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TENTAMEN I STATISTISK TEORI MED TILLÄMPNINGAR II
2022-03-21

Skrivtid: 14.00 − 19.00.
Godkända hjälpmedel: Miniräknare, språklexikon, formelsamling (bifogas) och statistiska tabeller (bifogas)
Tentamen består av fem uppgifter. För full poäng på en uppgift krävs tydliga, utförliga och väl motiverade lösningar.
Kortfattade svar läggs ut strax efter tentamen på Athena

Uppgift 1 (20 poäng)

Ett slumpmässigt urval om 236 patienter med konstaterad Covid 19 fick genomgå ett hemtest för sjukdomen. Av dessa gav testen positivt utslag i 189 fall.

a) Beräkna ett 99%-igt konfidensintervall för andelen positiva hemtest bland patienter med Covid 19.

b) Beräkna hur stort urval som skulle krävas för att längden av ett 99%-igt konfidensintervall inte ska överstiga 0.1. Antag vid beräkningen att den sanna andelen är okänd.

c) Hur påverkas erforderlig urvalsstorlek i b)-uppgiften om den statistiska felmarginalen minskar?

d) Hur påverkas erforderlig urvalsstorlek i b)-uppgiften om konfidensgraden minskar?

Uppgift 2. (20 poäng)
$Y_1, Y_2, \ldots, Y_n$ är n stycken oberoende stokastiska variabler som alla är Bernoullifördelade med parametern p. Betrakta följande estimatorer av p

$$\widehat{p}_1 = Y_1, \qquad \widehat{p}_2 = \frac{Y_1 + Y_2}{2} \qquad \widehat{p}_n = \frac{\sum_{i=1}^{n} Y_i}{n}.$$

a) Bestäm samplingfördelningarna för $\widehat{p}_1$ och $\widehat{p}_2$.

b) Är $\widehat{p}_1$ och $\widehat{p}_2$ väntevärdesriktiga estimatorer av p?

c) Beräkna varianserna för $\widehat{p}_1$ och $\widehat{p}_2$ och ange effektiviteten för $\widehat{p}_1$ relativt $\widehat{p}_2$. Kommentera resultatet.

d) Är $\widehat{p}_n$ en konsistent estimator av p?

Uppgift 3. (20 poäng)

Vid konstruktion av vindkraftverk och planering av vindkraftparker används bl.a. den så kallade Weibullfördelningen för att modellera vindhastighet. Weibullfördelningen har två parametrar, k och λ. I en viss tillämpning är parametern $k = 2$ medan parametern λ är okänd. Då $k = 2$ är täthetsfunktionen

$$f(y) = \frac{2y}{\lambda^2} e^{-(y/\lambda)^2}, \; 0 < y < \infty$$

och väntevärdet och variansen är

$$\begin{aligned} E(Y) &= \lambda \frac{\sqrt{\pi}}{2} \\ V(Y) &= \lambda^2 \left(1 - \frac{\pi}{4}\right) \end{aligned}$$

För att uppskatta λ gjordes n stokastiskt oberoende observationer, $Y_1, Y_2, \ldots, Y_n$.

a) Bestäm momentestimatorn av λ.

b) Är momentestimatorn av λ en väntevärdesriktig estimator?

c) Bestäm maximum likelihoodestimatorn av λ.

Uppgift 4 (20 poäng)
Man vill undersöka om energiförbrukningen är olika under sommaren som under vintern. Två medelvärden av energiförbrukningen per dag (i kWh) räknades ut för tio personer, ett medelvärde för sommarmånaderna och ett medelvärde för vintermånaderna, se tabell nedan

Person	1	2	3	4	5	6	7	8	9	10
Sommar	1,69	1,57	2,57	2,10	2,22	1,59	2,09	1,86	1,63	1,49
Vinter	1,65	1,74	1,74	2,02	2,36	1,09	2,09	1,63	1,56	1,36

a) Testa med ett lämpligt icke-parametriskt test om energiförbrukningen är lika under sommaren som under vintern. Ange nödvändiga antaganden, sätt upp lämpliga hypoteser och dra slutsats. Använd signifikansnivån 0,05.

b) Testa med ett lämpligt t-test om energiförbrukningen är lika under sommaren som under vintern. Ange nödvändiga antaganden, sätt upp lämpliga hypoteser och dra slutsats. Använd signifikansnivån 0,05.

Uppgift 5. (20 poäng)

Tiden mellan att två radioaktiva partiklar träffar en viss yta anses vara exponentialfördelad med väntevärdet β. Under normala förhållanden är tiden mellan två träffar i medeltal β_0 men vid en förhöjd strålnivå är tiden mellan två träffar kortare än β_0. För att undersöka om strålningsnivån kan anses vara normal vill man därför testa hypotesen $H_0 : \beta = \beta_0$ mot $H_a : \beta < \beta_0$ med hjälp av n stokastiskt oberoende observationer, $Y_1, Y_2, \ldots, Y_n$, på tider mellan två träffar av de radioaktiva partiklarna. Konstruera ett likformigt starkaste test för test av hypoteserna genom att ange en teststatistika och för vilka värden som anger förkastelseområdet (du behöver inte ange ett siffervärde för det kritiska värdet). Undersök om teststatistikan kan standardiseras och, i så fall, om fördelningen för den standardiserade teststatistikan kan approximeras med någon fördelning om n är stort.

Formelblad - Statistisk teori med tillämpningar

Väntevärde och varians för en stokastisk variabel

- För en diskret stokastisk variabel Y med sannolikhetsfördelning $p(y)$

$$E(Y) = \sum_y yp(y) \quad (1)$$

$$V(Y) = \sum_y [y - E(Y)]^2 p(y) = \sum_y y^2 p(y) - [E(Y)]^2 \quad (2)$$

- För en kontinuerlig stokastisk variabel Y med täthetsfunktion $f(y)$

$$E(Y) = \int_{-\infty}^{\infty} yf(y)\,dy \quad (3)$$

$$V(Y) = \int_{-\infty}^{\infty} [y - E(Y)]^2 f(y)\,dy = \int_{-\infty}^{\infty} y^2 f(y)\,dy - [E(Y)]^2 \quad (4)$$

Väntevärde för en funktion av stokastisk variabel $g(Y)$

- För en diskret stokastisk variabel Y med sannolikhetsfördelning $p(y)$

$$E[g(Y)] = \sum_{\text{alla } y} g(y)\,p(y) \quad (5)$$

- För en kontinuerlig stokastisk variabel Y med täthetsfunktion $f(y)$

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y)\,f(y)\,dy \quad (6)$$

Transformationsmetoden

Låt $U = h(Y)$, där $h(y)$ är antingen strängt växande eller strängt avtagande för alla y sådana att $f_Y(y) > 0$, då har $U = h(Y)$ täthetsfunktion

$$f_U(u) = f_Y\left[h^{-1}(u)\right]\left|\frac{dh^{-1}}{du}\right|, \qquad \text{där } \frac{dh^{-1}}{du} = \frac{d\left[h^{-1}(u)\right]}{du} \quad (7)$$

Ordningsvärden

Då $Y_{(1)} = \min(Y_1, Y_2, ..., Y_n)$ och $Y_{(n)} = \max(Y_1, Y_2, ..., Y_n)$ så har $Y_{(1)}$ respektive $Y_{(n)}$ täthetsfunktionerna

$$f_{Y_{(1)}}(y) = g_{(1)}(y) = n\left[1 - F_Y(y)\right]^{n-1} f_Y(y) \tag{8}$$

$$f_{Y_{(n)}}(y) = g_{(n)}(y) = n\left[F_Y(y)\right]^{n-1} f_Y(y) \tag{9}$$

samt fördelningsfunktionerna

$$F_{Y_{(1)}}(y) = 1 - \left[1 - F_Y(y)\right]^n \tag{10}$$

$$F_{Y(n)}(y) = \left[F_Y(y)\right]^n \tag{11}$$

Väntevärde för en funktion av två stokastiska variabler $g(Y_1, Y_2)$

- För två diskreta stokastiska variabler Y_1 och Y_2 med simultan sannolikhetsfördelning $p(y_1, y_2)$
$$E\left[g(Y_1, Y_2)\right] = \sum_{\text{alla } y_1} \sum_{\text{alla } y_2} g(y_1, y_2)\, p(y_1, y_2) \tag{12}$$

- För två kontinuerliga stokastiska variabler Y_1 och Y_2 med simultan täthetsfunktion $f(y_1, y_2)$
$$E\left[g(Y_1, Y_2)\right] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(y_1, y_2)\, f(y_1, y_2)\, dy_1 dy_2 \tag{13}$$

Marginalfördelning för Y_1

$$p_1(y_1) = \sum_{y_2} p(y_1, y_2) \tag{14}$$

där $p(y_1, y_2)$ är den simultana sannolikhetsfördelningen för Y_1 och Y_2

Marginaltäthet för Y_1

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_2 \tag{15}$$

där $f(y_1, y_2)$ är den simultana täthetsfunktionen för Y_1 och Y_2

Betingad sannolikhetsfördelning för Y_1 givet Y_2

$$p(y_1 | y_2) = \frac{p(y_1, y_2)}{p_2(y_2)} \qquad p_2(y_2) > 0 \tag{16}$$

där $p(y_1, y_2)$ är den simultana sannolikhetsfördelningen för Y_1 och Y_2 och $p_2(y_2)$ är marginalfördelningen för Y_2

Betingad täthetsfunktion för Y_1 givet Y_2

$$f(y_1 | y_2) = \frac{f(y_1, y_2)}{f_2(y_2)} \qquad f_2(y_2) > 0 \tag{17}$$

där $f(y_1, y_2)$ är den simultana täthetsfunktionen för Y_1 och Y_2 och $f_2(y_2)$ är marginaltätheten för Y_2

