Stockholm University

Department of Statistics
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Exam: Multivariate Analysis, Advanced level, 7.5 ECTS credits

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The exam consists of 4 exercises giving a total of 50 points. In order to get full score for an exercise provide detailed and well motivated solutions. In order to pass the exam at least 25 points are needed.

Exercise 1. (10p)

(a) Let $\boldsymbol{x}^{\mathrm{T}} = (x_1, x_2, x_3) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu}^{\mathrm{T}} = (5, 10, 2)$ and

$$\boldsymbol{\Sigma} = \begin{pmatrix} 4 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}.$$

(a) What is the conditional distribution of x_2 and x_3 given x_1?

(b) Let $\boldsymbol{z}^{\mathrm{T}} = (x_1 + x_2, x_1 - x_2)$. What is the conditional distribution of $\boldsymbol{z}$ given x_3?

(c) What is the distribution of $\boldsymbol{Ax}$, where

$$\boldsymbol{A} = \begin{pmatrix} \frac{1}{2} & -1 & \frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}?$$

(d) Let $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n$ be *iid* $N(a\mathbf{1}_p, \boldsymbol{\Sigma})$, where a is a scalar, $\boldsymbol{\Sigma}$ is known and $\mathbf{1}_p$ is a p-vector of ones. Consider the following two random variables

$$t_1 = \frac{\mathbf{1}_p^{\mathrm{T}}\bar{\boldsymbol{x}}}{\mathbf{1}_p^{\mathrm{T}}\mathbf{1}_p} \text{ and } t_2 = \frac{\mathbf{1}_p^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\bar{\boldsymbol{x}}}{\mathbf{1}_p^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{1}_p},$$

where $\bar{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i$.
Find $E(t_i)$ and $Var(t_i)$, $i = 1, 2$.

Exercise 2. (10p)

Eight men received 2.0 mg/kg of a certain drug. Their change in blood sugar level and blood pressure (systolic and diastolic) were recorded:

Blood sugar	30	90	-10	10	30	60	0	40
Blood pressure (systolic)	-8	7	-2	0	-2	0	-2	1
Blood pressure (diastolic)	-1	6	4	2	5	3	4	2

Source: Srivastava (2002). Methods of Multivariate Statistics.

(a) Test the hypothesis $\boldsymbol{\mu} = \mathbf{0}$ vs $H_1 : \boldsymbol{\mu} \neq \mathbf{0}$. Use $\alpha = 0.05$.

(b) Construct and plot a 95% joint confidence ellipse for the population mean vector $\boldsymbol{\mu}^{\mathrm{T}} = (\mu_1, \mu_2) = (E(X_1), E(X_2))$, where X_1 and X_2 are systolic and diastolic blood pressure, respectively.

Exercise 3. (15p)

Pokémon Go is an augmented reality mobile game developed and published by Niantic in collaboration with The Pokémon Company. It uses the mobile device GPS to locate, capture, battle, and train virtual creatures, called Pokémon, which appear as if they are in the player's real-world location. Each Pokémon has various characteristics that determine their power and usefullness. Players over the whole world are interested in the following variables: capture rate, flee rate and spawn chance (the average spawns per 10000). Primary type and Combat Power (CP) are believed to be important for capture rate, flee rate and spawn chance.

(a) Create a new variable CP-class with the following values:

$$CP - class = \begin{cases} 1 & \text{if } CP \leq 900; \\ 2 & \text{if } 900 < CP \leq 1600; \\ 3 & \text{if } 1600 < CP \leq 2300; \\ 4 & \text{if } 2300 < CP \leq 3000; \\ 5 & \text{if } CP > 3000; \end{cases}$$

(b) Test whether the capture rate, flee rate and spawn chance depend on the primary type of the Pokémon and the CP-class to which it is belongs. Formulate your model with necessary assumptions. The model should contain ONLY main effects!

(c) If the hypothesis in Part (b) is rejected, find the linear combinations of mean components most responsible for rejecting H_0.

(d) Provide appropriate plots to illustrate the obtained results.

(e) What conclusions can you draw from your data analysis?

Exercise 4. (15p)

Waugh (1942) analyzed price and consumption data on meat for the years 1921 through 1940. The datafile comprices Beef prices (PRICEB), hog prices (PRICEH), per capita consumption for beef (CONSUMPB) and consumption for pork (CONSUMPH) which are available for all 20 years. One would like to measure the association between livestock prices and meat consumption.

