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Inference Theory

WRITTEN EXAMINATION
Thursday January 16, 2025

Allowed tools: Calculator, handwritten notes in terms of one A4 paper, where it is allowed to write on both sides.

Passing rate: 50% of overall total, which is 100 points. There are five questions worth 20 points each.

For the maximum number of points on each problem detailed, well motivated and clear solutions are required.

In the problems 1–3 we will work with an observed random sample $x_1, \ldots, x_n$ from iid $X_1, \ldots, X_n$ generated from a beta distribution with pfd

$$f(x|\theta) = \theta(\theta + 1)x^{\theta-1}(1 - x), \ \ 0 \leq x \leq 1, \ \ \theta > 0.$$

1. (a) Does this pdf belong to an exponential family of distributions? Why/why not?

   (b) Find a sufficient statistic for θ.

   (c) Is your sufficient statistic complete? Why/why not?

2. (a) Derive the MLE $\hat{\theta}$ of θ, that is, do the derivation up to the point that you get a quadratic equation in θ. You do not need to find the actual solution(s). Is the equation related to the sufficient statistic?

   (b) Derive the asymptotic variance of $\hat{\theta}$.

3. (a) Derive the method of moments estimator $\tilde{\theta}$ of θ.

   (b) Derive the asymptotic distribution of $\tilde{\theta}$.

4. Let 9.3, 14.1, 13.2, 20.6, 12.3, 10.4 be observations independently generated from a distribution with pdf

$$f(x|\theta) = 2e^{-2(x-\theta)}, \quad x \geq \theta, \quad \theta > 0.$$

Compute an exact 90% two-sided symmetric confidence interval for θ.

5. Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with mean λ. Derive the most powerful test of $H_0 : \lambda = 2$ vs. $H_1 : \lambda = 5$, that is, present the structure of the rejection region.