Betingat väntevärde för $g(Y_1)$ givet Y_2

$$E[g(Y_1) | Y_2 = y_2] = \sum_{\text{alla } y_1} g(y_1)\, p(y_1 | y_2) \tag{18}$$

där $p(y_1 | y_2)$ är den betingade sannolikhetsfördelningen för Y_1 givet Y_2

$$E[g(Y_1) | Y_2 = y_2] = \int_{-\infty}^{\infty} g(y_1)\, f(y_1 | y_2)\, dy_1 \tag{19}$$

där $f(y_1 | y_2)$ är den betingade täthetsfunktionen för Y_1 givet Y_2

Kovarians mellan två stokastiska variabler Y_1 och Y_2

$$Cov(Y_1, Y_2) = E[(Y_1 - E(Y_1))(Y_2 - E(Y_2))] = E(Y_1 Y_2) - E(Y_1) E(Y_2) \tag{20}$$

Korrelation mellan två stokastiska variabler Y_1 och Y_2

$$\rho = \frac{Cov(Y_1, Y_2)}{\sigma_1 \sigma_2} \tag{21}$$

där $\sigma_1 = \sqrt{V(Y_1)}$ och $\sigma_2 = \sqrt{V(Y_2)}$

Multinomialfördelningen

$$p(y_1, y_2, \ldots, y_k) = \frac{n!}{y_1! y_2! \cdot \ldots \cdot y_k!} p_1^{y_1} p_2^{y_2} \cdot \ldots \cdot p_k^{y_k} \tag{22}$$

Potenser

För reella tal x, y och positiva tal a, b gäller

i) $a^x a^y = a^{x+y}$

ii) $\frac{a^x}{a^y} = a^{x-y}$

iii) $(ab)^x = a^x b^x$

iv) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

v) $\frac{1}{a^x} = a^{-x}$

vi) $(a^x)^y = a^{xy}$

vii) $a^0 = 1$

viii) $a^{\frac{1}{n}} = \sqrt[n]{a}$, där n är ett positivt heltal

Naturliga logaritmen

För positiva tal x, y gäller

i) $e^x = y \Leftrightarrow x = \ln(y)$

ii) $\ln(xy) = \ln(x) + \ln(y)$

iii) $\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

iv) $\ln(x^p) = p\ln(x)$

Derivator

i) $\frac{d}{dx}k = 0$

ii) $\frac{d}{dx}x^n = nx^{n-1}$

iii) $\frac{d}{dx}e^{ax} = ae^{ax}$

iv) $\frac{d}{dx}\ln(x) = \frac{1}{x},\ x > 0$

v) $\frac{d}{dx}(k \cdot f(x)) = kf'(x)$

vi) Deriveringsregeln för en summa

$$\frac{d}{dx}\left(f\left(x\right)+g\left(x\right)\right)=f'\left(x\right)+g'\left(x\right)$$

vii) Deriveringsregeln för en produkt

$$\frac{d}{dx}\left(f\left(x\right)\cdot g\left(x\right)\right)=f'\left(x\right)g\left(x\right)+f\left(x\right)g'\left(x\right)$$

viii) Deriveringsregeln för en kvot

$$\frac{d}{dx}\left(\frac{f\left(x\right)}{g\left(x\right)}\right)=\frac{f'\left(x\right)g\left(x\right)-f\left(x\right)g'\left(x\right)}{\left(g\left(x\right)\right)^{2}}$$

ix) För den sammansatta funktionen $f\left(x\right)=h\left(g\left(x\right)\right)$ gäller

$$\frac{d}{dx}f\left(x\right)=h'\left(g\left(x\right)\right)\cdot g'\left(x\right)$$

Integraler

i) $\int x^{n}dx=\frac{1}{n+1}x^{n+1}+C,\ n\neq-1$

ii) $\int e^{ax}dx=\frac{1}{a}e^{ax}+C,\ a\neq0$

iii) $\int\frac{1}{x}dx=\ln\left|x\right|,\ x>0$

iv) $\int kf\left(x\right)dx=k\int f\left(x\right)dx$

v) $\int\left(f\left(x\right)+g\left(x\right)\right)dx=\int f\left(x\right)dx+\int g\left(x\right)dx$

vi) Partiell integration

$$\int f\left(x\right)g\left(x\right)dx=F\left(x\right)g\left(x\right)-\int F\left(x\right)g'\left(x\right)dx$$

vii) Bestämda integraler

$$\int\limits_{a}^{b}f\left(x\right)dx=\left[F\left(x\right)\right]_{a}^{b}=F\left(b\right)-F\left(a\right)$$

Formelblad 2 - Statistisk teori med tillämpningar

Estimatorer

i) $\overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$

ii) $S^2 = \frac{\sum_{i=1}^{n}\left(Y_i - \overline{Y}\right)^2}{n-1} = \frac{\sum_{i=1}^{n} {Y_i}^2 - n\left(\overline{Y}\right)^2}{n-1}$

iii) $S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$

Konfidensintervall

i) $\widehat{\theta} \pm z_{a/2}\sigma_{\widehat{\theta}}$

ii) $\overline{Y} \pm t_{\alpha/2}\frac{S}{\sqrt{n}}$

iii) $\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2}}, \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}\right)$

iv) $\widehat{\theta}_1 - \widehat{\theta}_2 \pm z_{a/2}\sigma_{\left(\widehat{\theta}_1 - \widehat{\theta}_2\right)}$

v) $\overline{Y}_1 - \overline{Y}_2 \pm t_{\alpha/2}S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Antal observationer

i) $n = z^2_{\frac{\alpha}{2}}\frac{\sigma^2}{B^2}$

ii) $n = \frac{(z_\alpha + z_\beta)^2\sigma^2}{(\mu_a - \mu_0)^2}$

Testvariabler

i) $Z = \frac{\widehat{\theta} - \theta_0}{\sigma_{\widehat{\theta}}}$

ii) $T = \frac{\overline{Y} - \mu_0}{S/\sqrt{n}}$

iii) $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$

iv) $Z = \frac{\widehat{\theta}_1 - \widehat{\theta}_2 - D_0}{\sigma_{(\widehat{\theta}_1 - \widehat{\theta}_2)}}$

v) $T = \frac{\overline{Y}_1 - \overline{Y}_2 - D_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

vi) $Z = \frac{M - n/2}{(1/2)\sqrt{n}}$

vii) $Z = \frac{T^+ - [n(n+1)/4]}{\sqrt{n(n+1)(2n+1)/24}}$

viii) $Z = \frac{U - (n_1 n_2/2)}{\sqrt{n_1 n_2 (n_1 + n_2 + 1)/12}}, \quad U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - W$

ix) $Z = \frac{R - E(R)}{\sqrt{V(R)}}, \quad E(R) = \frac{2n_1 n_2}{n_1 + n_2} + 1, \quad V(R) = \frac{2n_1 n_2 (2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}$

x) $r_S = \frac{\sum_{i=1}^n R(x_i)R(y_i) - \frac{1}{n}\left[\sum_{i=1}^n R(x_i)\right]\left[\sum_{i=1}^n R(y_i)\right]}{\sqrt{\left\{\sum_{i=1}^n [R(x_i)]^2 - \frac{1}{n}\left[\sum_{i=1}^n R(x_i)\right]^2\right\}\left\{\sum_{i=1}^n [R(y_i)]^2 - \frac{1}{n}\left[\sum_{i=1}^n R(y_i)\right]^2\right\}}}$

xi) $r_S = 1 - \frac{6\sum_{i=1}^n d_i^2}{n(n^2-1)}$

Likelihood

$$L(y_1, y_2, ..., y_n | \theta) = \begin{cases} \prod_{i=1}^{n} p(y_i | \theta) & \text{om } Y_1, Y_2, ..., Y_n \text{ är oberoende s.v. med sannolikhetsfunktion } p(y | \theta) \\ \prod_{i=1}^{n} f(y_i | \theta) & \text{om } Y_1, Y_2, ..., Y_n \text{ är oberoende s.v. med täthetsfunktion } f(y | \theta) \end{cases} \tag{23}$$

Bayesiansk inferens

$$g * (\theta \mid y) = \frac{L(y \mid \theta) g(\theta)}{\int_\theta L(y \mid \theta) g(\theta) d\theta}, \tag{24}$$

där $g(\theta)$ är apriorifördelningen, $L(\theta \mid y)$ är likelihooden, och $g^*(\theta \mid y)$ är aposteriorifördelningen.

Discrete Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y};$ $y = 0, 1, \ldots, n$	np	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$p(y) = p(1-p)^{y-1};$ $y = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}};$ $y = 0, 1, \ldots, n$ if $n \le r$, $y = 0, 1, \ldots, r$ if $n > r$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$	
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!};$ $y = 0, 1, 2, \ldots$	λ	λ	$\exp[\lambda(e^t - 1)]$
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r};$ $y = r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$

Continuous Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}; \theta_1 \le y \le \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$	$\frac{e^{t\theta_2} - e^{t\theta_1}}{t(\theta_2 - \theta_1)}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right]$ $-\infty < y < +\infty$	μ	σ^2	$\exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$
Exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}; \quad \beta > 0$ $0 < y < \infty$	β	β^2	$(1 - \beta t)^{-1}$
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Chi-square	$f(y) = \frac{(y)^{(v/2)-1} e^{-y/2}}{2^{v/2}\Gamma(v/2)};$ $y^2 > 0$	v	$2v$	$(1-2t)^{-v/2}$
Beta	$f(y) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1}(1-y)^{\beta-1};$ $0 < y < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	does not exist in closed form

