Stockholms universitet

OBS! Läs noga igenom anvisningarna i tentamen, t.ex. hur du ska skriva svaren. Det är ditt ansvar som student att följa de anvisningar som ges.

NOTE! Read the examination instructions carefully, e.g. how to write the answers. It is your responsibility as a student to follow the given instructions.

Skriv din anonymiseringskod och dagens datum på allt material du lämnar in.
(*Enter your anonymization code and todays date on all submitted materials*)

Anonymiseringskod (*Anonymization code*)	3	1	1	-	0	0	0	3	-	0	C	D

Datum (*Date* YYYY-MM-DD)	2022-05-02	Plats nr. (*Seat No.*)	6

Kurs/Kurskod (*Course/Course code*)	ST4301
Kursmoment (*Course component*)	11IT Inferensteori tentamen

Fylls i av tentamensvärd (*To be filled in by invigilator*)

Direkt i skrivning: (kryss)		Svarsblankett: (kryss)		Lösa svarsblad: (antal)	11
Lämnat in blankt: (kryss)		Dator: (kryss)			

Inlämningstid: 12:56 Signatur tentamensvärd: A. Moregård

Fylls i av lärare/examinator (*To be filled in by teacher/examinator*)

Betyg:	C	Poäng:	72

Signatur rättande lärare/examinator: [signature]

Regler i skrivsalen

- Följ tentamensvärds anvisningar.
- Väskor och ytterkläder ska placeras på anvisad plats.
- Placera ID-handling väl synlig på bordet framför dig.
- Ingen student får lämna skrivsalen under de första 30 minuterna.
- Endast en student i taget får besöka toaletten. Vid toalettbesök skriv ditt namn och klockslag på avsedd lista. Efter toalettbesöket ska du åter ange klockslag på listan.
- Elektronisk utrustning som mobiltelefon eller Smartwatch ska vara avstängd och placerad på anvisad plats.
- Under tentamen gäller tystnad – det är förbjudet att prata, eller på annat sätt kommunicera, med andra studenter under pågående tentamen.
- Innan tentamenshandlingarna lämnas in; skriv sidnummer, anonymiseringskod och datum på alla inlämnade papper.

Om något är oklart – fråga gärna tentamensvärden. Lycka till!

Rules in the examination hall

- Follow the invigilator's instructions.
- Bags and outerwear must be placed at the designated place.
- Place your ID document clearly visible on the table in front of you.
- No student may leave the examination hall for the first 30 minutes.
- Only one student at a time may visit the toilet. Before visiting the toilet, write your name and time on the intended list. After the toilet visit, enter the time on the list again.
- Electronic equipment such as a mobile phone or Smartwatch must be switched off and placed at the designated place.
- During the exam, silence applies – you are not allowed to talk, or otherwise communicate, with other students during the exam.
- Before submitting the examination documents; remember to write the page number, anonymization code, and date on all papers.

Please do not hesitate to ask the invigilator if anything is unclear. Good luck!

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Uppgg.nr.: (Task no.) 1

Lärarens kommentar: (Teacher's note)

1. According to Theorem 6.2.10, a distribution
a) belongs to an exponential family if it can be expressed in the following way:

$$f(y|\theta) = h(y)\, c(\theta) \exp\left(\sum_{i=1}^{k} w_i(\theta)\, t_i(y)\right)$$

Here, $k=1$

For $Y_i \overset{iid}{\sim} Po(\lambda)$, $0<\lambda<\infty$

$$f(y|\lambda) = \frac{e^{-\lambda} \lambda^{y}}{y!},$$

Joint density is:

$$L = \frac{e^{-n\lambda} \lambda^{\sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n} y_i!}$$, which can be rewritten to

$$L = \frac{e^{-n\lambda}\, e^{\ln(\lambda) \sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n} y_i!}$$

Now we see that $h(y) = \frac{1}{\prod_{i=1}^{n} y_i!}$ $c(\lambda) = e^{-n\lambda}$

$w(\lambda) = \ln(\lambda)$ $t(y) = \sum y_i$

Thus, according to theorem 6.2.10, the Poisson family is an exponential family of distributions

b) From a), we have that $t(y) = \sum y_i$ which according to Theorem 6.2.10 is then a sufficient statistic for λ.

Or instead, using Theorem 6.2.6 (Factorization Theorem):

$$L = \frac{e^{-n\lambda} \lambda^{\sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n} x_i!}$$

If this can be written as $f(y|\lambda) = g(T(x)|\lambda)\, h(x)$ then $T(x)$ is sufficient for λ.

