

# FORMULA SHEET

## CHAPTER 1

	With replacement	Without replacement
Ordered	$\frac{n!}{(n-r)!}$	$n^r$
Unordered	$\binom{n}{r}$	$\binom{n+r-1}{r}$

### Bonferroni's Inequality

$$P\left(\bigcap_{i=1}^n A_i\right) \geq \sum_{i=1}^n P(A_i) - (n-1)$$

## CHAPTER 2

### Univariate transformations:

$X$  is a discrete r.v. Let  $Y = g(X)$ , then  $Y$  has the following pmf:

$$f_Y(y) = \sum_{x \in g^{-1}(y)} f_X(x)$$

$X$  is a continuous r.v with domain  $\mathcal{X}$ . Let  $Y = g(X)$

Probability density function of $Y$	
If $g(x)$ is monotone for $x \in \mathcal{X}$	$f_Y(y) = f_X(g^{-1}(y)) \left  \frac{d}{dy} g^{-1}(y) \right $
If $g(x)$ is not monotone for $x \in \mathcal{X}$ . But $g_i(x) = g(x)$ for $x \in A_i$ is monotone on $A_i$ .	$f_Y(y) = \sum_{i=1}^k f_X(g_i^{-1}(y)) \left  \frac{d}{dy} g_i^{-1}(y) \right $

### Cumulative density function of $Y$

If  $g(x) \uparrow\uparrow$  for all  $x \in \mathcal{X}$

$$F_Y(y) = F_X(g^{-1}(y))$$

If  $g(x) \downarrow\downarrow$  for all  $x \in \mathcal{X}$

$$F_Y(y) = 1 - F_X(g^{-1}(y))$$

The expected value of a r.v  $g(X)$ :

$$E[g(X)] \begin{cases} \int_{-\infty}^{\infty} g(x) f_X(x) dx & , \text{if } X \text{ is continuous} \\ \sum_{x \in \mathcal{X}} g(x) f_X(x) & , \text{if } X \text{ is discrete} \end{cases}$$

The moment generating function for a r.v  $X$  is given by:

$$M_X(t) = E(e^{tX})$$

The  $n$ :th moment of  $X$ :

$$E(X^n) = M_X^{(n)}(0) = \frac{d^n}{dt^n} M_X(t) \Big|_{t=0}$$

**Leibnitz Formula (Change order of  $\int$  and  $\frac{d}{dx}$ )**

$$\frac{d}{d\theta} \int_{a(\theta)}^{b(\theta)} f(x, \theta) dx = f(b(\theta), \theta) \cdot \frac{d}{d\theta} b(\theta) - f(a(\theta), \theta) \frac{d}{d\theta} a(\theta) + \int_{a(\theta)}^{b(\theta)} \frac{d}{d\theta} f(x, \theta) dx$$

## CHAPTER 3

**Chebyshev's Inequality.** Let  $X$  be a random variable,  $g(x) \geq 0, \forall x \geq 0$  and  $\forall r > 0$

$$P(g(X) \geq r) \leq \frac{E[g(X)]}{r}$$

**Exponential families:**

A family of pdf's or pmf's is called an exponential family if it can be expressed as:

$$f(x|\theta) = h(x)c(\theta) \exp\left\{\sum_{i=1}^k w_i(\theta) t_i(x)\right\}$$

**Binomial Theorem:**

$$(x+y)^n = \sum_{i=0}^n \binom{n}{i} x^i y^{n-i}$$

**Gamma function:**

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt ; \Gamma(\alpha+1) = \alpha\Gamma(\alpha) ; \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} ; \text{If } \alpha \text{ is an integer: } \Gamma(n) = (n-1)!$$

	Discrete	Continuous
Joint pdf/pmf	$P((X, Y) \in A) = \sum_{(x,y) \in A} f(x, y)$	$P((X, Y) \in A) = \iint_A f(x, y) dx dy$
Marginal pdf/pmf	$f_X(x) = \sum_y f_{X,Y}(x, y)$ $f_Y(y) = \sum_x f_{X,Y}(x, y)$	$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$ $f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$
Conditional pdf/pmf	$f(y x) = \frac{f(x, y)}{f_X(x)}$	$f(x y) = \frac{f(x, y)}{f_Y(y)}$

**Bivariate transformations:**

Let  $(X, Y) \sim f(x, y)$ . Suppose the functions  $u = g_1(x, y)$  and  $v = g_2(x, y)$  have the inverse functions  $x = h_1(u, v)$  and  $y = h_2(u, v)$ .

