases} \sum_y g(y)f(y|x) & \text{, discrete case} \\ \int_{-\infty}^{\infty} g(y)f(y|x)dy & \text{, continuous case} \end{cases}$$

Conditional variance of Y given $X = x$

$$V[Y|x] = E[Y^2|x] - E[Y|x]^2$$

Lemma 4.2.7: Let $(X,Y) \sim f(x,y)$. X and Y are independent if and only if there exist functions $g(x)$ and $h(y)$ s.t $f(x,y) = g(x)h(y)\ \forall x,y \in R$

Hierarchical Models:
If X and Y are any two random variables then:

$$E(X) = E[E(X|Y)]$$
$$V(X) = E[V(X|Y)] + V[E(X|Y)]$$

Jensen's inequality: For any r.v X, if $g(x)$ is a convex function then:

$$E[g(X)] \geq g[E(X)]$$

Cauchy-Schwartz inequality: For any two r.v X and Y

$$|E(XY)| \leq E[|XY|] \leq (E|X|^2)^{1/2}(E|Y|^2)^{1/2}$$

Covariance/Correlation

$$Cov(X,Y) = E(XY) - E(X)E(Y) \quad ; \quad Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X\sigma_Y}$$

CHAPTER 5

Convergence in distribution	$X_n \xrightarrow{d} X$	$\Leftrightarrow$	$\lim_{n\to\infty} F_{X_n}(x) = F_X(x)$
Convergence in probability	$X_n \xrightarrow{p} X$	$\Leftrightarrow$	$\lim_{n\to\infty} P(\omega: \|X_n - X\| < \epsilon) = 1 \quad, \forall \epsilon > 0$
Almost sure convergence	$X_n \xrightarrow{a.s.} X$	$\Leftrightarrow$	$P\left(\omega: \lim_{n\to\infty} \|X_n - X\| < \epsilon\right) = 1 \quad, \forall \epsilon > 0$

Delta Method:

If Y_n is a sequence of random variables that satisfies $\sqrt{n}(Y_n - \theta) \xrightarrow{d} N(0,\sigma^2)$. Then for a given function g and a specific value of θ (where $g'(\theta)$ exists):

$$\sqrt{n}[g(Y_n) - g(\theta)] \xrightarrow{d} N(0, \sigma^2[g'(\theta)]^2)$$

EXTRA (USEFULL SUMS)

$$\sum_{k=0}^{\infty} p^k = \frac{1}{1-p} \qquad \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \qquad \sum_{x=0}^{n-1} ak^x = a\frac{k^n - 1}{k-1} \quad k \neq 1$$

$$\sum_{n=0}^{\infty} ak^n = \frac{a}{1-k} \quad |k| < 1 \qquad \sum_{x=1}^{n} x = \frac{n(n+1)}{2} \qquad \sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$$

Table of Common Distributions

Discrete Distributions

Bernoulli(p)

pmf $P(X=x|p)=p^x(1-p)^{1-x};\quad x=0,1;\quad 0\le p\le 1$

mean and variance $\mathrm{E}X=p,\quad \mathrm{Var}\,X=p(1-p)$

mgf $M_X(t)=(1-p)+pe^t$

Binomial(n,p)

pmf $P(X=x|n,p)=\binom{n}{x}p^x(1-p)^{n-x};\quad x=0,1,2,\ldots,n;\quad 0\le p\le 1$

mean and variance $\mathrm{E}X=np,\quad \mathrm{Var}\,X=np(1-p)$

mgf $M_X(t)=[pe^t+(1-p)]^n$

notes Related to Binomial Theorem (Theorem 3.2.2). The *multinomial* distribution (Definition 4.6.2) is a multivariate version of the binomial distribution.

Discrete uniform

pmf $P(X=x|N)=\frac{1}{N};\quad x=1,2,\ldots,N;\quad N=1,2,\ldots$

mean and variance $\mathrm{E}X=\frac{N+1}{2},\quad \mathrm{Var}\,X=\frac{(N+1)(N-1)}{12}$

mgf $M_X(t)=\frac{1}{N}\sum_{i=1}^{N}e^{it}$

Geometric(p)

pmf $P(X=x|p)=p(1-p)^{x-1};\quad x=1,2,\ldots;\quad 0\le p\le 1$

mean and variance $\mathrm{E}X=\frac{1}{p},\quad \mathrm{Var}\,X=\frac{1-p}{p^2}$

mgf	$M_X(t) = \frac{pe^t}{1-(1-p)e^t}, \quad t < -\log(1-p)$
notes	$Y = X - 1$ is negative binomial$(1, p)$. The distribution is *memoryless*: $P(X > s \mid X > t) = P(X > s - t)$.