Fördelningsfunktionen för binomialfördelningen med parametrarna *n* och *p*

		p							
n	*x*	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
2	0	0.90250	0.81000	0.72250	0.64000	0.56250	0.49000	0.36000	0.25000
	1	0.99750	0.99000	0.97750	0.96000	0.93750	0.91000	0.84000	0.75000
3	0	0.85737	0.72900	0.61412	0.51200	0.42188	0.34300	0.21600	0.12500
	1	0.99275	0.97200	0.93925	0.89600	0.84375	0.78400	0.64800	0.50000
	2	0.99987	0.99900	0.99663	0.99200	0.98438	0.97300	0.93600	0.87500
4	0	0.81451	0.65610	0.52201	0.40960	0.31641	0.24010	0.12960	0.06250
	1	0.98598	0.94770	0.89048	0.81920	0.73828	0.65170	0.47520	0.31250
	2	0.99952	0.99630	0.98802	0.97280	0.94922	0.91630	0.82080	0.68750
	3	0.99999	0.99990	0.99949	0.99840	0.99609	0.99190	0.97440	0.93750
5	0	0.77378	0.59049	0.44371	0.32768	0.23730	0.16807	0.07776	0.03125
	1	0.97741	0.91854	0.83521	0.73728	0.63281	0.52822	0.33696	0.18750
	2	0.99884	0.99144	0.97339	0.94208	0.89648	0.83692	0.68256	0.50000
	3	0.99997	0.99954	0.99777	0.99328	0.98437	0.96922	0.91296	0.81250
	4	1.00000	0.99999	0.99992	0.99968	0.99902	0.99757	0.98976	0.96875
6	0	0.73509	0.53144	0.37715	0.26214	0.17798	0.11765	0.04666	0.01563
	1	0.96723	0.88574	0.77648	0.65536	0.53394	0.42018	0.23328	0.10938
	2	0.99777	0.98415	0.95266	0.90112	0.83057	0.74431	0.54432	0.34375
	3	0.99991	0.99873	0.99411	0.98304	0.96240	0.92953	0.82080	0.65625
	4	1.00000	0.99995	0.99960	0.99840	0.99536	0.98906	0.95904	0.89063
	5	1.00000	1.00000	0.99999	0.99994	0.99976	0.99927	0.99590	0.98438
7	0	0.69834	0.47830	0.32058	0.20972	0.13348	0.08235	0.02799	0.00781
	1	0.95562	0.85031	0.71658	0.57672	0.44495	0.32942	0.15863	0.06250
	2	0.99624	0.97431	0.92623	0.85197	0.75641	0.64707	0.41990	0.22656
	3	0.99981	0.99727	0.98790	0.96666	0.92944	0.87396	0.71021	0.50000
	4	0.99999	0.99982	0.99878	0.99533	0.98712	0.97120	0.90374	0.77344
	5	1.00000	0.99999	0.99993	0.99963	0.99866	0.99621	0.98116	0.93750
	6	1.00000	1.00000	1.00000	0.99999	0.99994	0.99978	0.99836	0.99219
8	0	0.66342	0.43047	0.27249	0.16777	0.10011	0.05765	0.01680	0.00391
	1	0.94276	0.81310	0.65718	0.50332	0.36708	0.25530	0.10638	0.03516
	2	0.99421	0.96191	0.89479	0.79692	0.67854	0.55177	0.31539	0.14453
	3	0.99963	0.99498	0.97865	0.94372	0.88618	0.80590	0.59409	0.36328
	4	0.99998	0.99957	0.99715	0.98959	0.97270	0.94203	0.82633	0.63672
	5	1.00000	0.99998	0.99976	0.99877	0.99577	0.98871	0.95019	0.85547
	6	1.00000	1.00000	0.99999	0.99992	0.99962	0.99871	0.99148	0.96484
	7	1.00000	1.00000	1.00000	1.00000	0.99998	0.99993	0.99934	0.99609
9	0	0.63025	0.38742	0.23162	0.13422	0.07508	0.04035	0.01008	0.00195
	1	0.92879	0.77484	0.59948	0.43621	0.30034	0.19600	0.07054	0.01953
	2	0.99164	0.94703	0.85915	0.73820	0.60068	0.46283	0.23179	0.08984
	3	0.99936	0.99167	0.96607	0.91436	0.83427	0.72966	0.48261	0.25391
	4	0.99997	0.99911	0.99437	0.98042	0.95107	0.90119	0.73343	0.50000
	5	1.00000	0.99994	0.99937	0.99693	0.99001	0.97471	0.90065	0.74609
	6	1.00000	1.00000	0.99995	0.99969	0.99866	0.99571	0.97497	0.91016
	7	1.00000	1.00000	1.00000	0.99998	0.99989	0.99957	0.99620	0.98047
	8	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99974	0.99805

		p							
n	*x*	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
10	0	0.59874	0.34868	0.19687	0.10737	0.05631	0.02825	0.00605	0.00098
	1	0.91386	0.73610	0.54430	0.37581	0.24403	0.14931	0.04636	0.01074
	2	0.98850	0.92981	0.82020	0.67780	0.52559	0.38278	0.16729	0.05469
	3	0.99897	0.98720	0.95003	0.87913	0.77588	0.64961	0.38228	0.17188
	4	0.99994	0.99837	0.99013	0.96721	0.92187	0.84973	0.63310	0.37695
	5	1.00000	0.99985	0.99862	0.99363	0.98027	0.95265	0.83376	0.62305
	6	1.00000	0.99999	0.99987	0.99914	0.99649	0.98941	0.94524	0.82813
	7	1.00000	1.00000	0.99999	0.99992	0.99958	0.99841	0.98771	0.94531
	8	1.00000	1.00000	1.00000	1.00000	0.99997	0.99986	0.99832	0.98926
	9	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99990	0.99902
11	0	0.56880	0.31381	0.16734	0.08590	0.04224	0.01977	0.00363	0.00049
	1	0.89811	0.69736	0.49219	0.32212	0.19710	0.11299	0.03023	0.00586
	2	0.98476	0.91044	0.77881	0.61740	0.45520	0.31274	0.11892	0.03271
	3	0.99845	0.98147	0.93056	0.83886	0.71330	0.56956	0.29628	0.11328
	4	0.99989	0.99725	0.98411	0.94959	0.88537	0.78970	0.53277	0.27441
	5	0.99999	0.99970	0.99734	0.98835	0.96567	0.92178	0.75350	0.50000
	6	1.00000	0.99998	0.99968	0.99803	0.99244	0.97838	0.90065	0.72559
	7	1.00000	1.00000	0.99997	0.99976	0.99881	0.99571	0.97072	0.88672
	8	1.00000	1.00000	1.00000	0.99998	0.99987	0.99942	0.99408	0.96729
	9	1.00000	1.00000	1.00000	1.00000	0.99999	0.99995	0.99927	0.99414
	10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99996	0.99951
12	0	0.54036	0.28243	0.14224	0.06872	0.03168	0.01384	0.00218	0.00024
	1	0.88164	0.65900	0.44346	0.27488	0.15838	0.08503	0.01959	0.00317
	2	0.98043	0.88913	0.73582	0.55835	0.39068	0.25282	0.08344	0.01929
	3	0.99776	0.97436	0.90779	0.79457	0.64878	0.49252	0.22534	0.07300
	4	0.99982	0.99567	0.97608	0.92744	0.84236	0.72366	0.43818	0.19385
	5	0.99999	0.99946	0.99536	0.98059	0.94560	0.88215	0.66521	0.38721
	6	1.00000	0.99995	0.99933	0.99610	0.98575	0.96140	0.84179	0.61279
	7	1.00000	1.00000	0.99993	0.99942	0.99722	0.99051	0.94269	0.80615
	8	1.00000	1.00000	0.99999	0.99994	0.99961	0.99831	0.98473	0.92700
	9	1.00000	1.00000	1.00000	1.00000	0.99996	0.99979	0.99719	0.98071
	10	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99968	0.99683
	11	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99976
13	0	0.51334	0.25419	0.12091	0.05498	0.02376	0.00969	0.00131	0.00012
	1	0.86458	0.62134	0.39828	0.23365	0.12671	0.06367	0.01263	0.00171
	2	0.97549	0.86612	0.69196	0.50165	0.33260	0.20248	0.05790	0.01123
	3	0.99690	0.96584	0.88200	0.74732	0.58425	0.42061	0.16858	0.04614
	4	0.99971	0.99354	0.96584	0.90087	0.79396	0.65431	0.35304	0.13342
	5	0.99998	0.99908	0.99247	0.96996	0.91979	0.83460	0.57440	0.29053
	6	1.00000	0.99990	0.99873	0.99300	0.97571	0.93762	0.77116	0.50000
	7	1.00000	0.99999	0.99984	0.99875	0.99435	0.98178	0.90233	0.70947
	8	1.00000	1.00000	0.99998	0.99983	0.99901	0.99597	0.96792	0.86658
	9	1.00000	1.00000	1.00000	0.99998	0.99987	0.99935	0.99221	0.95386
	10	1.00000	1.00000	1.00000	1.00000	0.99999	0.99993	0.99868	0.98877
	11	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99986	0.99829
	12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99988

		p							
n	*x*	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
14	0	0.48767	0.22877	0.10277	0.04398	0.01782	0.00678	0.00078	0.00006
	1	0.84701	0.58463	0.35667	0.19791	0.10097	0.04748	0.00810	0.00092
	2	0.96995	0.84164	0.64791	0.44805	0.28113	0.16084	0.03979	0.00647
	3	0.99583	0.95587	0.85349	0.69819	0.52134	0.35517	0.12431	0.02869
	4	0.99957	0.99077	0.95326	0.87016	0.74153	0.58420	0.27926	0.08978
	5	0.99997	0.99853	0.98847	0.95615	0.88833	0.78052	0.48585	0.21198
	6	1.00000	0.99982	0.99779	0.98839	0.96173	0.90672	0.69245	0.39526
	7	1.00000	0.99998	0.99967	0.99760	0.98969	0.96853	0.84986	0.60474
	8	1.00000	1.00000	0.99996	0.99962	0.99785	0.99171	0.94168	0.78802
	9	1.00000	1.00000	1.00000	0.99995	0.99966	0.99833	0.98249	0.91022
	10	1.00000	1.00000	1.00000	1.00000	0.99996	0.99975	0.99609	0.97131
	11	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99939	0.99353
	12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99994	0.99908
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99994
15	0	0.46329	0.20589	0.08735	0.03518	0.01336	0.00475	0.00047	0.00003
	1	0.82905	0.54904	0.31859	0.16713	0.08018	0.03527	0.00517	0.00049
	2	0.96380	0.81594	0.60423	0.39802	0.23609	0.12683	0.02711	0.00369
	3	0.99453	0.94444	0.82266	0.64816	0.46129	0.29687	0.09050	0.01758
	4	0.99939	0.98728	0.93829	0.83577	0.68649	0.51549	0.21728	0.05923
	5	0.99995	0.99775	0.98319	0.93895	0.85163	0.72162	0.40322	0.15088
	6	1.00000	0.99969	0.99639	0.98194	0.94338	0.86886	0.60981	0.30362
	7	1.00000	0.99997	0.99939	0.99576	0.98270	0.94999	0.78690	0.50000
	8	1.00000	1.00000	0.99992	0.99922	0.99581	0.98476	0.90495	0.69638
	9	1.00000	1.00000	0.99999	0.99989	0.99921	0.99635	0.96617	0.84912
	10	1.00000	1.00000	1.00000	0.99999	0.99988	0.99933	0.99065	0.94077
	11	1.00000	1.00000	1.00000	1.00000	0.99999	0.99991	0.99807	0.98242
	12	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99972	0.99631
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99951
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997
16	0	0.44013	0.18530	0.07425	0.02815	0.01002	0.00332	0.00028	0.00002
	1	0.81076	0.51473	0.28390	0.14074	0.06348	0.02611	0.00329	0.00026
	2	0.95706	0.78925	0.56138	0.35184	0.19711	0.09936	0.01834	0.00209
	3	0.99300	0.93159	0.78989	0.59813	0.40499	0.24586	0.06515	0.01064
	4	0.99914	0.98300	0.92095	0.79825	0.63019	0.44990	0.16657	0.03841
	5	0.99992	0.99670	0.97646	0.91831	0.81035	0.65978	0.32884	0.10506
	6	0.99999	0.99950	0.99441	0.97334	0.92044	0.82469	0.52717	0.22725
	7	1.00000	0.99994	0.99894	0.99300	0.97287	0.92565	0.71606	0.40181
	8	1.00000	0.99999	0.99984	0.99852	0.99253	0.97433	0.85773	0.59819
	9	1.00000	1.00000	0.99998	0.99975	0.99836	0.99287	0.94168	0.77275
	10	1.00000	1.00000	1.00000	0.99997	0.99971	0.99843	0.98086	0.89494
	11	1.00000	1.00000	1.00000	1.00000	0.99996	0.99973	0.99510	0.96159
	12	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99906	0.98936
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99987	0.99791
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99974
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998