The joint pdf for  $(U, V)$  is given by:

$$f_{U,V}(u, v) = f_{X,Y}(h_1(u, v), h_2(u, v)) |J|$$

Where:

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u}$$

**The expected value of  $g(X, Y)$**

$$E(g(X, Y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f(x, y) dx dy$$

**Conditional expected value of  $g(Y)$  given  $X = x$**

$$E[g(Y)|x] = \begin{cases} \sum_y g(y) f(y|x) & , \text{discrete case} \\ \int_{-\infty}^{\infty} g(y) f(y|x) dy & , \text{continuous case} \end{cases}$$

**Conditional variance of  $Y$  given  $X = x$**

$$V[Y|x] = E[Y^2|x] - E[Y|x]^2$$

**Lemma 4.2.7:** Let  $(X, Y) \sim f(x, y)$ .  $X$  and  $Y$  are independent if and only if there exist functions  $g(x)$  and  $h(y)$  s.t  $f(x, y) = g(x)h(y) \forall x, y \in R$

**Hierarchical Models:**

If  $X$  and  $Y$  are any two random variables then:

$$E(X) = E[E(X|Y)]$$

$$V(X) = E[V(X|Y)] + V[E(X|Y)]$$

**Jensen's inequality:** For any r.v  $X$ , if  $g(x)$  is a convex function then:

$$E[g(X)] \geq g[E(X)]$$

**Cauchy-Schwartz Inequality:** For any two r.v  $X$  and  $Y$

$$|E(XY)| \leq E[|XY|] \leq (E[X^2])^{1/2} (E[Y^2])^{1/2}$$

**Covariance/Correlation**

$$Cov(X, Y) = E(XY) - E(X)E(Y) \quad ; \quad Corr(X, Y) = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

Convergence in distribution	$X_n \xrightarrow{d} X$	$\Leftrightarrow$	$\lim_{n \rightarrow \infty} F_{X_n}(x) = F_X(x)$
Convergence in probability	$X_n \xrightarrow{p} X$	$\Leftrightarrow$	$\lim_{n \rightarrow \infty} P(\omega:  X_n - X  < \epsilon) = 1, \forall \epsilon > 0$
Almost sure convergence	$X_n \xrightarrow{a.s.} X$	$\Leftrightarrow$	$P(\omega: \lim_{n \rightarrow \infty}  X_n - X  < \epsilon) = 1, \forall \epsilon > 0$

**Delta Method:**

If  $Y_n$  is a sequence of random variables that satisfies  $\sqrt{n}(Y_n - \theta) \xrightarrow{d} N(0, \sigma^2)$ . Then for a given function  $g$  and a specific value of  $\theta$  (where  $g'(\theta)$  exists):

$$\sqrt{n}[g(Y_n) - g(\theta)] \xrightarrow{d} N(0, \sigma^2 [g'(\theta)]^2)$$

**EXTRA (USEFULL SUMS)**

$$\sum_{k=0}^{\infty} p^k = \frac{1}{1-p}$$

$$\sum_{k=0}^{\infty} a k^n = \frac{1}{1-a} \quad |a| < 1$$

$$\sum_{k=0}^{\infty} a^k x^k = e^{ax}$$

$$\sum_{k=0}^{\infty} a^k x^k = a \frac{k^n - 1}{k - 1} \quad k \neq 1$$

$$\sum_{k=0}^{\infty} a k^n = \frac{a}{1-a} \quad |a| < 1$$

$$\sum_{x=0}^n x = \frac{n(n+1)}{2}$$

$$\sum_{x=0}^{n-1} a k^x = a \frac{k^n - 1}{k - 1} \quad k \neq 1$$

$$\sum_{x=1}^n x^2 = \frac{n(n+1)(2n+1)}{6}$$

# Table of Common Distributions

## Discrete Distributions

### Bernoulli( $p$ )

<i>pmf</i>	$P(X = x p) = p^x(1-p)^{1-x}; \quad x = 0, 1; \quad 0 \leq p \leq 1$
<i>mean and variance</i>	$EX = p, \quad \text{Var } X = p(1-p)$
<i>mgf</i>	$M_X(t) = (1-p) + pe^t$

