Stockholm University

Spring 2021
20210423

Department of Statistics

Exam: Multivariate Analysis, Advanced level, 7.5 ECTS credits

For questions about the content of the exam, contact the course coordinator on email tatjana.vonrosen@stat.su.se. Incoming e-mail questions are answered between 09.00 and 10.00.

If the course coordinator needs to send out information to all students during the exam, this is done to your registered email address. Therefore, check your email during the exam.

NOTE! The exam shall be submitted electronically via the department's web site **no later than 15.00 (3PM)**. **The system does not allow submission after deadline** which is a new setup for this semester. Therefore, start the submission well in advance. The last hour of the exam time is intended for arranging the electronic submission.

Please note that practical help is only available during the first hour of the exam by email expedition@stat.su.se. Carefully read the enclosed instructions for exam submission. There you find all the necessary information about submission, anonymous code, extended writing time etc. If you, despite the instructions have problems submitting the exam, email the exam to tenta@stat.su.se. However, this is only done in exceptional cases. Exams sent in by email after deadline will not be corrected.

NOTE! All forms of cooperation and plagiarism are prohibited. We go over all exams carefully to detect cheating. Suspected cheating is reported to the Disciplinary Board and can lead to suspension.

The exam consists of 4 exercises giving a total of 50 points. In order to get full score for an exercise provide detailed and well motivated solutions. In order to pass the exam at least 25 points are needed.

Exercise 1. (10p)

We consider the data collected in the empirical study where children in kindergarten were measured on various instruments to determine whether they could be classified as low risk or high risk with respect to having reading problems later on in school. The variables considered are word identification (**WI**), word comprehension (**WC**) and passage comprehension (**PC**). The data are provided in Table 1 (and in file Exercise1_23042021).

(a) For $\alpha = 0.05$, test the hypothesis than the mean performance on the three response variables is the same for both groups $H_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$ vs $H_1 : \boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_2$. Check the validity of the assumptions used in your analysis.
(b) Determine the 95% confidence region for $\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2$.
(c) Construct the 95% simultaneous confidence intervals for each $\mu_{1i} - \mu_{2i}$, $i = 1, 2, 3$.

Exercise 2. (10p)

The data in Table 2 (also stored in the file Exercise2_23042021.txt) comprises observations on three measures of motivation (denoted by MOTIV1, MOTIV2, MOTIV3) that are obtained for three socioeconomic status (SES) groups: lower, medium and high.

(a) Construct box-plots for variables MOTIV1, MOTIV2, MOTIV3 to compare different groups of SES. Discuss your plots.
(b) Conduct an appropriate analysis to test whether there are any SES group differences in motivation. Use $\alpha = 0.05$.
(c) In (b), state an appropriate model, hypotheses, assumptions. Check the validity of the assumptions.
(d) Discuss your findings.

Exercise 3. (15p)

The data (stored in the file Exercise3_23042021.txt) was collected on 11 subtests of the Wechsler Intelligence Scale for Children (WISC): information (INFO), similarities (SIMIL), arithmetic (ARITH), comprehension (COMP), vocabulary (VOCAB), digit span (DIGIT), picture completion (PICT), picture arrangement (PARANG), block design (BLOCK), object assembly (OBJECT) and coding (CODING). The grouping variable (PLAY) was preference for age of playmates, divided into children who have (1) preference for playmates younger than themselves, and (2) preference for playmates the same age as themselves or no preference.

(a) Plot the data.
(b) Conduct the profiles analysis for two groups.
(c) Discuss your findings.

Exercise 4. (15p)

Shin (1971) examined the relationship between creativity and achievement. He used Guilford's battery to obtain the following six creativity scores: ideational fluency (x_1), spontaneous flexibility (x_2), associational fluency (x_3), expressional fluency (x_4), originality (x_5), and elaboration (x_6). The Kropp test was used to obtain the following six achievement variables: knowledge (z_1), comprehension (z_2), application (z_3), analysis (z_4), synthesis (z_5) and evaluation (z_6).