		p							
n	x	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
17	0	0.41812	0.16677	0.06311	0.02252	0.00752	0.00233	0.00017	0.00001
	1	0.79223	0.48179	0.25245	0.11822	0.05011	0.01928	0.00209	0.00014
	2	0.94975	0.76180	0.51976	0.30962	0.16370	0.07739	0.01232	0.00117
	3	0.99120	0.91736	0.75561	0.54888	0.35302	0.20191	0.04642	0.00636
	4	0.99884	0.97786	0.90129	0.75822	0.57389	0.38869	0.12600	0.02452
	5	0.99988	0.99533	0.96813	0.89430	0.76531	0.59682	0.26393	0.07173
	6	0.99999	0.99922	0.99172	0.96234	0.89292	0.77522	0.44784	0.16615
	7	1.00000	0.99989	0.99826	0.98907	0.95976	0.89536	0.64051	0.31453
	8	1.00000	0.99999	0.99970	0.99742	0.98762	0.95972	0.80106	0.50000
	9	1.00000	1.00000	0.99996	0.99951	0.99690	0.98731	0.90810	0.68547
	10	1.00000	1.00000	1.00000	0.99992	0.99937	0.99676	0.96519	0.83385
	11	1.00000	1.00000	1.00000	0.99999	0.99990	0.99934	0.98941	0.92827
	12	1.00000	1.00000	1.00000	1.00000	0.99999	0.99990	0.99748	0.97548
	13	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99955	0.99364
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99994	0.99883
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99986
	16	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999
18	0	0.39721	0.15009	0.05365	0.01801	0.00564	0.00163	0.00010	0.00000
	1	0.77352	0.45028	0.22405	0.09908	0.03946	0.01419	0.00132	0.00007
	2	0.94187	0.73380	0.47966	0.27134	0.13531	0.05995	0.00823	0.00066
	3	0.98913	0.90180	0.72024	0.50103	0.30569	0.16455	0.03278	0.00377
	4	0.99845	0.97181	0.87944	0.71635	0.51867	0.33265	0.09417	0.01544
	5	0.99983	0.99358	0.95810	0.86708	0.71745	0.53438	0.20876	0.04813
	6	0.99998	0.99883	0.98818	0.94873	0.86102	0.72170	0.37428	0.11894
	7	1.00000	0.99983	0.99728	0.98372	0.94305	0.85932	0.56344	0.24034
	8	1.00000	0.99998	0.99949	0.99575	0.98065	0.94041	0.73684	0.40726
	9	1.00000	1.00000	0.99992	0.99909	0.99458	0.97903	0.86529	0.59274
	10	1.00000	1.00000	0.99999	0.99984	0.99876	0.99393	0.94235	0.75966
	11	1.00000	1.00000	1.00000	0.99998	0.99977	0.99857	0.97972	0.88106
	12	1.00000	1.00000	1.00000	1.00000	0.99997	0.99973	0.99425	0.95187
	13	1.00000	1.00000	1.00000	1.00000	1.00000	0.99996	0.99872	0.98456
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99979	0.99623
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99934
	16	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99993
	17	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
19	0	0.37735	0.13509	0.04560	0.01441	0.00423	0.00114	0.00006	0.00000
	1	0.75471	0.42026	0.19849	0.08287	0.03101	0.01042	0.00083	0.00004
	2	0.93345	0.70544	0.44132	0.23689	0.11134	0.04622	0.00546	0.00036
	3	0.98676	0.88500	0.68415	0.45509	0.26309	0.13317	0.02296	0.00221
	4	0.99799	0.96481	0.85556	0.67329	0.46542	0.28222	0.06961	0.00961
	5	0.99976	0.99141	0.94630	0.83694	0.66776	0.47386	0.16292	0.03178
	6	0.99998	0.99830	0.98367	0.93240	0.82512	0.66550	0.30807	0.08353
	7	1.00000	0.99973	0.99592	0.97672	0.92254	0.81803	0.48778	0.17964
	8	1.00000	0.99996	0.99916	0.99334	0.97125	0.91608	0.66748	0.32380
	9	1.00000	1.00000	0.99986	0.99842	0.99110	0.96745	0.81391	0.50000
	10	1.00000	1.00000	0.99998	0.99969	0.99771	0.98946	0.91153	0.67620
	11	1.00000	1.00000	1.00000	0.99995	0.99952	0.99718	0.96477	0.82036
	12	1.00000	1.00000	1.00000	0.99999	0.99992	0.99938	0.98844	0.91647
	13	1.00000	1.00000	1.00000	1.00000	0.99999	0.99989	0.99693	0.96822
	14	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99936	0.99039
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99990	0.99779
	16	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99964
	17	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99996
	18	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 3 Poisson Probabilities

$$P(Y \leq a) = \sum_{y=0}^{a} e^{-\lambda} \frac{\lambda^y}{y!}$$

λ \ a	0	1	2	3	4	5	6	7	8	9
0.02	0.980	1.000								
0.04	0.961	0.999	1.000							
0.06	0.942	0.998	1.000							
0.08	0.923	0.997	1.000							
0.10	0.905	0.995	1.000							
0.15	0.861	0.990	0.999	1.000						
0.20	0.819	0.982	0.999	1.000						
0.25	0.779	0.974	0.998	1.000						
0.30	0.741	0.963	0.996	1.000						
0.35	0.705	0.951	0.994	1.000						
0.40	0.670	0.938	0.992	0.999	1.000					
0.45	0.638	0.925	0.989	0.999	1.000					
0.50	0.607	0.910	0.986	0.998	1.000					
0.55	0.577	0.894	0.982	0.988	1.000					
0.60	0.549	0.878	0.977	0.997	1.000					
0.65	0.522	0.861	0.972	0.996	0.999	1.000				
0.70	0.497	0.844	0.966	0.994	0.999	1.000				
0.75	0.472	0.827	0.959	0.993	0.999	1.000				
0.80	0.449	0.809	0.953	0.991	0.999	1.000				
0.85	0.427	0.791	0.945	0.989	0.998	1.000				
0.90	0.407	0.772	0.937	0.987	0.998	1.000				
0.95	0.387	0.754	0.929	0.981	0.997	1.000				
1.00	0.368	0.736	0.920	0.981	0.996	0.999	1.000			
1.1	0.333	0.699	0.900	0.974	0.995	0.999	1.000			
1.2	0.301	0.663	0.879	0.966	0.992	0.998	1.000			
1.3	0.273	0.627	0.857	0.957	0.989	0.998	1.000			
1.4	0.247	0.592	0.833	0.946	0.986	0.997	0.999	1.000		
1.5	0.223	0.558	0.809	0.934	0.981	0.996	0.999	1.000		
1.6	0.202	0.525	0.783	0.921	0.976	0.994	0.999	1.000		
1.7	0.183	0.493	0.757	0.907	0.970	0.992	0.998	1.000		
1.8	0.165	0.463	0.731	0.891	0.964	0.990	0.997	0.999	1.000	
1.9	0.150	0.434	0.704	0.875	0.956	0.987	0.997	0.999	1.000	
2.0	0.135	0.406	0.677	0.857	0.947	0.983	0.995	0.999	1.000	

Table 3 (***Continued***)

λ \ a	0	1	2	3	4	5	6	7	8	9
2.2	0.111	0.355	0.623	0.819	0.928	0.975	0.993	0.998	1.000	
2.4	0.091	0.308	0.570	0.779	0.904	0.964	0.988	0.997	0.999	1.000
2.6	0.074	0.267	0.518	0.736	0.877	0.951	0.983	0.995	0.999	1.000
2.8	0.061	0.231	0.469	0.692	0.848	0.935	0.976	0.992	0.998	0.999
3.0	0.050	0.199	0.423	0.647	0.815	0.916	0.966	0.988	0.996	0.999
3.2	0.041	0.171	0.380	0.603	0.781	0.895	0.955	0.983	0.994	0.998
3.4	0.033	0.147	0.340	0.558	0.744	0.871	0.942	0.977	0.992	0.997
3.6	0.027	0.126	0.303	0.515	0.706	0.844	0.927	0.969	0.988	0.996
3.8	0.022	0.107	0.269	0.473	0.668	0.816	0.909	0.960	0.984	0.994
4.0	0.018	0.092	0.238	0.433	0.629	0.785	0.889	0.949	0.979	0.992
4.2	0.015	0.078	0.210	0.395	0.590	0.753	0.867	0.936	0.972	0.989
4.4	0.012	0.066	0.185	0.359	0.551	0.720	0.844	0.921	0.964	0.985
4.6	0.010	0.056	0.163	0.326	0.513	0.686	0.818	0.905	0.955	0.980
4.8	0.008	0.048	0.143	0.294	0.476	0.651	0.791	0.887	0.944	0.975
5.0	0.007	0.040	0.125	0.265	0.440	0.616	0.762	0.867	0.932	0.968
5.2	0.006	0.034	0.109	0.238	0.406	0.581	0.732	0.845	0.918	0.960
5.4	0.005	0.029	0.095	0.213	0.373	0.546	0.702	0.822	0.903	0.951
5.6	0.004	0.024	0.082	0.191	0.342	0.512	0.670	0.797	0.886	0.941
5.8	0.003	0.021	0.072	0.170	0.313	0.478	0.638	0.771	0.867	0.929
6.0	0.002	0.017	0.062	0.151	0.285	0.446	0.606	0.744	0.847	0.916