(a) Determine the sample canonical correlations.
(b) Find the sample canonical variates corresponding to significant (at the $\alpha = 0.05$ level) canonical correlations.
(c) Prepare a table showing the canonical variate coefficients (for "significant" canonical correlations) and the sample correlations the canonical variates with their component variables.
(d) Given the information in (c), interpret the canonical variates.
(e) Do the meat prise variables have something to say about the consumption variables? Do the consumption variables provide much information about the prise variables?
(f) What proportion of the total sample variance of the first set (meat prises) is explained by the canonical variate $\hat{\boldsymbol{U}}_1$? What proportion of the total sample variance of the second set (consumption variables) is explained by the canonical variate $\hat{\boldsymbol{V}}_1$? Discuss your answers.

Data for Exercise 3.

obs	name	stamina	atk	def	capture rate	flee rate	spawn chance	primary type	cp
1	Bulbasaur	90	126	126	0.16	0.1	69	Grass	1079
2	Ivysaur	120	156	158	0.08	0.07	4.2	Grass	1643
3	Venusaur	160	198	200	0.04	0.05	1.7	Grass	2598
4	Charmander	78	128	108	0.16	0.1	25.3	Fire	962
5	Charmeleon	116	160	140	0.08	0.07	1.2	Fire	1568
6	Charizard	156	212	182	0.04	0.05	0.31	Fire	2620
7	Squirtle	88	112	142	0.16	0.1	58	Water	1015
8	Wartortle	118	144	176	0.08	0.07	3.4	Water	1594
9	Blastoise	158	186	222	0.04	0.05	0.67	Water	2560
10	Caterpie	90	62	66	0.4	0.2	303.2	Bug	446
11	Metapod	100	56	86	0.2	0.09	18.7	Bug	481
12	Butterfree	120	144	144	0.1	0.06	2.2	Bug	1465
13	Weedle	80	68	64	0.4	0.2	712	Bug	452
14	Kakuna	90	62	82	0.2	0.09	44	Bug	488
15	Beedrill	130	144	130	0.1	0.06	5.1	Bug	1450
16	Pidgey	80	94	90	0.4	0.2	1598	Normal	684
17	Pidgeotto	126	126	122	0.2	0.09	102	Normal	1232
18	Pidgeot	166	170	166	0.1	0.06	13	Normal	2106
19	Rattata	60	92	86	0.4	0.2	1305	Normal	585
20	Raticate	110	146	150	0.16	0.07	41	Normal	1454
21	Spearow	80	102	78	0.4	0.15	473	Normal	691
22	Fearow	130	168	146	0.16	0.07	15	Normal	1758
23	Ekans	70	112	112	0.4	0.15	227	Poison	830
24	Arbok	120	166	166	0.16	0.07	7.2	Poison	1779
25	Pikachu	70	124	108	0.16	0.1	21	Electric	894
26	Raichu	120	200	154	0.08	0.06	0.76	Electric	2042
27	Sandshrew	100	90	114	0.4	0.1	111	Ground	804
28	Sandslash	150	150	172	0.16	0.06	3.7	Ground	1823
29	Nidoran	110	100	104	0.4	0.15	138	Poison	882
30	Nidorina	140	132	136	0.2	0.07	8.8	Poison	1414
31	Nidoqueen	180	184	190	0.1	0.05	1.2	Poison	2502
32	Nidoran	92	110	94	0.4	0.15	131	Poison	849
33	Nidorino	122	142	128	0.2	0.07	8.3	Poison	1382
34	Nidoking	162	204	170	0.1	0.05	1.7	Poison	2492
35	Clefairy	140	116	124	0.24	0.1	92	Fairy	1209
36	Clefable	190	178	178	0.08	0.06	1.2	Fairy	2414
37	Vulpix	76	106	118	0.24	0.1	22	Fire	837
38	Ninetales	146	176	194	0.08	0.06	0.77	Fire	2203
39	Jigglypuff	230	98	54	0.4	0.1	39	Normal	924
40	Wigglytuff	280	168	108	0.16	0.06	1.8	Normal	2192
41	Zubat	80	88	90	0.4	0.2	652	Poison	647
42	Golbat	150	164	164	0.16	0.07	42	Poison	1935
43	Oddish	90	134	130	0.48	0.15	102	Grass	1156
44	Gloom	120	162	158	0.24	0.07	6.4	Grass	1701
45	Vileplume	150	202	190	0.12	0.05	0.97	Grass	2510
46	Paras	70	122	120	0.32	0.15	236	Bug	923
47	Parasect	120	162	170	0.16	0.07	7.4	Bug	1759
48	Venonat	120	108	118	0.4	0.15	228	Bug	1036