Data from 116 high school students yielded the following correlation matrix.

	x_1	x_2	x_3	x_4	x_5	x_6	z_1	z_2	z_3	z_4	z_5	z_6
x_1	1.000	.	.	.	.	.	.	.	.	.	.	.
x_2	0.710	1.000	.	.	.	.	.	.	.	.	.	.
x_3	0.120	0.120	1.000	.	.	.	.	.	.	.	.	.
x_4	0.340	0.450	0.430	1.000	.	.	.	.	.	.	.	.
x_5	0.270	0.330	0.240	0.330	1.000	.	.	.	.	.	.	.
x_6	0.210	0.110	0.420	0.460	0.320	1.000	.	.	.	.	.	.
z_1	0.130	0.270	0.210	0.390	0.270	0.380	1.000	.	.	.	.	.
z_2	0.180	0.240	0.150	0.360	0.330	0.260	0.620	1.000	.	.	.	.
z_3	0.080	0.140	0.090	0.250	0.130	0.230	0.440	0.660	1.000	.	.	.
z_4	0.100	0.160	0.090	0.250	0.120	0.280	0.580	0.660	0.640	1.000	.	.
z_5	0.130	0.230	0.420	0.500	0.410	0.470	0.460	0.470	0.370	0.530	1.000	.
z_6	0.080	0.150	0.360	0.280	0.210	0.260	0.300	0.240	0.190	0.290	0.580	1.000

Examine the association between the creativity and achievement variables via canonical correlation, and from the printout answer the following questions:

(a) How would you characterize the strength of the relationship between the two sets of variables from the simple correlations?
(b) How many of the canonical correlations are significant at the $\alpha = 0.05$ level?
(c) Interpret the canonical variates corresponding to the largest canonical correlation.
(d) Do the achievement variables have something to say about the creativity variables? Do the creativity variables provide much information about the achievement variables? Explain.
(e) Are the first canonical variates good summary measures of their respective sets of variables? Explain.

Table 1.

GROUP	WI	WC	PC
1	5.80	9.70	8.90
1	10.60	10.90	11.00
1	8.60	7.20	8.70
1	4.80	4.60	6.20
1	8.30	10.60	7.80
1	4.60	3.30	4.70
1	4.80	3.70	6.40
1	6.70	6.00	7.20
1	6.90	9.70	7.20
1	5.60	4.10	4.30
1	4.80	3.80	5.30
1	2.90	3.70	4.20
2	2.40	2.10	2.40
2	3.50	1.80	3.90
2	6.70	3.60	5.90
2	5.30	3.30	6.10
2	5.20	4.10	6.40
2	3.20	2.70	4.00
2	4.50	4.90	5.70
2	3.90	4.70	4.70
2	4.00	3.60	2.90
2	5.70	5.50	6.20
2	2.40	2.90	3.20
2	2.70	2.60	4.10

Table 2.