Hypergeometric

pmf	$P(X = x \mid N, M, K) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}; \quad x = 0, 1, 2, \ldots, K;$ $M - (N - K) \le x \le M; \quad N, M, K \ge 0$
mean and variance	$\mathrm{E}X = \frac{KM}{N}, \quad \mathrm{Var}\, X = \frac{KM}{N}\frac{(N-M)(N-K)}{N(N-1)}$
notes	If $K \ll M$ and N, the range $x = 0, 1, 2, \ldots, K$ will be appropriate.

Negative binomial(r, p)

pmf	$P(X = x \mid r, p) = \binom{r+x-1}{x} p^r (1-p)^x; \quad x = 0, 1, \ldots; \quad 0 \le p \le 1$
mean and variance	$\mathrm{E}X = \frac{r(1-p)}{p}, \quad \mathrm{Var}\, X = \frac{r(1-p)}{p^2}$
mgf	$M_X(t) = \left(\frac{p}{1-(1-p)e^t}\right)^r, \quad t < -\log(1-p)$
notes	An alternate form of the pmf is given by $P(Y = y \mid r, p) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$, $y = r, r+1, \ldots$. The random variable $Y = X + r$. The negative binomial can be derived as a gamma mixture of Poissons. (See Exercise 4.32.)

Poisson(λ)

pmf	$P(X = x \mid \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; \quad x = 0, 1, \ldots; \quad 0 \le \lambda < \infty$
mean and variance	$\mathrm{E}X = \lambda, \quad \mathrm{Var}\, X = \lambda$
mgf	$M_X(t) = e^{\lambda(e^t - 1)}$

Continuous Distributions

Beta(α, β)

pdf	$f(x \mid \alpha, \beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \le x \le 1, \quad \alpha > 0, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \frac{\alpha}{\alpha+\beta}, \quad \mathrm{Var}\, X = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
mgf	$M_X(t) = 1 + \sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1}\frac{\alpha+r}{\alpha+\beta+r}\right)\frac{t^k}{k!}$
notes	The constant in the beta pdf can be defined in terms of gamma functions, $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$. Equation (3.2.18) gives a general expression for the moments.

Cauchy(θ, σ)

pdf	$f(x \mid \theta, \sigma) = \frac{1}{\pi\sigma}\frac{1}{1+\left(\frac{x-\theta}{\sigma}\right)^2}, \quad -\infty < x < \infty; \quad -\infty < \theta < \infty, \quad \sigma > 0$
mean and variance	do not exist
mgf	does not exist
notes	Special case of Student's t, when degrees of freedom = 1. Also, if X and Y are independent $\mathrm{n}(0,1)$, X/Y is Cauchy.

Chi squared(p)

pdf	$f(x \mid p) = \frac{1}{\Gamma(p/2)2^{p/2}} x^{(p/2)-1} e^{-x/2}; \quad 0 \le x < \infty; \quad p = 1, 2, \ldots$
mean and variance	$\mathrm{E}X = p, \quad \mathrm{Var}\, X = 2p$
mgf	$M_X(t) = \left(\frac{1}{1-2t}\right)^{p/2}, \quad t < \frac{1}{2}$
notes	Special case of the gamma distribution.

Double exponential(μ, σ)

pdf	$f(x \mid \mu, \sigma) = \frac{1}{2\sigma} e^{-\lvert x-\mu \rvert/\sigma}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$
mean and variance	$\mathrm{E}X = \mu, \quad \mathrm{Var}\, X = 2\sigma^2$
mgf	$M_X(t) = \frac{e^{\mu t}}{1-(\sigma t)^2}, \quad \lvert t \rvert < \frac{1}{\sigma}$
notes	Also known as the *Laplace* distribution.

Exponential(β)

pdf $f(x|\beta) = \frac{1}{\beta}e^{-x/\beta}, \quad 0 \le x < \infty, \quad \beta > 0$

mean and variance $\mathrm{E}X = \beta, \quad \mathrm{Var}\, X = \beta^2$

mgf $M_X(t) = \frac{1}{1-\beta t}, \quad t < \frac{1}{\beta}$

notes Special case of the gamma distribution. Has the *memoryless* property. Has many special cases: $Y = X^{1/\gamma}$ is *Weibull*, $Y = \sqrt{2X/\beta}$ is *Rayleigh*, $Y = \alpha - \gamma \log(X/\beta)$ is *Gumbel*.