Continued on next page

Poäng: (Points) 20

Uppg.nr.:
(*Task no.*)

Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

1 b) continued

So, given $\frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n} x_i!}$, we can see that

$g(T(y)|\lambda) = e^{-n\lambda}\lambda^{T(y)}$

$T(y) = \sum_{i=1}^{n} y_i$ So $\sum_{i=1}^{n} y_i$ is a sufficient statistic for λ.

$h(x) = \frac{1}{\prod_{i=1}^{n} y_i!}$

c) Want to determine $\hat{\theta}_{ML}$ where $\theta = \ln(\lambda)$

According to Theorem 7.2.10 (Invariance property of MLEs) we have that if $\hat{\theta}$ is the MLE of θ, then for any function $\tau(\theta)$, the MLE of $\tau(\theta)$ is $\tau(\hat{\theta})$.

In this case it means that the MLE of θ, ie $\hat{\theta}_{ML}$, is $\ln(\hat{\lambda}_{ML})$

Finding $\hat{\lambda}_{ML}$:

Joint density, from previous tasks, is:

$L = \frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n} y_i!}$, $\ell = \ln(L) = -n\lambda + \ln(\lambda)\sum_{i=1}^{n} y_i + \ln\left(\frac{1}{\prod_{i=1}^{n} y_i!}\right)$

$\ell' = \frac{d}{d\lambda}\ell = -n + \frac{\sum_{i=1}^{n} y_i}{\lambda}$ setting derivative equal to zero gives that

$\frac{\sum_{i=1}^{n} y_i}{\lambda} = n \longrightarrow \hat{\lambda}_{ML} = \frac{\sum_{i=1}^{n} y_i}{n} = \bar{x}$

Plugging this into $\theta = \ln(\hat{\lambda}_{ML})$ we get that

$\hat{\theta}_{ML} = \ln(\bar{x})$

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Lärarens
kommentar:
(*Teacher's*
note)

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Uppg.nr.: (Task no.) 1

Lärarens kommentar: (Teacher's note)

d) Want to determine the asymptotic distribution of $\sqrt{n}(\hat{\theta}-\theta)$ using the Delta Method.

$Var(y)=\sigma^2=\lambda$

By CLT we have that

$\sqrt{n}(Y_n-\lambda) \xrightarrow{d} N(0,\lambda)$

$\hat{\theta}_{ML} = \ln(\bar{x})$

$\sqrt{n}(\ln(\bar{x})-\ln(\lambda)) \xrightarrow{d} N\left(0, \lambda\left[\frac{d}{d\lambda}\ln(\lambda)\right]^2\right)$

$\left[\frac{d}{d\lambda}\ln(\lambda)\right]^2 = \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$

This gives that

$\sqrt{n}(\ln(\bar{x})-\ln(\lambda)) \xrightarrow{d} N\left(0, \lambda\cdot\frac{1}{\lambda^2}\right)$ such that

$\sqrt{n}(\ln(\bar{x})-\ln(\lambda)) \xrightarrow{d} N\left(0, \frac{1}{\lambda}\right)$

Poäng: (Points)

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Uppgg.nr.: (Task no.) 2

Lärarens kommentar: (Teacher's note)

2. A cat was allowed to choose from three different dishes. Experiment repeated N times.

The # of times the cat chose each meal is Y_1, Y_2, Y_3

The joint is

$$p(y_1,y_2,y_3) = \binom{N}{y_1\ y_2\ y_3}\theta_1^{y_1}\theta_2^{y_2}\theta_3^{y_3}$$

$\theta_1, \theta_2, \theta_3$ are probabilities that the cat chooses either meal.

know that $\theta_1+\theta_2+\theta_3=1$, such that $\theta_3=1-\theta_1-\theta_2$

Also know that $Y_1+Y_2+Y_3=N$, such that $Y_3=N-Y_1-Y_2$

Want to construct a LRT to test

$H_0: \theta_1=\theta_2=\theta_3$ vs. H_a: At least one of θ_i $i=1,2,3$ different from the other two

$$LRT = \frac{\sup_{\Theta_0} L(\theta|y)}{\sup_{\Theta} L(\theta|y)},$$ want to find MLEs of $\theta_1, \theta_2, \theta_3$.