λ \ a	10	11	12	13	14	15	16
2.8	1.000						
3.0	1.000						
3.2	1.000						
3.4	0.999	1.000					
3.6	0.999	1.000					
3.8	0.998	0.999	1.000				
4.0	0.997	0.999	1.000				
4.2	0.996	0.999	1.000				
4.4	0.994	0.998	0.999	1.000			
4.6	0.992	0.997	0.999	1.000			
4.8	0.990	0.996	0.999	1.000			
5.0	0.986	0.995	0.998	0.999	1.000		
5.2	0.982	0.993	0.997	0.999	1.000		
5.4	0.977	0.990	0.996	0.999	1.000		
5.6	0.972	0.988	0.995	0.998	0.999	1.000	
5.8	0.965	0.984	0.993	0.997	0.999	1.000	
6.0	0.957	0.980	0.991	0.996	0.999	0.999	1.000

Table 3 (*Continued*)

λ \ a	0	1	2	3	4	5	6	7	8	9
6.2	0.002	0.015	0.054	0.134	0.259	0.414	0.574	0.716	0.826	0.902
6.4	0.002	0.012	0.046	0.119	0.235	0.384	0.542	0.687	0.803	0.886
6.6	0.001	0.010	0.040	0.105	0.213	0.355	0.511	0.658	0.780	0.869
6.8	0.001	0.009	0.034	0.093	0.192	0.327	0.480	0.628	0.755	0.850
7.0	0.001	0.007	0.030	0.082	0.173	0.301	0.450	0.599	0.729	0.830
7.2	0.001	0.006	0.025	0.072	0.156	0.276	0.420	0.569	0.703	0.810
7.4	0.001	0.005	0.022	0.063	0.140	0.253	0.392	0.539	0.676	0.788
7.6	0.001	0.004	0.019	0.055	0.125	0.231	0.365	0.510	0.648	0.765
7.8	0.000	0.004	0.016	0.048	0.112	0.210	0.338	0.481	0.620	0.741
8.0	0.000	0.003	0.014	0.042	0.100	0.191	0.313	0.453	0.593	0.717
8.5	0.000	0.002	0.009	0.030	0.074	0.150	0.256	0.386	0.523	0.653
9.0	0.000	0.001	0.006	0.021	0.055	0.116	0.207	0.324	0.456	0.587
9.5	0.000	0.001	0.004	0.015	0.040	0.089	0.165	0.269	0.392	0.522
10.0	0.000	0.000	0.003	0.010	0.029	0.067	0.130	0.220	0.333	0.458
	10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
6.2	0.949	0.975	0.989	0.995	0.998	0.999	1.000			
6.4	0.939	0.969	0.986	0.994	0.997	0.999	1.000			
6.6	0.927	0.963	0.982	0.992	0.997	0.999	0.999	1.000		
6.8	0.915	0.955	0.978	0.990	0.996	0.998	0.999	1.000		
7.0	0.901	0.947	0.973	0.987	0.994	0.998	0.999	1.000		
7.2	0.887	0.937	0.967	0.984	0.993	0.997	0.999	0.999	1.000	
7.4	0.871	0.926	0.961	0.980	0.991	0.996	0.998	0.999	1.000	
7.6	0.854	0.915	0.954	0.976	0.989	0.995	0.998	0.999	1.000	
7.8	0.835	0.902	0.945	0.971	0.986	0.993	0.997	0.999	1.000	
8.0	0.816	0.888	0.936	0.966	0.983	0.992	0.996	0.998	0.999	1.000
8.5	0.763	0.849	0.909	0.949	0.973	0.986	0.993	0.997	0.999	0.999
9.0	0.706	0.803	0.876	0.926	0.959	0.978	0.989	0.995	0.998	0.999
9.5	0.645	0.752	0.836	0.898	0.940	0.967	0.982	0.991	0.996	0.998
10.0	0.583	0.697	0.792	0.864	0.917	0.951	0.973	0.986	0.993	0.997
	20	**21**	**22**							
8.5	1.000									
9.0	1.000									
9.5	0.999	1.000								
10.0	0.998	0.999	1.000							

Table 3 (***Continued***)

λ \ a	0	1	2	3	4	5	6	7	8	9
10.5	0.000	0.000	0.002	0.007	0.021	0.050	0.102	0.179	0.279	0.397
11.0	0.000	0.000	0.001	0.005	0.015	0.038	0.079	0.143	0.232	0.341
11.5	0.000	0.000	0.001	0.003	0.011	0.028	0.060	0.114	0.191	0.289
12.0	0.000	0.000	0.001	0.002	0.008	0.020	0.046	0.090	0.155	0.242
12.5	0.000	0.000	0.000	0.002	0.005	0.015	0.035	0.070	0.125	0.201
13.0	0.000	0.000	0.000	0.001	0.004	0.011	0.026	0.054	0.100	0.166
13.5	0.000	0.000	0.000	0.001	0.003	0.008	0.019	0.041	0.079	0.135
14.0	0.000	0.000	0.000	0.000	0.002	0.006	0.014	0.032	0.062	0.109
14.5	0.000	0.000	0.000	0.000	0.001	0.004	0.010	0.024	0.048	0.088
15.0	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.018	0.037	0.070

	10	11	12	13	14	15	16	17	18	19
10.5	0.521	0.639	0.742	0.825	0.888	0.932	0.960	0.978	0.988	0.994
11.0	0.460	0.579	0.689	0.781	0.854	0.907	0.944	0.968	0.982	0.991
11.5	0.402	0.520	0.633	0.733	0.815	0.878	0.924	0.954	0.974	0.986
12.0	0.347	0.462	0.576	0.682	0.772	0.844	0.899	0.937	0.963	0.979
12.5	0.297	0.406	0.519	0.628	0.725	0.806	0.869	0.916	0.948	0.969
13.0	0.252	0.353	0.463	0.573	0.675	0.764	0.835	0.890	0.930	0.957
13.5	0.211	0.304	0.409	0.518	0.623	0.718	0.798	0.861	0.908	0.942
14.0	0.176	0.260	0.358	0.464	0.570	0.669	0.756	0.827	0.883	0.923
14.5	0.145	0.220	0.311	0.413	0.518	0.619	0.711	0.790	0.853	0.901
15.0	0.118	0.185	0.268	0.363	0.466	0.568	0.664	0.749	0.819	0.875

	20	21	22	23	24	25	26	27	28	29
10.5	0.997	0.999	0.999	1.000						
11.0	0.995	0.998	0.999	1.000						
11.5	0.992	0.996	0.998	0.999	1.000					
12.0	0.988	0.994	0.997	0.999	0.999	1.000				
12.5	0.983	0.991	0.995	0.998	0.999	0.999	1.000			
13.0	0.975	0.986	0.992	0.996	0.998	0.999	1.000			
13.5	0.965	0.980	0.989	0.994	0.997	0.998	0.999	1.000		
14.0	0.952	0.971	0.983	0.991	0.995	0.997	0.999	0.999	1.000	
14.5	0.936	0.960	0.976	0.986	0.992	0.996	0.998	0.999	0.999	1.000
15.0	0.917	0.947	0.967	0.981	0.989	0.994	0.997	0.998	0.999	1.000

Table 3 (***Continued***)

λ \ a	4	5	6	7	8	9	10	11	12	13
16	0.000	0.001	0.004	0.010	0.022	0.043	0.077	0.127	0.193	0.275
17	0.000	0.001	0.002	0.005	0.013	0.026	0.049	0.085	0.135	0.201
18	0.000	0.000	0.001	0.003	0.007	0.015	0.030	0.055	0.092	0.143
19	0.000	0.000	0.001	0.002	0.004	0.009	0.018	0.035	0.061	0.098
20	0.000	0.000	0.000	0.001	0.002	0.005	0.011	0.021	0.039	0.066
21	0.000	0.000	0.000	0.000	0.001	0.003	0.006	0.013	0.025	0.043
22	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.008	0.015	0.028
23	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.009	0.017
24	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.005	0.011
25	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.006

λ \ a	14	15	16	17	18	19	20	21	22	23
16	0.368	0.467	0.566	0.659	0.742	0.812	0.868	0.911	0.942	0.963
17	0.281	0.371	0.468	0.564	0.655	0.736	0.805	0.861	0.905	0.937
18	0.208	0.287	0.375	0.469	0.562	0.651	0.731	0.799	0.855	0.899
19	0.150	0.215	0.292	0.378	0.469	0.561	0.647	0.725	0.793	0.849
20	0.105	0.157	0.221	0.297	0.381	0.470	0.559	0.644	0.721	0.787
21	0.072	0.111	0.163	0.227	0.302	0.384	0.471	0.558	0.640	0.716
22	0.048	0.077	0.117	0.169	0.232	0.306	0.387	0.472	0.556	0.637
23	0.031	0.052	0.082	0.123	0.175	0.238	0.310	0.389	0.472	0.555
24	0.020	0.034	0.056	0.087	0.128	0.180	0.243	0.314	0.392	0.473
25	0.012	0.022	0.038	0.060	0.092	0.134	0.185	0.247	0.318	0.394