MOTIV1	MOTIV2	MOTIV3	SES	MOTIV1	MOTIV2	MOTIV3	SES	MOTIV1	MOTIV2	MOTIV3	SES
11.432	42.858	18.577	2	15.719	26.191	25.722	2	7.145	35.715	24.293	3
28.58	45.239	41.441	3	52.873	71.43	55.731	3	25.722	38.096	25.722	1
38.583	69.049	51.444	1	51.444	52.382	50.015	1	17.148	47.62	18.577	2
41.441	73.811	55.731	2	11.432	33.334	21.435	2	7.145	30.953	7.145	3
25.722	54.763	30.009	3	25.722	57.144	37.154	3	30.009	57.144	27.151	1
20.006	26.191	12.861	2	45.728	64.287	51.444	1	38.583	57.144	52.873	2
8.574	40.477	8.574	2	38.583	19.048	41.441	2	42.87	59.525	57.16	3
14.29	54.763	12.861	3	42.87	50.001	51.444	3	40.012	69.049	77.166	1
22.864	42.858	31.438	1	32.867	47.62	42.87	1	48.586	64.287	64.305	2
60.018	61.906	60.018	2	34.296	57.144	34.296	2	25.722	47.62	34.296	3
28.58	59.525	34.296	3	24.293	50.001	30.009	3	24.293	26.191	28.58	1
1.429	26.191	12.861	1	45.728	69.049	67.163	1	18.577	35.715	31.438	2
18.577	16.667	21.435	2	15.719	54.763	18.577	2	18.577	47.62	24.293	3
18.577	26.191	42.87	3	25.722	38.096	10.003	3	34.296	40.477	41.441	1
21.435	30.953	17.148	1	12.861	28.572	10.003	1	22.864	47.62	24.293	2
31.438	30.953	35.725	2	58.589	73.811	75.737	2	24.293	28.572	17.148	3
38.583	45.239	40.012	3	32.867	54.763	38.583	3	24.293	45.239	21.435	1
38.583	64.287	34.296	1	12.861	30.953	8.574	1	25.722	66.668	37.154	2
25.722	52.382	35.725	2	30.009	69.049	38.583	2	20.006	45.239	22.864	3
35.725	35.715	27.151	3	28.58	19.048	21.435	3	42.87	66.668	57.16	1
32.867	52.382	44.299	1	78.595	76.192	68.592	1	22.864	33.334	27.151	2
20.006	19.048	31.438	2	30.009	45.239	37.154	2	58.589	76.192	68.592	3
32.867	33.334	28.58	3	47.157	21.429	44.299	3	25.722	28.572	28.58	1
8.574	30.953	8.574	1	28.58	61.906	21.435	1	30.009	28.572	42.87	2
44.299	52.382	50.015	2	18.577	26.191	30.009	2	15.719	50.001	37.154	3
15.719	50.001	27.151	3	25.722	42.858	20.006	3	10.003	52.382	21.435	1
31.438	47.62	42.87	1	18.577	26.191	17.148	1	51.444	71.43	60.018	3
34.296	59.525	40.012	2	61.447	66.668	61.447	2	21.435	61.906	32.867	1
37.154	59.525	54.302	3	38.583	47.62	47.157	3	44.299	61.906	52.873	2
21.435	35.715	14.29	1	21.435	33.334	22.864	1	71.45	78.573	71.45	3
32.867	54.763	25.722	2	21.435	47.62	17.148	2	20.006	38.096	28.58	1
27.151	59.525	42.87	3	34.296	69.049	55.731	3	25.722	30.953	30.009	2
31.438	40.477	20.006	1	42.87	59.525	55.731	1	52.873	59.525	60.018	3
38.583	40.477	48.586	2	50.015	71.43	54.302	2	37.154	30.953	42.87	1
28.58	73.811	50.015	3	41.441	42.858	37.154	3	27.151	23.81	51.444	2
58.589	71.43	57.16	1	57.16	66.668	75.737	1	7.145	16.667	18.577	3
22.864	35.715	5.716	2	27.151	40.477	40.012	2	17.148	28.572	24.293	1
12.861	33.334	10.003	3	25.722	59.525	42.87	3	20.006	35.715	31.438	2
32.867	52.382	37.154	1	24.293	73.811	61.447	1	55.731	69.049	61.447	3
48.586	69.049	64.305	2	24.293	64.287	48.586	2	47.157	59.525	62.876	1
10.003	40.477	14.29	3	12.861	28.572	7.145	3	37.154	69.049	55.731	2
7.145	19.048	5.716	1	31.438	66.668	50.015	1	10.003	52.382	20.006	3
31.438	50.001	37.154	2	18.577	57.144	27.151	2	55.731	66.668	51.444	1
44.299	59.525	50.015	3	28.58	52.382	42.87	3	32.867	59.525	50.015	2
18.577	52.382	30.009	1	18.577	45.239	17.148	1	38.583	76.192	45.728	3
28.58	52.382	18.577	2	14.29	30.953	30.009	2	7.145	57.144	27.151	1
48.586	54.763	51.444	1	21.435	35.715	30.009	2	47.157	80.954	61.447	2
47.157	64.287	55.731	2	18.577	40.477	12.861	3	60.018	76.192	58.589	3
21.435	28.572	27.151	1								

Table 3.