F

pdf $f(x|\nu_1,\nu_2) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}\frac{x^{(\nu_1-2)/2}}{\left(1+\left(\frac{\nu_1}{\nu_2}\right)x\right)^{(\nu_1+\nu_2)/2}};$
$0 \le x < \infty; \quad \nu_1,\nu_2 = 1,\ldots$

mean and variance $\mathrm{E}X = \frac{\nu_2}{\nu_2-2}, \quad \nu_2 > 2,$

$\mathrm{Var}\, X = 2\left(\frac{\nu_2}{\nu_2-2}\right)^2\frac{(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}, \quad \nu_2 > 4$

moments (mgf does not exist) $\mathrm{E}X^n = \frac{\Gamma\left(\frac{\nu_1+2n}{2}\right)\Gamma\left(\frac{\nu_2-2n}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\left(\frac{\nu_2}{\nu_1}\right)^n, \quad n < \frac{\nu_2}{2}$

notes Related to chi squared ($F_{\nu_1,\nu_2} = \left(\frac{\chi^2_{\nu_1}}{\nu_1}\right)\Big/\left(\frac{\chi^2_{\nu_2}}{\nu_2}\right)$, where the χ^2s are independent) and t ($F_{1,\nu} = t^2_\nu$).

Gamma(α,β)

pdf $f(x|\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}, \quad 0 \le x < \infty, \quad \alpha,\beta > 0$

mean and variance $\mathrm{E}X = \alpha\beta, \quad \mathrm{Var}\, X = \alpha\beta^2$

mgf $M_X(t) = \left(\frac{1}{1-\beta t}\right)^\alpha, \quad t < \frac{1}{\beta}$

notes Some special cases are exponential ($\alpha = 1$) and chi squared ($\alpha = p/2$, $\beta = 2$). If $\alpha = \frac{3}{2}$, $Y = \sqrt{X/\beta}$ is *Maxwell*. $Y = 1/X$ has the *inverted gamma distribution*. Can also be related to the Poisson (Example 3.2.1).

Logistic(μ,β)

pdf $f(x|\mu,\beta) = \frac{1}{\beta}\frac{e^{-(x-\mu)/\beta}}{[1+e^{-(x-\mu)/\beta}]^2}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \beta > 0$

mean and variance $\mathrm{E}X = \mu, \quad \mathrm{Var}\, X = \frac{\pi^2\beta^2}{3}$

mgf $M_X(t) = e^{\mu t}\Gamma(1-\beta t)\Gamma(1+\beta t), \quad |t| < \frac{1}{\beta}$

notes The cdf is given by $F(x|\mu,\beta) = \frac{1}{1+e^{-(x-\mu)/\beta}}$.

Lognormal(μ,σ^2)

pdf $f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\frac{e^{-(\log x-\mu)^2/(2\sigma^2)}}{x}, \quad 0 \le x < \infty, \quad -\infty < \mu < \infty,$
$\sigma > 0$

mean and variance $\mathrm{E}X = e^{\mu+(\sigma^2/2)}, \quad \mathrm{Var}\, X = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$

moments (mgf does not exist) $\mathrm{E}X^n = e^{n\mu+n^2\sigma^2/2}$

notes Example 2.3.5 gives another distribution with the same moments.

Normal(μ,σ^2)

pdf $f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma^2)}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty,$
$\sigma > 0$

mean and variance $\mathrm{E}X = \mu, \quad \mathrm{Var}\, X = \sigma^2$

mgf $M_X(t) = e^{\mu t+\sigma^2t^2/2}$

notes Sometimes called the *Gaussian* distribution.

Pareto(α,β)

pdf $f(x|\alpha,\beta) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad a < x < \infty, \quad \alpha > 0, \quad \beta > 0$

mean and variance $\mathrm{E}X = \frac{\beta\alpha}{\beta-1}, \quad \beta > 1, \quad \mathrm{Var}\, X = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}, \quad \beta > 2$

mgf does not exist

t

pdf $f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\left(\frac{x^2}{\nu}\right)\right)^{(\nu+1)/2}}, \quad -\infty < x < \infty, \quad \nu = 1,\ldots$

mean and variance $\mathrm{E}X = 0, \quad \nu > 1, \quad \mathrm{Var}\, X = \frac{\nu}{\nu-2}, \quad \nu > 2$

moments (mgf does not exist) $\mathrm{E}X^n = \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{\nu-n}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\nu^{n/2}$ if $n < \nu$ and even,
$\mathrm{E}X^n = 0$ if $n < \nu$ and odd.