As shown above θ_3 can be determined by the other two quantities, so only need to find θ_{1ML} & θ_{2ML}

Putting together all information we get

$$p(y_1,y_2,y_3) = \binom{N}{y_1\ y_2\ y_3}\theta_1^{y_1}\theta_2^{y_2}(1-\theta_1-\theta_2)^{N-y_1-y_2}$$

Taking the log of $p(y_1,y_2,y_3)$ we get.

$$\ln\binom{N}{y_1\ y_2\ y_3} + \ln(\theta_1)y_1 + \ln(\theta_2)y_2 + \ln(1-\theta_1-\theta_2)(N-y_1-y_2)$$

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Poäng: (Points) 20

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Uppg.nr.: (Task no.) 2

Lärarens kommentar: (Teacher's note)

2. cont.

Taking derivatives wrt Θ_1 and Θ_2 we get:

$$\frac{d}{d\Theta_1}\ln(p(y_1,y_2,y_3)) = \frac{y_1}{\Theta_1} - \frac{N-y_1-y_2}{1-\Theta_1-\Theta_2} = 0$$

$$\frac{d}{d\Theta_2}\ln(p(y_1,y_2,y_3)) = \frac{y_2}{\Theta_2} - \frac{N-y_1-y_2}{1-\Theta_1-\Theta_2} = 0$$

Can see that $\frac{y_1}{\Theta_1} = \frac{y_2}{\Theta_2} = \frac{N-y_1-y_2}{1-\Theta_1-\Theta_2}$

$$y_1/\Theta_1 = y_2/\Theta_2 \rightarrow \hat{\Theta}_1 = \frac{y_1\Theta_2}{y_2}, \quad \hat{\Theta}_2 = \frac{\Theta_1 y_2}{y_1}$$

$$\frac{y_1}{\frac{y_1\Theta_2}{y_2}} = \frac{N-y_1-y_2}{1-\frac{y_1\Theta_2}{y_2}-\Theta_2} \rightarrow \frac{y_1y_2}{y_1\Theta_2} = \frac{N-y_1-y_2}{1-\frac{y_1\Theta_2}{y_2}-\Theta_2}$$

$$\frac{y_2}{\Theta_2} = \frac{N-y_1-y_2}{\frac{y_2-y_1\Theta_2-\Theta_2y_2}{y_2}} \Rightarrow \frac{y_2}{\Theta_2} = \frac{y_2(N-y_1-y_2)}{y_2-y_1\Theta_2-\Theta_2y_2}$$

$$y_2(y_2-y_1\Theta_2-\Theta_2y_2) = y_2\Theta_2(N-y_1-y_2)$$

$$y_2-y_1\Theta_2-\Theta_2y_2 = \Theta_2(N-y_1-y_2)$$

$$\frac{y_2}{\Theta_2}-y_1-y_2 = N-y_1-y_2 \rightarrow \frac{y_2}{\Theta_2} = N$$

$\hat{\Theta}_2 = \frac{y_2}{N}$ By symmetry $\hat{\Theta}_1 = \frac{y_1}{N}$

$$\Theta_3 = 1-\hat{\Theta}_1-\hat{\Theta}_2 = 1-\frac{y_1}{N}-\frac{y_2}{N} \rightarrow \Theta_3\cdot N = N-y_1-y_2$$

$\Theta_3\cdot N = y_3 \rightarrow \hat{\Theta}_3 = \frac{y_3}{N}$ cont. on next page.

Poäng: (Points)

Tentamensservice/SU/typ_b

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Uppg.nr.: (Task no.) 2

Lärarens kommentar: (Teacher's note)

2. cont.

Now that the MLEs are found the LRT can be constructed

$$\lambda(y) = \frac{\binom{N}{y_1\, y_2\, y_3}\left(\frac{1}{3}\right)^N}{\binom{N}{y_1\, y_2\, y_3}\left(\frac{y_1}{N}\right)^{y_1}\left(\frac{y_2}{N}\right)^{y_2}\left(\frac{N-y_1-y_2}{N}\right)^{N-y_1-y_2}} =$$

$$= \frac{\left(\frac{1}{3}\right)^N}{\left(\frac{y_1}{N}\right)^{y_1}\left(\frac{y_2}{N}\right)^{y_2}\left(\frac{y_3}{N}\right)^{y_3}} = \frac{\left(\frac{1}{3}\right)^N N^{y_1} N^{y_2} N^{y_3}}{y_1^{y_1}\, y_2^{y_2}\, y_3^{y_3}} =$$

$$= \frac{\left(\frac{1}{3}\right)^N N^{y_1+y_2+y_3}}{y_1^{y_1}\, y_2^{y_2}\, y_3^{y_3}} = \frac{\left(\frac{N}{3}\right)^N}{y_1^{y_1}\, y_2^{y_2}\, y_3^{y_3}}$$

If $y_1 = y_2 = y_3$ then this ratio becomes 1. When any y_i $(i=1,2,3)$ is different from the other two, the ratio becomes < 1. The larger difference between the y's, the smaller it gets (approaches 0).