49	Venomoth	140	172	154	0.16	0.07	7.2	Bug	1903
50	Diglett	20	108	86	0.4	0.1	40	Ground	460
51	Dugtrio	70	148	140	0.16	0.06	1.4	Ground	1176
52	Meowth	80	104	94	0.4	0.15	86	Normal	761
53	Persian	130	156	146	0.16	0.07	2.2	Normal	1643
54	Psyduck	100	132	112	0.4	0.1	254	Water	1117
55	Golduck	160	194	176	0.16	0.06	8.7	Water	2403
56	Mankey	80	122	96	0.4	0.1	92	Fighting	884
57	Primeape	130	178	150	0.16	0.06	3.1	Fighting	1877
58	Growlithe	110	156	110	0.24	0.1	92	Fire	1344
59	Arcanine	180	230	180	0.08	0.06	1.7	Fire	3005
60	Poliwag	80	108	98	0.4	0.15	219	Water	801
61	Poliwhirl	130	132	132	0.2	0.07	13	Water	1350
62	Poliwrath	180	180	202	0.1	0.05	1.1	Water	2523
63	Abra	50	110	76	0.4	0.99	42	Psychic	604
64	Kadabra	80	150	112	0.2	0.07	2.7	Psychic	1140
65	Alakazam	110	186	152	0.1	0.05	0.73	Psychic	1826
66	Machop	140	118	96	0.4	0.1	49	Fighting	1097
67	Machoke	160	154	144	0.2	0.07	3.4	Fighting	1773
68	Machamp	180	198	180	0.1	0.05	0.68	Fighting	2612
69	Bellsprout	100	158	78	0.4	0.15	115	Grass	1125
70	Weepinbell	130	190	110	0.2	0.07	7.2	Grass	1736
71	Victreebel	160	222	152	0.1	0.05	0.59	Grass	2548
72	Tentacool	80	106	136	0.4	0.15	81	Water	911
73	Tentacruel	160	170	196	0.16	0.07	8.2	Water	2236
74	Geodude	80	106	118	0.4	0.1	119	Rock	855
75	Graveler	110	142	156	0.2	0.07	7.1	Rock	1443
76	Golem	160	176	198	0.1	0.05	0.47	Rock	2319
77	Ponyta	100	168	138	0.32	0.1	51	Fire	1526
78	Rapidash	130	200	170	0.12	0.06	1.1	Fire	2215
79	Slowpoke	180	110	110	0.4	0.1	105	Water	1227
80	Slowbro	190	184	198	0.16	0.06	3.6	Water	2615
81	Magnemite	50	128	138	0.4	0.1	71	Electric	897
82	Magneton	100	186	180	0.16	0.06	2.3	Electric	1893
83	Farfetchd	104	138	132	0.24	0.09	2.12	Normal	1272
84	Doduo	70	126	96	0.4	0.1	52	Normal	861
85	Dodrio	120	182	150	0.16	0.06	22	Normal	1849
86	Seel	130	104	138	0.4	0.09	28	Water	1114
87	Dewgong	180	156	192	0.16	0.06	1.3	Water	2161
88	Grimer	160	124	110	0.4	0.1	5.2	Poison	1293
89	Muk	210	180	188	0.16	0.06	0.31	Poison	2621
90	Shellder	60	120	112	0.4	0.1	52	Water	828
91	Cloyster	100	196	196	0.16	0.06	1.5	Water	2067
92	Gastly	60	136	82	0.32	0.1	79	Ghost	810
93	Haunter	90	172	118	0.16	0.07	5.2	Ghost	1390
94	Gengar	120	204	156	0.08	0.05	0.67	Ghost	2093
95	Onix	70	90	186	0.16	0.09	10	Rock	863
96	Drowzee	120	104	140	0.4	0.1	321	Psychic	1082
97	Hypno	170	162	196	0.16	0.06	10	Psychic	2199
98	Krabby	60	116	110	0.4	0.15	212	Water	797