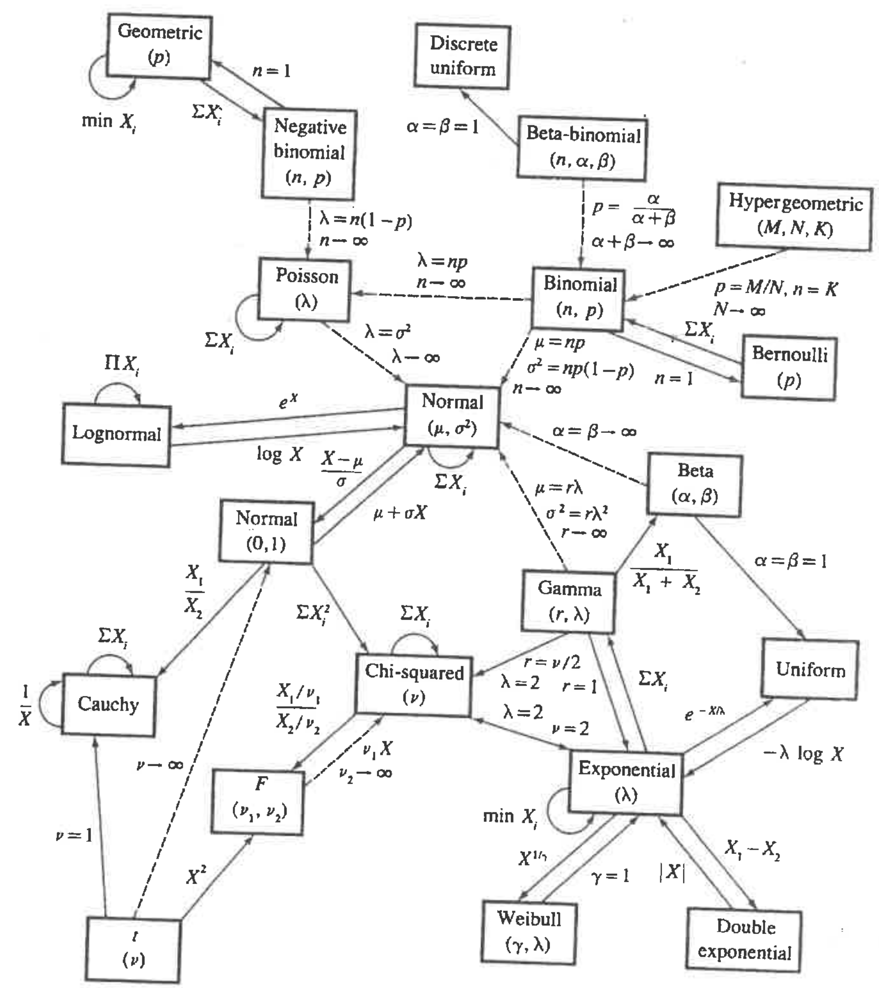
notes Related to F ($F_{1,\nu} = t^2_\nu$).

***Uniform*(a, b)**

pdf	$f(x\|a,b) = \frac{1}{b-a}, \quad a \le x \le b$
mean and variance	$\mathrm{E}X = \frac{b+a}{2}, \quad \mathrm{Var}\,X = \frac{(b-a)^2}{12}$
mgf	$M_X(t) = \frac{e^{bt}-e^{at}}{(b-a)t}$
notes	If $a=0$ and $b=1$, this is a special case of the beta ($\alpha=\beta=1$).

***Weibull*(γ, β)**

pdf	$f(x\|\gamma,\beta) = \frac{\gamma}{\beta}x^{\gamma-1}e^{-x^\gamma/\beta}, \quad 0 \le x < \infty, \quad \gamma > 0, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \beta^{1/\gamma}\Gamma\left(1+\frac{1}{\gamma}\right), \quad \mathrm{Var}\,X = \beta^{2/\gamma}\left[\Gamma\left(1+\frac{2}{\gamma}\right) - \Gamma^2\left(1+\frac{1}{\gamma}\right)\right]$
moments	$\mathrm{E}X^n = \beta^{n/\gamma}\Gamma\left(1+\frac{n}{\gamma}\right)$
notes	The mgf exists only for $\gamma \ge 1$. Its form is not very useful. A special case is exponential ($\gamma = 1$).



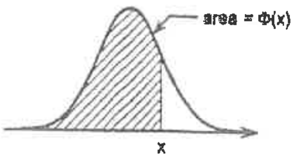
Relationships among common distributions. Solid lines represent transformations and special cases, dashed lines represent limits. Adapted from Leemis (1986).

Tabeller

Tabell 1. Standardiserad normalfördelning

$\Phi(x) = P(X \leq x)$ där $X \in N(0, 1)$

För negativa värden, utnyttja att $\Phi(x) = 1 - \Phi(-x)$



x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865									
3.1	.99903									
3.2	.99931									
3.3	.99952									
3.4	.99966									
3.5	.99977									
3.6	.99984									
3.7	.99989									
3.8	.99993									
3.9	.99995									
4.0	.99997									

Tabell 2. Normalfördelningens kvantiler

$P(X > \lambda_\alpha) = \alpha$ där $X \in N(0, 1)$

α	λ_α	α	λ_α
0.1	1.2816	0.001	3.0902
0.05	1.6449	0.0005	3.2905
0.025	1.9600	0.0001	3.7190
0.01	2.3263	0.00005	3.8906
0.005	2.5758	0.00001	4.2649