Theorem 10.3.1 (Asymptotic distribution of the LRT) states that under H_0:

$$-2\log(\lambda(y)) \xrightarrow{d} \chi^2_{df},$$

where df = # unknown param in denominator "minus" # unknown param in nominator. Here, under H_0 there are no unknown parameters, while there are two in the denominator (θ_1, θ_2). θ_3 is determined by θ_1 & θ_2

Cont. on next page.

Poäng: (Points)

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Lärarens kommentar: (Teacher's note)

2. cont.

So the large sample version of the LRT is

$$-2\cdot\left(\log\left(\frac{N}{3}\right)\cdot N - \log(y_1)y_1 - \log(y_2)y_2 - \log(y_3)y_3\right) =$$

$$= 2\left(\log(y_1)y_1 + \log(y_2)y_2 + \log(y_3)y_3 - \log\left(\frac{N}{3}\right)N\right)$$

Which $\xrightarrow{d} \chi^2_2$ under H_0 as $n \to \infty$

Poäng: (Points)

Uppg.nr.:
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Lärarens kommentar:
(*Teacher's note*)

Poäng:
(*Points*)

Uppg.nr.: (Task no.) 3

Lärarens kommentar: (Teacher's note)

3. $X \sim Binom(n, p)$ with $n=10$
and $p \in \{p; p = 0.2, 0.5\}$
$H_0: p=0.5$ vs. $H_a: p=0.2$
H_0 is rejected and H_a accepted if
$X_1 \leq 3$
Find power function of test
Power $= 1-\beta$ where $\beta = P(\text{accept } H_0 \mid H_a = \text{true})$
$f(X=x \mid n,p) = \binom{n}{x} p^x (1-p)^{n-x}$
Since we reject H_0 for $X_1 \leq 3$, we accept H_0 for $X_1 > 3$ $(X_1 \geq 4)$
Want to know $P(\text{accept } H_0 \mid H_a = \text{true})$, so want to calculate

$$P(X_1 \geq 4 \mid n=10, p=0,2) =$$
$$= \sum_{i=4}^{10} \binom{10}{x_i} \cdot 0,2^{x_i} (1-0,2)^{10-x_i}$$

Power is $1-\beta$ so power of the test is

$$1 - \sum_{i=4}^{10} \binom{10}{x_i} \cdot 0,2^{x_i}\, 0,8^{10-x_i}$$

If my calculations are correct this is around
$1 - 0,12087... \approx 0,88$ ok

Note that the power function is defined for all values in the parameter space! (-5)

Poäng: (Points) 15

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Uppg.nr.: (Task no.) 4

Lärarens kommentar: (Teacher's note)

4 $f(x) = \frac{x}{\theta^2} e^{\frac{-x^2}{2\theta^2}}$, $x > 0$, $\theta > 0$

$E[X] = \theta\sqrt{\frac{\pi}{2}}$ $\quad V(X) = \theta^2 \frac{4-\pi}{2}$

a) Derive $\hat{\theta}_{MoM}$, equate first sample moment with first population moment.

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \theta\sqrt{\frac{\pi}{2}} \rightarrow \hat{\theta}_{MoM} = \frac{\frac{1}{n}\sum_{i=1}^{n} x_i}{\sqrt{\pi/2}}$$ R

b) $$E\left[\hat{\theta}_{MoM}\right] = E\left[\frac{\frac{1}{n}\sum_{i=1}^{n} x_i}{\sqrt{\pi/2}}\right] = \frac{\frac{1}{n}\sum E[X]}{\sqrt{\pi/2}} =$$

$$= \frac{\frac{1}{n}\cdot n\cdot\theta\sqrt{\pi/2}}{\sqrt{\pi/2}} = \theta\cdot\frac{n}{n}\cdot\frac{\sqrt{\pi/2}}{\sqrt{\pi/2}} = \theta$$ R

The method of moments-estimator of θ is unbiased.

c) know that $\hat{\theta} \xrightarrow{d} N(\theta, i^{-1})$ as $n \rightarrow \infty$?

Where $i = \frac{I}{n}$ and I is the Fisher information.