### Binomial( $n, p$ )

<i>pmf</i>	$P(X = x n, p) = \binom{n}{x} p^x (1-p)^{n-x}; \quad x = 0, 1, 2, \dots, n; \quad 0 \leq p \leq 1$
<i>mean and variance</i>	$EX = np, \quad \text{Var } X = np(1-p)$
<i>mgf</i>	$M_X(t) = [pe^t + (1-p)]^n$

*notes* Related to Binomial Theorem (Theorem 3.2.2). The *multinomial* distribution (Definition 4.6.2) is a multivariate version of the binomial distribution.

### Discrete uniform

<i>pmf</i>	$P(X = x N) = \frac{1}{N}; \quad x = 1, 2, \dots, N; \quad N = 1, 2, \dots$
<i>mean and variance</i>	$EX = \frac{N+1}{2}, \quad \text{Var } X = \frac{(N+1)(N-1)}{12}$
<i>mgf</i>	$M_X(t) = \frac{1}{N} \sum_{i=1}^N e^{it}$

### Geometric( $p$ )

<i>pmf</i>	$P(X = x p) = p(1-p)^{x-1}; \quad x = 1, 2, \dots; \quad 0 \leq p \leq 1$
<i>mean and variance</i>	$EX = \frac{1}{p}, \quad \text{Var } X = \frac{1-p}{p^2}$

mgf  $M_X(t) = \frac{pe^t}{1-(1-p)e^t}, \quad t < -\log(1-p)$

notes  $Y = X - 1$  is negative binomial(1, p). The distribution is memoryless:  $P(X > s | X > t) = P(X > s - t)$ .

**Hypergeometric**

pmf  $P(X = x | N, M, K) = \frac{\binom{M}{x} \binom{N-M}{K-x}}{\binom{N}{K}}; \quad x = 0, 1, 2, \dots, K;$   
 $M - (N - K) \leq x \leq M; \quad N, M, K \geq 0$

mean and variance  $EX = \frac{KM}{N}, \quad \text{Var } X = \frac{KM}{N} \frac{(N-M)(N-K)}{N(N-1)}$

notes If  $K \ll M$  and  $N$ , the range  $x = 0, 1, 2, \dots, K$  will be appropriate.

**Negative binomial(r, p)**

pmf  $P(X = x | r, p) = \binom{r+x-1}{x} p^r (1-p)^x; \quad x = 0, 1, \dots; \quad 0 \leq p \leq 1$

mean and variance  $EX = \frac{r(1-p)}{p}, \quad \text{Var } X = \frac{r(1-p)}{p^2}$

mgf  $M_X(t) = \left( \frac{p}{1-(1-p)e^t} \right)^r, \quad t < -\log(1-p)$

notes An alternate form of the pmf is given by  $P(Y = y | r, p) = \binom{y-1}{r-1} p^r (1-p)^{y-r}, y = r, r+1, \dots$ . The random variable  $Y = X + r$ . The negative binomial can be derived as a gamma mixture of Poissons. (See Exercise 4.32.)

**Poisson(λ)**

pmf  $P(X = x | \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}; \quad x = 0, 1, \dots; \quad 0 \leq \lambda < \infty$

mean and variance  $EX = \lambda, \quad \text{Var } X = \lambda$

mgf  $M_X(t) = e^{\lambda(e^t-1)}$

**Continuous Distributions**

**Beta(α, β)**

pdf  $f(x | \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad 0 \leq x \leq 1, \quad \alpha > 0, \quad \beta > 0$

mean and variance  $EX = \frac{\alpha}{\alpha+\beta}, \quad \text{Var } X = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

mgf  $M_X(t) = 1 + \sum_{k=1}^{\infty} \left( \prod_{r=0}^{k-1} \frac{\alpha+r}{\alpha+\beta+r} \right) \frac{t^k}{k!}$

notes The constant in the beta pdf can be defined in terms of gamma functions,  $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ . Equation (3.2.18) gives a general expression for the moments.