PLAY	INFO	COMP	ARITH	SIMIL	VOCAB	DIGIT	PICT	PARANG	BLOCK	OBJECT	CODING
2	8	7	13	9	12	9	6	11	12	7	9
2	9	6	8	7	11	12	6	8	7	12	14
2	13	18	11	16	15	6	18	8	11	12	9
2	8	11	6	12	9	7	13	4	7	12	11
2	11	7	15	12	10	12	6	12	10	5	10
2	6	13	7	8	11	6	14	9	14	14	10
2	10	8	8	14	9	9	10	11	10	9	6
2	9	10	8	11	9	11	10	12	9	13	13
2	11	15	13	14	12	7	15	12	10	10	4
2	12	11	8	11	13	13	14	7	11	9	10
.	9	5	7	11	13	7	13	10	15	15	11
2	9	15	8	15	13	8	11	13	11	11	7
1	12	11	9	17	17	7	17	12	11	11	5
1	5	6	5	8	6	5	11	8	10	11	5
2	7	6	8	9	8	12	7	10	9	8	7
1	7	8	7	7	8	5	8	5	9	12	8
2	12	13	12	11	15	9	11	11	11	14	15
1	8	6	7	5	6	8	6	7	6	5	11
2	13	16	10	15	14	5	10	9	13	14	10
2	8	6	6	5	7	7	10	6	9	5	11
.	8	4	8	13	10	7	9	8	4	11	5
.	10	15	10	12	10	15	9	12	7	11	10
2	10	10	10	12	13	14	14	15	12	9	6
2	13	9	11	12	11	8	11	9	11	11	7
2	8	10	9	11	8	11	8	6	12	11	4
1	10	7	10	5	12	14	10	9	3	3	10
2	11	9	8	11	12	6	10	5	11	8	8
1	15	6	11	10	16	8	12	10	11	13	14
2	10	10	10	12	10	9	13	13	12	10	10
1	8	11	8	7	5	6	14	10	5	8	8
2	5	8	6	3	11	9	10	5	10	9	5
2	10	10	11	12	8	8	10	12	11	6	10
1	19	18	19	19	19	16	13	10	19	13	6
2	11	8	5	11	8	5	12	10	5	13	5
2	10	8	9	13	12	4	12	9	11	12	7
2	9	8	5	8	10	7	7	5	7	8	4
1	7	10	11	10	9	8	8	7	7	6	9
2	5	7	6	10	9	7	7	9	9	10	5
.	10	11	8	8	10	7	8	5	9	15	11
1	9	9	6	6	12	7	9	10	10	8	6
.	8	15	9	9	11	8	10	9	11	7	12
2	8	9	9	8	10	10	7	7	6	8	8
2	10	9	10	10	8	9	8	11	8	10	9
1	5	6	11	6	8	7	6	10	9	7	8
1	10	10	11	8	12	6	13	10	9	12	10
2	10	7	12	12	17	9	13	12	5	12	7
2	5	7	7	10	8	8	9	11	12	11	12
1	12	10	9	6	15	12	9	7	13	11	12
1	14	15	14	13	13	8	11	15	13	12	6
1	9	9	7	7	9	9	9	9	11	14	10