Joint density for X is:

$L = \frac{\prod x_i}{\theta^{2n}} e^{-\sum x_i^2/2\theta^2}$ $\quad \ell = \log(L)$

$\ell = \sum\log(x_i) - 2n\log(\theta) - \frac{\sum x_i^2}{2\theta^2}$

$\ell' = \frac{-2n}{\theta} + \frac{\sum x_i^2}{\theta^3}$

$E[(\ell')^2] = E\left[\frac{4n^2}{\theta^2} + \frac{(\sum x_i^2)^2}{\theta^6}\right] = \frac{4n^2}{\theta^2} + \frac{(\sum E[X^2])^2}{\theta^6} =$

Wanted to make a normal approximation based on $n \rightarrow \infty$ but ran out of time. Would have used it to create

$P(\sqrt{\sigma^2}\cdot z_{\alpha/2} < \theta < \sqrt{\sigma^2}\cdot z_{\alpha/2}) = 100(1-\alpha)$ CI. ?

Poäng: (Points) 10

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Uppg.nr.: (Task no.) 5

Lärarens kommentar: (Teacher's note)

5. Two variables:

$X \sim \exp(\beta)$

$Y \sim \exp(\gamma\beta)$, $\gamma > 1$

$X_1, X_2, \ldots, X_n$ are observations on times between hits of particles outside a box

$Y_1, Y_2, \ldots, Y_n$ are observations on times between hits of particles inside a box

All variables are independent

a) Want to construct LRT for $H_0: \gamma = 2$ vs
$H_a: \gamma \neq 2$

$$f(x|\beta) = \frac{1}{\beta} e^{-\frac{x}{\beta}} \qquad f(y|\gamma\beta) = \frac{1}{\gamma\beta} e^{\frac{-y}{\gamma\beta}}$$ (+.5)

Need MLE of γ:

Joint density for y is:

$$L = \frac{1}{(\gamma\beta)^n} e^{-\sum_{i=1}^{n} y_i/\gamma\beta} \qquad \log(L) = \ell$$

$$\ell = \log(1) - \log(\gamma\beta)\cdot n - \frac{\sum_{i=1}^{n} y_i}{\gamma\beta}$$

$$\frac{d}{d\gamma}\ell = \frac{-n}{\gamma\beta} + \frac{\sum_{i=1}^{n} y_i}{\gamma^2\beta} \longrightarrow \frac{\sum_{i=1}^{n} y_i}{\gamma^2\beta} = \frac{n}{\gamma\beta} \rightarrow \frac{\sum_{i=1}^{n} y_i}{\gamma} = n$$

$$\hat{\gamma}_{ML} = \frac{\sum_{i=1}^{n} y_i}{n}$$

So LRT is:

$$\lambda(y) = \frac{\frac{1}{(2\beta)^n} e^{\frac{-\Sigma y_i}{2\beta}}}{\frac{1}{(\frac{1}{n}\Sigma y_i \beta)^n} e^{\frac{-\Sigma y_i}{\frac{1}{n}\Sigma y_i \beta}}} = \frac{(\frac{1}{n}\Sigma y_i \beta)^n}{(2\beta)^n} e^{-\frac{1}{2}\frac{\Sigma y_i}{\beta} + \frac{n}{\beta}\frac{\Sigma y_i}{\Sigma y_i}}$$

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Poäng: (Points) 7

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Lärarens kommentar: (Teacher's note)

5. cont.

$$\lambda(y) = \frac{\left(\frac{1}{n}\sum y_i\right)^n}{2^n} e^{\frac{-1}{2\beta}\sum y_i + \frac{n}{\beta}} =$$

$$= \left(\frac{1}{2}\bar{y}\right)^n e^{-\frac{1}{\beta}\left(\frac{\sum y_i}{2} - n\right)}$$

(+2 p)

b) According to Theorem 10.3.1:

$-2\log(\lambda(y)) \xrightarrow{d} \chi^2_{df}$ as $n \to \infty$ under H_0

$$\text{So } -2\left(\log\left(\tfrac{1}{2}\bar{y}\right)\cdot n - \frac{1}{\beta}\left(\frac{\sum y_i}{2} - n\right)\right) =$$

$$= \frac{2}{\beta}\left(\frac{\sum y_i}{2} - n\right) - 2\log\left(\tfrac{1}{2}\bar{y}\right)\cdot n$$

which follows a χ^2_1-distribution

$df = 2 - 1 = 1$

2 → # unknown param in denominator

1 → # unknown param under H_0.

Poäng: (Points)

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