**Cauchy(θ, σ)**

pdf  $f(x | \theta, \sigma) = \frac{1}{\pi\sigma} \frac{1}{1 + \left(\frac{x-\theta}{\sigma}\right)^2}, \quad -\infty < x < \infty; \quad -\infty < \theta < \infty, \quad \sigma > 0$

mean and variance do not exist

mgf does not exist

notes Special case of Student's  $t$ , when degrees of freedom = 1. Also, if  $X$  and  $Y$  are independent  $n(0, 1)$ ,  $X/Y$  is Cauchy.

**Chi squared(p)**

pdf  $f(x | p) = \frac{1}{\Gamma(p/2)2^{p/2}} x^{(p/2)-1} e^{-x/2}; \quad 0 \leq x < \infty; \quad p = 1, 2, \dots$

mean and variance  $EX = p, \quad \text{Var } X = 2p$

mgf  $M_X(t) = \left( \frac{1}{1-2t} \right)^{p/2}, \quad t < \frac{1}{2}$

notes Special case of the gamma distribution.

**Double exponential(μ, σ)**

pdf  $f(x | \mu, \sigma) = \frac{1}{2\sigma} e^{-|x-\mu|/\sigma}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$

mean and variance  $EX = \mu, \quad \text{Var } X = 2\sigma^2$

mgf  $M_X(t) = \frac{e^{\mu t}}{1-(\sigma t)^2}, \quad |t| < \frac{1}{\sigma}$

notes Also known as the Laplace distribution.

### Exponential( $\beta$ )

pdf  $f(x|\beta) = \frac{1}{\beta}e^{-x/\beta}$ ,  $0 \leq x < \infty$ ,  $\beta > 0$

mean and variance  $EX = \beta$ ,  $Var X = \beta^2$

mgf  $M_X(t) = \frac{1}{1-\beta t}$ ,  $t < \frac{1}{\beta}$

notes Special case of the gamma distribution. Has the *memoryless* property. Has many special cases:  $Y = X^{1/\gamma}$  is Weibull,  $Y = \sqrt{2X}/\beta$  is Rayleigh,  $Y = \alpha - \gamma \log(X/\beta)$  is Gumbel.

### F

pdf  $f(x|\nu_1, \nu_2) = \frac{\Gamma(\frac{\nu_1+\nu_2}{2})}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})} \left(\frac{\nu_2}{\nu_1}\right)^{\nu_1/2} \frac{x^{(\nu_1-2)/2}}{(1+(\frac{\nu_2}{\nu_1})x)^{(\nu_1+\nu_2)/2}}$ ,  
 $0 \leq x < \infty$ ;  $\nu_1, \nu_2 = 1, \dots$

mean and variance  $EX = \frac{\nu_2}{\nu_2-2}$ ,  $\nu_2 > 2$ ,

$Var X = 2 \left(\frac{\nu_2}{\nu_2-2}\right)^2 \frac{(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}$ ,  $\nu_2 > 4$

moments  $EX^n = \frac{\Gamma(\frac{\nu_1+2n}{2})\Gamma(\frac{\nu_2-2n}{2})}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})} \left(\frac{\nu_2}{\nu_1}\right)^n$ ,  $n < \frac{\nu_2}{2}$   
*(mgf does not exist)*

notes Related to chi squared ( $F_{\nu_1, \nu_2} = \left(\frac{\chi_{\nu_1}^2}{\nu_1}\right) / \left(\frac{\chi_{\nu_2}^2}{\nu_2}\right)$ , where the  $\chi^2$ s are independent) and  $t$  ( $F_{1, \nu} = t_n^2$ ).

### Gamma( $\alpha, \beta$ )

pdf  $f(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$ ,  $0 \leq x < \infty$ ,  $\alpha, \beta > 0$

mean and variance  $EX = \alpha\beta$ ,  $Var X = \alpha\beta^2$

mgf  $M_X(t) = \left(\frac{1}{1-\beta t}\right)^\alpha$ ,  $t < \frac{1}{\beta}$

notes Some special cases are exponential ( $\alpha = 1$ ) and chi squared ( $\alpha = p/2$ ,  $\beta = 2$ ). If  $\alpha = \frac{3}{2}$ ,  $Y = \sqrt{X}/\beta$  is Maxwell.  $Y = 1/X$  has the *inverted gamma distribution*. Can also be related to the Poisson (Example 3.2.1).