1	12	8	9	9	10	11	10	11	8	9	5
1	10	10	11	15	12	10	7	9	4	10	4
1	12	9	13	13	12	11	15	14	16	13	8
2	12	11	14	14	15	9	13	14	16	12	13
1	8	8	9	7	9	4	10	10	10	6	6
2	6	12	7	2	10	12	8	9	10	8	9
2	12	14	11	18	16	11	13	14	10	13	8
2	13	12	10	15	11	12	13	10	13	12	2
.	10	14	11	15	14	16	13	11	15	13	12
1	5	9	8	10	11	8	11	11	12	12	9
2	4	10	8	9	10	4	10	11	11	13	9
1	10	13	10	11	12	6	13	11	13	12	8
1	10	9	8	13	11	8	13	7	7	14	5
2	5	8	8	8	7	9	12	8	10	10	5
1	9	8	9	9	8	11	10	11	8	10	8
2	10	6	8	7	3	6	10	15	10	17	4
1	7	9	9	7	9	9	10	11	9	9	12
1	5	9	12	8	10	11	12	8	12	11	8
2	13	14	14	16	10	14	14	12	13	11	9
1	8	8	8	10	9	8	13	10	10	10	6
1	5	11	7	12	10	7	13	14	12	14	13
2	8	10	10	5	11	7	9	9	8	11	5
1	4	11	7	4	9	.	8	6	8	8	13
2	9	.	8	9	8	7	10	13	8	10	12
2	10	10	13	9	9	8	11	11	13	13	7
2	8	9	8	5	6	4	9	9	9	8	7
2	9	12	10	11	12	13	13	13	12	16	12
2	3	6	6	12	6	7	8	12	6	11	4
2	8	11	8	12	10	6	10	9	11	17	9
1	4	12	5	9	10	12	10	9	8	10	10
2	8	8	8	6	8	7	14	11	11	11	7
2	5	7	6	4	9	6	12	15	14	12	7
1	9	10	9	10	9	8	14	15	9	13	9
2	10	9	8	10	11	7	12	11	12	15	6
2	10	11	7	10	10	11	10	13	12	12	8
2	9	7	9	8	6	10	8	11	10	10	8
.	10	11	10	11	12	6	12	13	14	15	6
2	9	10	8	8	9	12	10	11	9	9	7
2	7	11	7	12	9	9	15	12	15	14	6
2	12	14	8	12	8	13	11	13	8	13	13
2	9	7	9	10	11	8	8	10	11	9	9
1	13	17	14	15	15	14	14	8	10	13	12
2	15	16	16	14	16	12	11	11	18	18	12
1	6	10	7	9	9	8	12	14	15	11	11
1	13	10	10	12	14	11	10	11	8	9	14
2	9	12	7	8	9	8	13	6	9	8	7
2	7	8	9	11	8	10	10	10	11	11	9
.	12	10	9	11	11	12	9	8	11	16	5
2	11	11	9	9	9	8	16	14	12	19	12
1	11	7	6	9	7	8	14	12	10	13	10
1	4	6	9	9	6	6	11	7	11	7	8
2	13	12	13	17	12	13	15	14	11	9	11
1	4	3	9	5	2	3	8	14	8	7	7
.	14	11	12	12	18	13	12	12	11	8	7

1	6	11	5	11	8	9	5	9	7	9	10
2	10	8	11	11	6	12	12	10	10	6	10
1	10	10	10	10	9	5	10	11	10	13	9
2	11	11	9	15	14	9	12	9	12	14	10
2	11	9	8	12	14	8	13	10	13	9	6
1	11	10	10	9	10	8	12	11	11	12	12
1	9	12	11	11	10	11	14	13	12	12	9
2	10	14	13	15	10	5	14	13	14	12	11
1	7	10	10	11	10	9	14	8	10	11	6
1	8	6	5	9	6	8	10	10	11	10	8
2	11	12	7	11	11	6	18	12	8	11	5
1	6	8	8	16	11	11	12	10	10	14	9
2	13	.	17	12	15	12	13	7	15	12	12
1	11	9	10	7	9	7	13	6	12	9	11
2	13	16	11	18	13	6	12	11	13	10	6
1	15	15	12	14	19	7	16	9	14	11	8
2	7	9	7	11	12	9	14	10	11	13	7
1	10	9	7	10	9	9	12	11	9	11	6