99	Kingler	110	178	168	0.16	0.07	6.2	Water	1836
100	Voltorb	80	102	124	0.4	0.1	65	Electric	845
101	Electrode	120	150	174	0.16	0.06	2	Electric	1657
102	Exeggcute	120	110	132	0.4	0.1	78	Grass	1107
103	Exeggutor	190	232	164	0.16	0.06	1.4	Grass	2976
104	Cubone	100	102	150	0.32	0.1	61	Ground	1013
105	Marowak	120	140	202	0.12	0.06	2	Ground	1668
106	Hitmonlee	100	148	172	0.16	0.09	2	Fighting	1503
107	Hitmonchan	100	138	204	0.16	0.09	2.2	Fighting	1527
108	Lickitung	180	126	160	0.16	0.09	1.1	Normal	1638
109	Koffing	80	136	142	0.4	0.1	20	Poison	1160
110	Weezing	130	190	198	0.16	0.06	1.6	Poison	2266
111	Rhyhorn	160	110	116	0.4	0.1	63	Ground	1190
112	Rhydon	210	166	160	0.16	0.06	2.2	Ground	2259
113	Chansey	500	40	60	0.16	0.09	1.3	Normal	679
114	Tangela	130	164	152	0.32	0.09	22.8	Grass	1752
115	Kangaskhan	210	142	178	0.16	0.09	0.86	Normal	2057
116	Horsea	60	122	100	0.4	0.1	113	Water	800
117	Seadra	110	176	150	0.16	0.06	3.4	Water	1725
118	Goldeen	90	112	126	0.4	0.15	218	Water	972
119	Seaking	160	172	160	0.16	0.07	8	Water	2058
120	Staryu	60	130	128	0.4	0.15	195	Water	944
121	Starmie	120	194	192	0.16	0.06	3.4	Water	2197
122	MrMime	80	154	196	0.24	0.09	0.31	Psychic	1505
123	Scyther	140	176	180	0.24	0.09	14	Bug	2088
124	Jynx	130	172	134	0.24	0.09	35	Ice	1728
125	Electabuzz	130	198	160	0.24	0.09	7.4	Electric	2134
126	Magmar	130	214	158	0.24	0.09	10	Fire	2281
127	Pinsir	130	184	186	0.24	0.09	99	Bug	2137
128	Tauros	150	148	184	0.24	0.09	12	Normal	1857
129	Magikarp	40	42	84	0.56	0.15	478	Water	264
130	Gyarados	190	192	196	0.08	0.07	0.32	Water	2708
131	Lapras	260	186	190	0.16	0.09	0.6	Water	3002
132	Ditto	96	110	110	0.16	0.1	0.01	Normal	926
133	Eevee	110	114	128	0.32	0.1	275	Normal	1084
134	Vaporeon	260	186	168	0.12	0.06	1.4	Water	2836
135	Jolteon	130	192	174	0.12	0.06	1.2	Electric	2155
136	Flareon	130	238	178	0.12	0.06	1.7	Fire	2662
137	Porygon	130	156	158	0.32	0.09	1.2	Normal	1703
138	Omanyte	70	132	160	0.32	0.09	14	Rock	1127
139	Omastar	140	180	202	0.12	0.05	0.61	Rock	2249
140	Kabuto	60	148	142	0.32	0.09	10	Rock	1112
141	Kabutops	120	190	190	0.12	0.05	0.32	Rock	2145
142	Aerodactyl	160	182	162	0.16	0.09	1.8	Rock	2180
143	Snorlax	320	180	180	0.16	0.09	1.6	Normal	3135
144	Dratini	82	128	110	0.32	0.09	30	Dragon	990
145	Dragonair	122	170	152	0.08	0.06	2	Dragon	1760
146	Dragonite	182	250	212	0.04	0.05	0.11	Dragon	3525

Data for Exercise 4.

year	PRICEB	PRICEH	CONSUMPB	CONSUMPH
1921	8.41	8.73	55.7	65.0
1922	8.68	9.26	59.2	65.9
1923	8.22	6.60	59.8	74.5
1924	8.24	7.23	59.9	74.7
1925	8.64	10.04	60.0	67.3
1926	7.63	9.95	60.7	64.6
1927	9.50	8.32	54.9	68.2
1928	11.41	7.56	49.0	71.3
1929	10.49	7.93	49.8	69.8
1930	9.84	8.51	48.9	67.0
1931	9.20	7.03	48.5	68.3
1932	10.58	6.05	46.7	70.6
1933	8.92	6.48	51.4	69.9
1934	9.52	6.55	55.8	64.2
1935	12.76	11.53	53.5	48.6
1936	9.47	10.62	58.8	55.6
1937	11.37	9.93	55.3	55.9
1938	10.10	8.70	54.5	58.3
1939	9.88	6.66	54.5	64.4
1940	9.91	5.42	55.2	72.5