### Logistic( $\mu, \beta$ )

pdf  $f(x|\mu, \beta) = \frac{1}{\beta} \frac{e^{-(x-\mu)/\beta}}{[1+e^{-(x-\mu)/\beta}]^2}$ ,  $-\infty < x < \infty$ ,  $-\infty < \mu < \infty$ ,  $\beta > 0$

mean and variance  $EX = \mu$ ,  $Var X = \frac{\pi^2 \beta^2}{3}$

mgf  $M_X(t) = e^{\mu t} \Gamma(1 - \beta t) \Gamma(1 + \beta t)$ ,  $|t| < \frac{1}{\beta}$

notes The cdf is given by  $F(x|\mu, \beta) = \frac{1}{1+e^{-(x-\mu)/\beta}}$ .

### Lognormal( $\mu, \sigma^2$ )

pdf  $f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \frac{e^{-(\log x - \mu)^2 / (2\sigma^2)}}{x}$ ,  $0 \leq x < \infty$ ,  $-\infty < \mu < \infty$ ,  $\sigma > 0$

mean and variance  $EX = e^{\mu + (\sigma^2)/2}$ ,  $Var X = e^{2(\mu + \sigma^2)} - e^{2\mu + \sigma^2}$

moments  $EX^n = e^{n\mu + n^2 \sigma^2 / 2}$   
*(mgf does not exist)*

notes Example 2.3.5 gives another distribution with the same moments.\*

### Normal( $\mu, \sigma^2$ )

pdf  $f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2 / (2\sigma^2)}$ ,  $-\infty < x < \infty$ ,  $-\infty < \mu < \infty$ ,  $\sigma > 0$

mean and variance  $EX = \mu$ ,  $Var X = \sigma^2$

mgf  $M_X(t) = e^{\mu t + \sigma^2 t^2 / 2}$

notes Sometimes called the *Gaussian* distribution.

### Pareto( $\alpha, \beta$ )

pdf  $f(x|\alpha, \beta) = \frac{\beta \alpha^\beta}{x^{\beta+1}}$ ,  $a < x < \infty$ ,  $\alpha > 0$ ,  $\beta > 0$

mean and variance  $EX = \frac{\beta \alpha}{\beta-1}$ ,  $\beta > 1$ ,  $Var X = \frac{\beta \alpha^2}{(\beta-1)^2(\beta-2)}$ ,  $\beta > 2$

mgf does not exist

### t

pdf  $f(x|\nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\sqrt{\nu\pi}} \frac{1}{(1+(\frac{x^2}{\nu}))^{(\nu+1)/2}}$ ,  $-\infty < x < \infty$ ,  $\nu = 1, \dots$

mean and variance  $EX = 0$ ,  $\nu > 1$ ,  $Var X = \frac{\nu}{\nu-2}$ ,  $\nu > 2$

moments  $EX^n = \frac{\Gamma(\frac{\nu+1}{2})\Gamma(\frac{\nu-n}{2})}{\sqrt{\pi}\Gamma(\frac{\nu}{2})} \nu^{n/2}$  if  $n < \nu$  and even,  
 $EX^n = 0$  if  $n < \nu$  and odd.

notes Related to  $F$  ( $F_{1, \nu} = t_n^2$ ).

**Uniform(a, b)**

pdf  $f(x|a, b) = \frac{1}{b-a}, \quad a \leq x \leq b$

mean and variance  $EX = \frac{b+a}{2}, \quad \text{Var } X = \frac{(b-a)^2}{12}$

mfg  $M_X(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$

notes If  $a = 0$  and  $b = 1$ , this is a special case of the beta ( $\alpha = \beta = 1$ ).