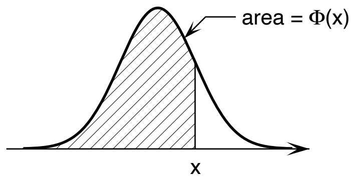
λ \ a	24	25	26	27	28	29	30	31	32	33
16	0.978	0.987	0.993	0.996	0.998	0.999	0.999	1.000		
17	0.959	0.975	0.985	0.991	0.995	0.997	0.999	0.999	1.000	
18	0.932	0.955	0.972	0.983	0.990	0.994	0.997	0.998	0.999	1.000
19	0.893	0.927	0.951	0.969	0.980	0.988	0.993	0.996	0.998	0.999
20	0.843	0.888	0.922	0.948	0.966	0.978	0.987	0.992	0.995	0.997
21	0.782	0.838	0.883	0.917	0.944	0.963	0.976	0.985	0.991	0.994
22	0.712	0.777	0.832	0.877	0.913	0.940	0.959	0.973	0.983	0.989
23	0.635	0.708	0.772	0.827	0.873	0.908	0.936	0.956	0.971	0.981
24	0.554	0.632	0.704	0.768	0.823	0.868	0.904	0.932	0.953	0.969
25	0.473	0.553	0.629	0.700	0.763	0.818	0.863	0.900	0.929	0.950

λ \ a	34	35	36	37	38	39	40	41	42	43
19	0.999	1.000								
20	0.999	0.999	1.000							
21	0.997	0.998	0.999	0.999	1.000					
22	0.994	0.996	0.998	0.999	0.999	1.000				
23	0.988	0.993	0.996	0.997	0.999	0.999	1.000			
24	0.979	0.987	0.992	0.995	0.997	0.998	0.999	0.999	1.000	
25	0.966	0.978	0.985	0.991	0.991	0.997	0.998	0.999	0.999	1.000

Tabeller

Tabell 1. Standardiserad normalfördelning

$\Phi(x) = P(X \leq x)$ där $X \in N(0, 1)$
För negativa värden, utnyttja att $\Phi(x) = 1 - \Phi(-x)$

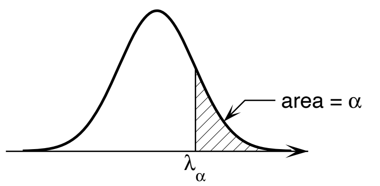


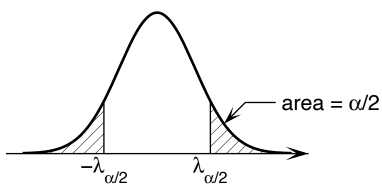
x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865									
3.1	.99903									
3.2	.99931									
3.3	.99952									
3.4	.99966									
3.5	.99977									
3.6	.99984									
3.7	.99989									
3.8	.99993									
3.9	.99995									
4.0	.99997									

Tabell 2. Normalfördelningens kvantiler

$P(X > \lambda_\alpha) = \alpha$ där $X \in N(0, 1)$

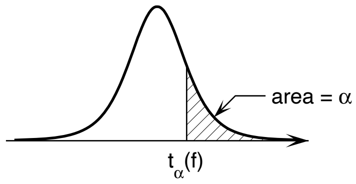
α	λ_α	α	λ_α
0.1	1.2816	0.001	3.0902
0.05	1.6449	0.0005	3.2905
0.025	1.9600	0.0001	3.7190
0.01	2.3263	0.00005	3.8906
0.005	2.5758	0.00001	4.2649





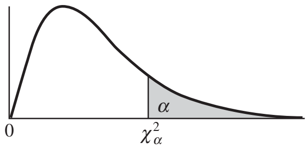
Tabell 3. t**-fördelningen**

$P(X > t_\alpha(f)) = \alpha$ där $X \in t(f)$



f	α 0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.08	6.31	12.71	31.82	63.66	318.31	636.62
2	1.89	2.92	4.30	6.96	9.92	22.33	31.60
3	1.64	2.35	3.18	4.54	5.84	10.21	12.92
4	1.53	2.13	2.78	3.75	4.60	7.17	8.61
5	1.48	2.02	2.57	3.36	4.03	5.89	6.87
6	1.44	1.94	2.45	3.14	3.71	5.21	5.96
7	1.41	1.89	2.36	3.00	3.50	4.79	5.41
8	1.40	1.86	2.31	2.90	3.36	4.50	5.04
9	1.38	1.83	2.26	2.82	3.25	4.30	4.78
10	1.37	1.81	2.23	2.76	3.17	4.14	4.59
11	1.36	1.80	2.20	2.72	3.11	4.02	4.44
12	1.36	1.78	2.18	2.68	3.05	3.93	4.32
13	1.35	1.77	2.16	2.65	3.01	3.85	4.22
14	1.35	1.76	2.14	2.62	2.98	3.79	4.14
15	1.34	1.75	2.13	2.60	2.95	3.73	4.07
16	1.34	1.75	2.12	2.58	2.92	3.69	4.01
17	1.33	1.74	2.11	2.57	2.90	3.65	3.97
18	1.33	1.73	2.10	2.55	2.88	3.61	3.92
19	1.33	1.73	2.09	2.54	2.86	3.58	3.88
20	1.33	1.72	2.09	2.53	2.85	3.55	3.85
21	1.32	1.72	2.08	2.52	2.83	3.53	3.82
22	1.32	1.72	2.07	2.51	2.82	3.50	3.79
23	1.32	1.71	2.07	2.50	2.81	3.48	3.77
24	1.32	1.71	2.06	2.49	2.80	3.47	3.75
25	1.32	1.71	2.06	2.49	2.79	3.45	3.73
26	1.31	1.71	2.06	2.48	2.78	3.43	3.71
27	1.31	1.70	2.05	2.47	2.77	3.42	3.69
28	1.31	1.70	2.05	2.47	2.76	3.41	3.67
29	1.31	1.70	2.05	2.46	2.76	3.40	3.66
30	1.31	1.70	2.04	2.46	2.75	3.39	3.65
40	1.30	1.68	2.02	2.42	2.70	3.31	3.55
60	1.30	1.67	2.00	2.39	2.66	3.23	3.46
120	1.29	1.66	1.98	2.36	2.62	3.16	3.37
∞	1.28	1.64	1.96	2.33	2.58	3.09	3.29

Table 6 Percentage Points of the χ^2 Distributions



df	$\chi^2_{0.995}$	$\chi^2_{0.990}$	$\chi^2_{0.975}$	$\chi^2_{0.950}$	$\chi^2_{0.900}$
1	0.0000393	0.0001571	0.0009821	0.0039321	0.0157908
2	0.0100251	0.0201007	0.0506356	0.102587	0.210720
3	0.0717212	0.114832	0.215795	0.351846	0.584375
4	0.206990	0.297110	0.484419	0.710721	1.063623
5	0.411740	0.554300	0.831211	1.145476	1.61031
6	0.675727	0.872085	1.237347	1.63539	2.20413
7	0.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100	67.3276	70.0648	74.2219	77.9295	82.3581

Table 6 (***Continued***)

$\chi^2_{0.100}$	$\chi^2_{0.050}$	$\chi^2_{0.025}$	$\chi^2_{0.010}$	$\chi^2_{0.005}$	df
2.70554	3.84146	5.02389	6.63490	7.87944	1
4.60517	5.99147	7.37776	9.21034	10.5966	2
6.25139	7.81473	9.34840	11.3449	12.8381	3
7.77944	9.48773	11.1433	13.2767	14.8602	4
9.23635	11.0705	12.8325	15.0863	16.7496	5
10.6446	12.5916	14.4494	16.8119	18.5476	6
12.0170	14.0671	16.0128	18.4753	20.2777	7
13.3616	15.5073	17.5346	20.0902	21.9550	8
14.6837	16.9190	19.0228	21.6660	23.5893	9
15.9871	18.3070	20.4831	23.2093	25.1882	10
17.2750	19.6751	21.9200	24.7250	26.7569	11
18.5494	21.0261	23.3367	26.2170	28.2995	12
19.8119	22.3621	24.7356	27.6883	29.8194	13
21.0642	23.6848	26.1190	29.1413	31.3193	14
22.3072	24.9958	27.4884	30.5779	32.8013	15
23.5418	26.2962	28.8454	31.9999	34.2672	16
24.7690	27.5871	30.1910	33.4087	35.7185	17
25.9894	28.8693	31.5264	34.8053	37.1564	18
27.2036	30.1435	32.8523	36.1908	38.5822	19
28.4120	31.4104	34.1696	37.5662	39.9968	20
29.6151	32.6705	35.4789	38.9321	41.4010	21
30.8133	33.9244	36.7807	40.2894	42.7956	22
32.0069	35.1725	38.0757	41.6384	44.1813	23
33.1963	36.4151	39.3641	42.9798	45.5585	24
34.3816	37.6525	40.6465	44.3141	46.9278	25
35.5631	38.8852	41.9232	45.6417	48.2899	26
36.7412	40.1133	43.1944	46.9630	49.6449	27
37.9159	41.3372	44.4607	48.2782	50.9933	28
39.0875	42.5569	45.7222	49.5879	52.3356	29
40.2560	43.7729	46.9792	50.8922	53.6720	30
51.8050	55.7585	59.3417	63.6907	66.7659	40
63.1671	67.5048	71.4202	76.1539	79.4900	50
74.3970	79.0819	83.2976	88.3794	91.9517	60
85.5271	90.5312	95.0231	100.425	104.215	70
96.5782	101.879	106.629	112.329	116.321	80
107.565	113.145	118.136	124.116	128.299	90
118.498	124.342	129.561	135.807	140.169	100

From "Tables of the Percentage Points of the χ^2-Distribution." *Biometrika*, Vol. 32 (1941), pp. 188–189, by Catherine M. Thompson.

Table 8 Distribution Function of *U*

$P(U \leq U_0)$; U_0 is the argument; $n_1 \leq n_2$; $3 \leq n_2 \leq 10$.

$n_2 = 3$

	n_1		
U_0	1	2	3
0	.25	.10	.05
1	.50	.20	.10
2		.40	.20
3		.60	.35
4			.50

$n_2 = 4$

	n_1			
U_0	1	2	3	4
0	.2000	.0667	.0286	.0143
1	.4000	.1333	.0571	.0286
2	.6000	.2667	.1143	.0571
3		.4000	.2000	.1000
4		.6000	.3143	.1714
5			.4286	.2429
6			.5714	.3429
7				.4429
8				.5571

Table 8 (*Continued*)

$n_2 = 5$

	n_1				
U_0	1	2	3	4	5
0	.1667	.0476	.0179	.0079	.0040
1	.3333	.0952	.0357	.0159	.0079
2	.5000	.1905	.0714	.0317	.0159
3		.2857	.1250	.0556	.0278
4		.4286	.1964	.0952	.0476
5		.5714	.2857	.1429	.0754
6			.3929	.2063	.1111
7			.5000	.2778	.1548
8				.3651	.2103
9				.4524	.2738
10				.5476	.3452
11					.4206
12					.5000

$n_2 = 6$

	n_1					
U_0	1	2	3	4	5	6
0	.1429	.0357	.0119	.0048	.0022	.0011
1	.2857	.0714	.0238	.0095	.0043	.0022
2	.4286	.1429	.0476	.0190	.0087	.0043
3	.5714	.2143	.0833	.0333	.0152	.0076
4		.3214	.1310	.0571	.0260	.0130
5		.4286	.1905	.0857	.0411	.0206
6		.5714	.2738	.1286	.0628	.0325
7			.3571	.1762	.0887	.0465
8			.4524	.2381	.1234	.0660
9			.5476	.3048	.1645	.0898
10				.3810	.2143	.1201
11				.4571	.2684	.1548
12				.5429	.3312	.1970
13					.3961	.2424
14					.4654	.2944
15					.5346	.3496
16						.4091
17						.4686
18						.5314

Table 8 (***Continued***)

$n_2 = 7$

	n_1						
U_0	1	2	3	4	5	6	7
0	.1250	.0278	.0083	.0030	.0013	.0006	.0003
1	.2500	.0556	.0167	.0061	.0025	.0012	.0006
2	.3750	.1111	.0333	.0121	.0051	.0023	.0012
3	.5000	.1667	.0583	.0212	.0088	.0041	.0020
4		.2500	.0917	.0364	.0152	.0070	.0035
5		.3333	.1333	.0545	.0240	.0111	.0055
6		.4444	.1917	.0818	.0366	.0175	.0087
7		.5556	.2583	.1152	.0530	.0256	.0131
8			.3333	.1576	.0745	.0367	.0189
9			.4167	.2061	.1010	.0507	.0265
10			.5000	.2636	.1338	.0688	.0364
11				.3242	.1717	.0903	.0487
12				.3939	.2159	.1171	.0641
13				.4636	.2652	.1474	.0825
14				.5364	.3194	.1830	.1043
15					.3775	.2226	.1297
16					.4381	.2669	.1588
17					.5000	.3141	.1914
18						.3654	.2279
19						.4178	.2675
20						.4726	.3100
21						.5274	.3552
22							.4024
23							.4508
24							.5000

Table 8 (*Continued*)

$n_2 = 8$

	n_1							
U_0	1	2	3	4	5	6	7	8
0	.1111	.0222	.0061	.0020	.0008	.0003	.0002	.0001
1	.2222	.0444	.0121	.0040	.0016	.0007	.0003	.0002
2	.3333	.0889	.0242	.0081	.0031	.0013	.0006	.0003
3	.4444	.1333	.0424	.0141	.0054	.0023	.0011	.0005
4	.5556	.2000	.0667	.0242	.0093	.0040	.0019	.0009
5		.2667	.0970	.0364	.0148	.0063	.0030	.0015
6		.3556	.1394	.0545	.0225	.0100	.0047	.0023
7		.4444	.1879	.0768	.0326	.0147	.0070	.0035
8		.5556	.2485	.1071	.0466	.0213	.0103	.0052
9			.3152	.1414	.0637	.0296	.0145	.0074
10			.3879	.1838	.0855	.0406	.0200	.0103
11			.4606	.2303	.1111	.0539	.0270	.0141
12			.5394	.2848	.1422	.0709	.0361	.0190
13						.0906	.0469	.0249
14						1142	.0603	.0325
15						1412	.0760	.0415
16						1725	.0946	.0524
17						2068	.1159	.0652
18						2454	.1405	.0803
19						2864	.1678	.0974
20						3310	.1984	.1172
21						3773	.2317	.1393
22						4259	.2679	.1641
23						4749	.3063	.1911
24						.5251	.3472	.2209
25							.3894	.2527
26							.4333	.2869
27							.4775	.3227
28							.5225	.3605
29								.3992
30								.4392
31								.4796
32								.5204

Table 8 (***Continued***)

$n_2 = 9$

	n_1								
U_0	1	2	3	4	5	6	7	8	9
0	.1000	.0182	.0045	.0014	.0005	.0002	.0001	.0000	.0000
1	.2000	.0364	.0091	.0028	.0010	.0004	.0002	.0001	.0000
2	.3000	.0727	.0182	.0056	.0020	.0008	.0003	.0002	.0001
3	.4000	.1091	.0318	.0098	.0035	.0014	.0006	.0003	.0001
4	.5000	.1636	.0500	.0168	.0060	.0024	.0010	.0005	.0002
5		.2182	.0727	.0252	.0095	.0038	.0017	.0008	.0004
6		.2909	.1045	.0378	.0145	.0060	.0026	.0012	.0006
7		.3636	.1409	.0531	.0210	.0088	.0039	.0019	.0009
8		.4545	.1864	.0741	.0300	.0128	.0058	.0028	.0014
9		.5455	.2409	.0993	.0415	.0180	.0082	.0039	.0020
10			.3000	.1301	.0559	.0248	.0115	.0056	.0028
11			.3636	.1650	.0734	.0332	.0156	.0076	.0039
12			.4318	.2070	.0949	.0440	.0209	.0103	.0053
13			.5000	.2517	.1199	.0567	.0274	.0137	.0071
14				.3021	.1489	.0723	.0356	.0180	.0094
15				.3552	.1818	.0905	.0454	.0232	.0122
16				.4126	.2188	.1119	.0571	.0296	.0157
17				.4699	.2592	.1361	.0708	.0372	.0200
18				.5301	.3032	.1638	.0869	.0464	.0252
19					.3497	.1942	.1052	.0570	.0313
20					.3986	.2280	.1261	.0694	.0385
21					.4491	.2643	.1496	.0836	.0470
22					.5000	.3035	.1755	.0998	.0567
23						.3445	.2039	.1179	.0680
24						.3878	.2349	.1383	.0807
25						.4320	.2680	.1606	.0951
26						.4773	.3032	.1852	.1112
27						.5227	.3403	.2117	.1290
28							.3788	.2404	.1487
29							.4185	.2707	.1701
30							.4591	.3029	.1933
31							.5000	.3365	.2181
32								.3715	.2447
33								.4074	.2729
34								.4442	.3024
35								.4813	.3332
36								.5187	.3652
37									.3981
38									.4317
39									.4657
40									.5000

Table 8 (*Continued*)

$n_2 = 10$

	n_1									
U_0	1	2	3	4	5	6	7	8	9	10
0	.0909	.0152	.0035	.0010	.0003	.0001	.0001	.0000	.0000	.0000
1	.1818	.0303	.0070	.0020	.0007	.0002	.0001	.0000	.0000	.0000
2	.2727	.0606	.0140	.0040	.0013	.0005	.0002	.0001	.0000	.0000
3	.3636	.0909	.0245	.0070	.0023	.0009	.0004	.0002	.0001	.0000
4	.4545	.1364	.0385	.0120	.0040	.0015	.0006	.0003	.0001	.0001
5	.5455	.1818	.0559	.0180	.0063	.0024	.0010	.0004	.0002	.0001
6		.2424	.0804	.0270	.0097	.0037	.0015	.0007	.0003	.0002
7		.3030	.1084	.0380	.0140	.0055	.0023	.0010	.0005	.0002
8		.3788	.1434	.0529	.0200	.0080	.0034	.0015	.0007	.0004
9		.4545	.1853	.0709	.0276	.0112	.0048	.0022	.0011	.0005
10		.5455	.2343	.0939	.0376	.0156	.0068	.0031	.0015	.0008
11			.2867	.1199	.0496	.0210	.0093	.0043	.0021	.0010
12			.3462	.1518	.0646	.0280	.0125	.0058	.0028	.0014
13			.4056	.1868	.0823	.0363	.0165	.0078	.0038	.0019
14			.4685	.2268	.1032	.0467	.0215	.0103	.0051	.0026
15			.5315	.2697	.1272	.0589	.0277	.0133	.0066	.0034
16				.3177	.1548	.0736	.0351	.0171	.0086	.0045
17				.3666	.1855	.0903	.0439	.0217	.0110	.0057
18				.4196	.2198	.1099	.0544	.0273	.0140	.0073
19				.4725	.2567	.1317	.0665	.0338	.0175	.0093
20				.5275	.2970	.1566	.0806	.0416	.0217	.0116
21					.3393	.1838	.0966	.0506	.0267	.0144
22					.3839	.2139	.1148	.0610	.0326	.0177
23					.4296	.2461	.1349	.0729	.0394	.0216
24					.4765	.2811	.1574	.0864	.0474	.0262
25					.5235	.3177	.1819	.1015	.0564	.0315
26						.3564	.2087	.1185	.0667	.0376
27						.3962	.2374	.1371	.0782	.0446
28						.4374	.2681	.1577	.0912	.0526
29						.4789	.3004	.1800	.1055	.0615
30						.5211	.3345	.2041	.1214	.0716
31							.3698	.2299	.1388	.0827
32							.4063	.2574	.1577	.0952
33							.4434	.2863	.1781	.1088
34							.4811	.3167	.2001	.1237
35							.5189	.3482	.2235	.1399
36								.3809	.2483	.1575
37								.4143	.2745	.1763
38								.4484	.3019	.1965
39								.4827	.3304	.2179

Table 8 (***Continued***)

$n_2 = 10$

	n_1									
U_0	1	2	3	4	5	6	7	8	9	10
40								.5173	.3598	.2406
41									.3901	.2644
42									.4211	.2894
43									.4524	.3153
44									.4841	.3421
45									.5159	.3697
46										.3980
47										.4267
48										.4559
49										.4853
50										.5147

Computed by M. Pagano, Department of Statistics, University of Florida.