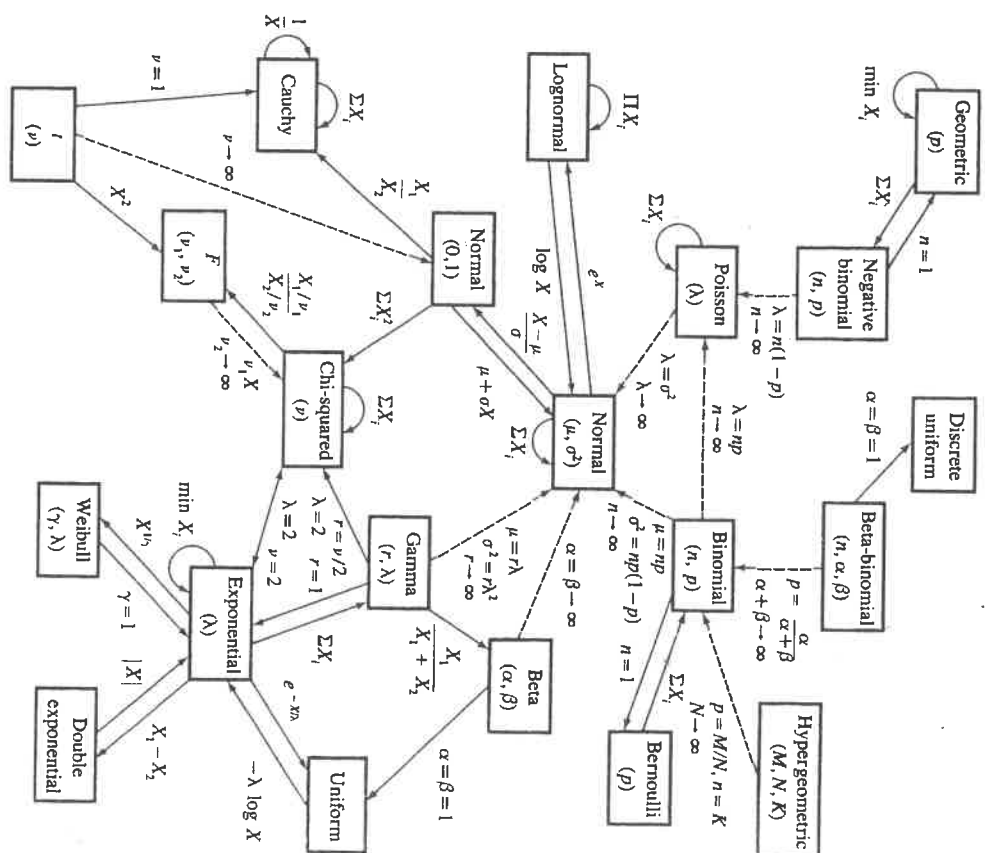
**Weibull( $\gamma, \beta$ )**

pdf  $f(x|\gamma, \beta) = \frac{\gamma}{\beta} x^{\gamma-1} e^{-x^\gamma/\beta}, \quad 0 \leq x < \infty, \quad \gamma > 0, \quad \beta > 0$

mean and variance  $EX = \beta^{1/\gamma} \Gamma\left(1 + \frac{1}{\gamma}\right), \quad \text{Var } X = \beta^{2/\gamma} \left[ \Gamma\left(1 + \frac{2}{\gamma}\right) - \Gamma^2\left(1 + \frac{1}{\gamma}\right) \right]$

moments  $EX^n = \beta^{n/\gamma} \Gamma\left(1 + \frac{n}{\gamma}\right)$

notes The mfg exists only for  $\gamma \geq 1$ . Its form is not very useful. A special case is exponential ( $\gamma = 1$ ).



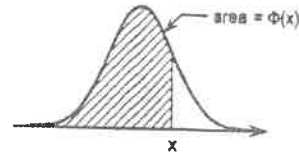
Relationships among common distributions. Solid lines represent transformations and special cases, dashed lines represent limits. Adapted from Leemis (1986).

## Tabeller

Tabell 1. Standardiserad normalfördelning

$\Phi(x) = P(X \leq x)$  där  $X \in N(0, 1)$

För negativa värden, utnyttja att  $\Phi(x) = 1 - \Phi(-x)$



x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861

Tabell 2. Normalfördelningens kvantiler

$P(X > \lambda_\alpha) = \alpha$  där  $X \in N(0, 1)$

$\alpha$	$\lambda_\alpha$	$\alpha$	$\lambda_\alpha$
0.1	1.2816	0.001	3.0902
0.05	1.6449	0.0005	3.2905
0.025	1.9600	0.0001	3.7190
0.01	2.3263	0.00005	3.8906
0.005	2.5758	0.00001	4.2649