Table 9 Critical Values of *T* in the Wilcoxon Matched-Pairs, Signed-Ranks Test; *n* = 5(1)50

One-sided	Two-sided	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
$P = .05$	$P = .10$	1	2	4	6	8	11
$P = .025$	$P = .05$		1	2	4	6	8
$P = .01$	$P = .02$			0	2	3	5
$P = .005$	$P = .01$				0	2	3
One-sided	**Two-sided**	$n = 11$	$n = 12$	$n = 13$	$n = 14$	$n = 15$	$n = 16$
$P = .05$	$P = .10$	14	17	21	26	30	36
$P = .025$	$P = .05$	11	14	17	21	25	30
$P = .01$	$P = .02$	7	10	13	16	20	24
$P = .005$	$P = .01$	5	7	10	13	16	19
One-sided	**Two-sided**	$n = 17$	$n = 18$	$n = 19$	$n = 20$	$n = 21$	$n = 22$
$P = .05$	$P = .10$	41	47	54	60	68	75
$P = .025$	$P = .05$	35	40	46	52	59	66
$P = .01$	$P = .02$	28	33	38	43	49	56
$P = .005$	$P = .01$	23	28	32	37	43	49
One-sided	**Two-sided**	$n = 23$	$n = 24$	$n = 25$	$n = 26$	$n = 27$	$n = 28$
$P = .05$	$P = .10$	83	92	101	110	120	130
$P = .025$	$P = .05$	73	81	90	98	107	117
$P = .01$	$P = .02$	62	69	77	85	93	102
$P = .005$	$P = .01$	55	68	68	76	84	92

Table 9 (*Continued*)

One-sided	Two-sided	$n = 29$	$n = 30$	$n = 31$	$n = 32$	$n = 33$	$n = 34$
$P = .05$	$P = .10$	141	152	163	175	188	201
$P = .025$	$P = .05$	127	137	148	159	171	183
$P = .01$	$P = .02$	111	120	130	141	151	162
$P = .005$	$P = .01$	100	109	118	128	138	149
One-sided	Two-sided	$n = 35$	$n = 36$	$n = 37$	$n = 38$	$n = 39$	
$P = .05$	$P = .10$	214	228	242	256	271	
$P = .025$	$P = .05$	195	208	222	235	250	
$P = .01$	$P = .02$	174	186	198	211	224	
$P = .005$	$P = .01$	160	171	183	195	208	
One-sided	Two-sided	$n = 40$	$n = 41$	$n = 42$	$n = 43$	$n = 44$	$n = 45$
$P = .05$	$P = .10$	287	303	319	336	353	371
$P = .025$	$P = .05$	264	279	295	311	327	344
$P = .01$	$P = .02$	238	252	267	281	297	313
$P = .005$	$P = .01$	221	234	248	262	277	292
One-sided	Two-sided	$n = 46$	$n = 47$	$n = 48$	$n = 49$	$n = 50$	
$P = .05$	$P = .10$	389	408	427	446	466	
$P = .025$	$P = .05$	361	379	397	415	434	
$P = .01$	$P = .02$	329	345	362	380	398	
$P = .005$	$P = .01$	307	323	339	356	373	

From "Some Rapid Approximate Statistical Procedures" (1964), 28, F. Wilcoxon and R. A. Wilcox.

Table 10 Distribution of the Total Number of Runs R in Samples of Size (n_1, n_2); $P(R \leq a)$

	a								
(n_1, n_2)	2	3	4	5	6	7	8	9	10
(2, 3)	.200	.500	.900	1.000					
(2, 4)	.133	.400	.800	1.000					
(2, 5)	.095	.333	.714	1.000					
(2, 6)	.071	.286	.643	1.000					
(2, 7)	.056	.250	.583	1.000					
(2, 8)	.044	.222	.533	1.000					
(2, 9)	.036	.200	.491	1.000					
(2, 10)	.030	.182	.455	1.000					
(3, 3)	.100	.300	.700	.900	1.000				
(3, 4)	.057	.200	.543	.800	.971	1.000			
(3, 5)	.036	.143	.429	.714	.929	1.000			
(3, 6)	.024	.107	.345	.643	.881	1.000			
(3, 7)	.017	.083	.283	.583	.833	1.000			
(3, 8)	.012	.067	.236	.533	.788	1.000			
(3, 9)	.009	.055	.200	.491	.745	1.000			
(3, 10)	.007	.045	.171	.455	.706	1.000			
(4, 4)	.029	.114	.371	.629	.886	.971	1.000		
(4, 5)	.016	.071	.262	.500	.786	.929	.992	1.000	
(4, 6)	.010	.048	.190	.405	.690	.881	.976	1.000	
(4, 7)	.006	.033	.142	.333	.606	.833	.954	1.000	
(4, 8)	.004	.024	.109	.279	.533	.788	.929	1.000	
(4, 9)	.003	.018	.085	.236	.471	.745	.902	1.000	
(4, 10)	.002	.014	.068	.203	.419	.706	.874	1.000	
(5, 5)	.008	.040	.167	.357	.643	.833	.960	.992	1.000
(5, 6)	.004	.024	.110	.262	.522	.738	.911	.976	.998
(5, 7)	.003	.015	.076	.197	.424	.652	.854	.955	.992
(5, 8)	.002	.010	.054	.152	.347	.576	.793	.929	.984
(5, 9)	.001	.007	.039	.119	.287	.510	.734	.902	.972
(5, 10)	.001	.005	.029	.095	.239	.455	.678	.874	.958
(6, 6)	.002	.013	.067	.175	.392	.608	.825	.933	.987
(6, 7)	.001	.008	.043	.121	.296	.500	.733	.879	.966
(6, 8)	.001	.005	.028	.086	.226	.413	.646	.821	.937
(6, 9)	.000	.003	.019	.063	.175	.343	.566	.762	.902
(6, 10)	.000	.002	.013	.047	.137	.288	.497	.706	.864
(7, 7)	.001	.004	.025	.078	.209	.383	.617	.791	.922
(7, 8)	.000	.002	.015	.051	.149	.296	.514	.704	.867
(7, 9)	.000	.001	.010	.035	.108	.231	.427	.622	.806
(7, 10)	.000	.001	.006	.024	.080	.182	.355	.549	.743
(8, 8)	.000	.001	.009	.032	.100	.214	.405	.595	.786
(8, 9)	.000	.001	.005	.020	.069	.157	.319	.500	.702
(8, 10)	.000	.000	.003	.013	.048	.117	.251	.419	.621
(9, 9)	.000	.000	.003	.012	.044	.109	.238	.399	.601
(9, 10)	.000	.000	.002	.008	.029	.077	.179	.319	.510
(10, 10)	.000	.000	.001	.004	.019	.051	.128	.242	.414

Table 10 (*Continued*)

(n_1, n_2)	*a*									
	11	12	13	14	15	16	17	18	19	20
(2, 3)										
(2, 4)										
(2, 5)										
(2, 6)										
(2, 7)										
(2, 8)										
(2, 9)										
(2, 10)										
(3, 3)										
(3, 4)										
(3, 5)										
(3, 6)										
(3, 7)										
(3, 8)										
(3, 9)										
(3, 10)										
(4, 4)										
(4, 5)										
(4, 6)										
(4, 7)										
(4, 8)										
(4, 9)										
(4, 10)										
(5, 5)										
(5, 6)	1.000									
(5, 7)	1.000									
(5, 8)	1.000									
(5, 9)	1.000									
(5, 10)	1.000									
(6, 6)	.998	1.000								
(6, 7)	.992	.999	1.000							
(6, 8)	.984	.998	1.000							
(6, 9)	.972	.994	1.000							
(6, 10)	.958	.990	1.000							
(7, 7)	.975	.996	.999	1.000						
(7, 8)	.949	.988	.998	1.000	1.000					
(7, 9)	.916	.975	.994	.999	1.000					
(7, 10)	.879	.957	.990	.998	1.000					
(8, 8)	.900	.968	.991	.999	1.000	1.000				
(8, 9)	.843	.939	.980	.996	.999	1.000	1.000			
(8, 10)	.782	.903	.964	.990	.998	1.000	1.000			
(9, 9)	.762	.891	.956	.988	.997	1.000	1.000	1.000		
(9, 10)	.681	.834	.923	.974	.992	.999	1.000	1.000	1.000	
(10, 10)	.586	.758	.872	.949	.981	.996	.999	1.000	1.000	1.000

From "Tables for Testing Randomness of Grouping in a Sequence of Alternatives," C. Eisenhart and F. Swed, *Annals of Mathematical Statistics*, Volume 14 (1943).

Table 11 Critical Values of Spearman's Rank Correlation Coefficient

n	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
5	0.900	—	—	—
6	0.829	0.886	0.943	—
7	0.714	0.786	0.893	—
8	0.643	0.738	0.833	0.881
9	0.600	0.683	0.783	0.833
10	0.564	0.648	0.745	0.794
11	0.523	0.623	0.736	0.818
12	0.497	0.591	0.703	0.780
13	0.475	0.566	0.673	0.745
14	0.457	0.545	0.646	0.716
15	0.441	0.525	0.623	0.689
16	0.425	0.507	0.601	0.666
17	0.412	0.490	0.582	0.645
18	0.399	0.476	0.564	0.625
19	0.388	0.462	0.549	0.608
20	0.377	0.450	0.534	0.591
21	0.368	0.438	0.521	0.576
22	0.359	0.428	0.508	0.562
23	0.351	0.418	0.496	0.549
24	0.343	0.409	0.485	0.537
25	0.336	0.400	0.475	0.526
26	0.329	0.392	0.465	0.515
27	0.323	0.385	0.456	0.505
28	0.317	0.377	0.448	0.496
29	0.311	0.370	0.440	0.487
30	0.305	0.364	0.432	0.478

From "Distribution of Sums of Squares of Rank Differences for Small Samples," E. G. Olds, *Annals of Mathematical Statistics*, Volume 9 (1938).