STOCKHOLMS UNIVERSITET
Statistiska institutionen
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HOME EXAM, ECONOMETRICS II
2021-06-01

Time for examination: 9.00-14.00. The exam shall be submitted electronically via the department's web site no later than 15.00. The system does not allow submission after deadline. Therefore, start the submission well in advance. The last hour of the exam time is intended for arranging the electronic submission.
All necessary information about submission, anonymous code, extended writing time, etc. can be found in a separate file. If you, despite the instructions, have problems submitting the exam, email the exam to tenta@stat.su.se. However, this is only done in exceptional cases. Exams sent in by email after deadline will not be corrected.

For questions regarding the submission, email to: expedition@stat.su.se. Practical help is only available during the first hour of the exam.
For questions regarding the content of the exam, email to: edgar.bueno@stat.su.se . Incoming email questions are answered continuously during the exam.
If the course coordinator needs to send out information to all students during the exam, this is done to your registered email address. Therefore, check your email during the exam.

Note: The exam should be written individually. All types of collaborations and/or help from others are strictly forbidden. Suspected cheating is reported to the Disciplinary Board and can lead to suspension

Allowed tools: Pocket calculator, computer, course books and lecture notes.

The exam consists of 4 independent problems. Well motivated and clear solutions are required for full scoring on exercises 2 to 4. Don't forget to state any necessary assumptions or conditions where needed. No motivation is required in exercise 1.
Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description. Answers may be given in English or Swedish.

Good luck!

Problem 1. (20 points) Multiple choice. Each sub-question has four options and *one single correct answer*. Please mark clearly your chosen option. Marking more than one alternative will invalidate the results for that sub-question. No motivation is required.

1. Assume that the model $y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \cdots + \beta_k x_{tk} + u_t$ holds. Let $x_t = (x_{t1}, \cdots, x_{tk})$ be the values of all explanatory variables in period t; and X be the matrix that collects all independent variables for all time periods. Which of the following is **not** correct regarding contemporaneous exogeneity, $E(u_t|x_t) = 0$, and strict exogeneity, $E(u_t|X) = 0$:
   (a) Strict exogeneity implies contemporaneous exogeneity.
   (b) Contemporaneous exogeneity is sufficient for consistency of the OLS estimators.
   (c) Strict exogeneity is required for unbiasedness of the OLS estimators.
   (d) Contemporaneous exogeneity implies that future values of the explanatory variables are uncorrelated with current values of the error term.

2. Which of the following is **not** correct regarding smoothing methods:
   (a) Simple exponential smoothing is generally adequate for time series that do not exhibit a positive or negative trend.
   (b) Second-order exponential smoothing is equivalent to Holt's method using $\gamma = \alpha$.
   (c) Simple exponential smoothing is generally inadequate for time series that exhibit a positive or negative trend.
   (d) Simple exponential smoothing is a particular case of Holt's method.

3. Which of the following is correct regarding exponential smoothing for seasonal data:
   (a) The multiplicative seasonal model is adequate when the period increases over time.
   (b) The additive seasonal model is adequate when the period increases over time.
   (c) The multiplicative seasonal model is adequate when the seasonal pattern increases over time.
   (d) The additive seasonal model is adequate when the seasonal pattern increases over time.

4. A time series that follows an ARIMA model has been observed up to time $T = 100$. Which of the following is correct regarding the variance of forecasts for periods 110 and 120, $V(\hat{y}_{110})$ and $V(\hat{y}_{120})$, respectively:
   (a) $V(\hat{y}_{120})$ may be smaller than $V(\hat{y}_{110})$ only if the model is a pure AR model.
   (b) $V(\hat{y}_{120})$ may be smaller than $V(\hat{y}_{110})$ only if the model is integrated of order zero.
   (c) $V(\hat{y}_{120})$ may be smaller than $V(\hat{y}_{110})$ only if the model is a pure MA model.
   (d) $V(\hat{y}_{120})$ cannot be smaller than $V(\hat{y}_{110})$.

5. Which of the following is correct regarding differencing and smoothing a time series:
   (a) Smoothing methods do not necessarily remove the trend.
   (b) Differencing is typically carried out in order to remove the trend. It will simultaneously smoothen the time series.
   (c) Smoothing methods will simultaneously smoothen and remove the trend.
   (d) None of the above is correct.

6. Which of the following is correct regarding $ARIMA(p, d, q) \times (P, D, Q)$ models.
   (a) For monthly time series, it is common in practice to use $P = 12$ and $Q = 12$.
   (b) They take into account possible correlations between errors.
   (c) Models with $D > d$ do not allow for estimation of parameters.
   (d) They are adequate for heteroskedastic time series.

Problem 2. (30 points) Consider the time series:

Time, t	1	2	3	4	5	6	7	8	9	10
Data, y_t	5	7	8	12	14	14	17	19	18	20

a) Smooth the data using simple exponential smoothing with $\lambda = 0.3$ and $\tilde{y}_0$=5. Provide a point-prediction for periods 11, 12 and 13.

b) Smooth the data using Holt's double exponential smoothing with $\alpha = 0.3$ and $\gamma = 0.5$. Initialize the level at 5 and the trend at 1. Provide a point-prediction for periods 11, 12 and 13.

c) The actual values for the next three periods are now observed: $y_{11} = 24$, $y_{12} = 23$ and $y_{13} = 25$. Calculate the Mean Error, Mean Absolute Error and the root of the Mean Square Error of the forecasts in (a) and (b).

Problem 3. (20 points) Consider the model

$$y_t = -0.5 + a_t + 0.5\,a_{t-2} - 0.25\,a_{t-4}$$

where $\{a_t\}$ is Gaussian white noise with mean 0 and variance 1 (i.e. $a_t \sim N(0,1)$ and $Cov(a_t, a_s) = 0$ for all $t \neq s$).

a) Calculate $E(y_t)$ and $V(y_t)$.

b) Calculate the autocorrelation coefficients at lags 2, 3 and 4 (ρ_2, ρ_3 and ρ_4).

c) Show that, for all $k > 4$, the correlation coefficient at lag k, ρ_k, is equal to zero.

d) The time series has been observed up to time T. It is known, therefore, that $a_{T-3} = 1$, $a_{T-2} = -0.1$, $a_{T-1} = -0.75$ and $a_T = -1.25$. Compute the forecasts for periods $T+1$, $T+2$, $T+3$, $T+4$ and $T+5$.

Problem 4. (30 points) Consider the following five time series processes where $\{\epsilon_t\}$ is Gaussian white noise with mean 0 and variance 1 (i.e. $\varepsilon_t \sim N(0,1)$ and $Cov(\epsilon_t, \epsilon_s) = 0$ for all $t \neq s$):

$$y_t = y_{t-1} + \varepsilon_t \quad (1)$$
$$y_t = a_t, \quad \text{where } a_t = \varepsilon_t\sqrt{h_t}, \text{ and } h_t = 1 + 0.9a_{t-1}^2 \quad (2)$$
$$y_t = 4 - 0.95y_{t-1} + \varepsilon_t \quad (3)$$
$$y_t = 2 + \varepsilon_t - 0.75\varepsilon_{t-1} + 0.45\varepsilon_{t-2} \quad (4)$$
$$y_t = 1 + 0.95y_{t-1} - 0.78y_{t-2} + \varepsilon_t \quad (5)$$

and the plots in Figure 1.

(a) What are the processes in equation (1)–(5) called? Also, write them in terms of the back-shift and/or difference operator.

(b) Match each time series process in (1)–(5) to a figure in (a)–(e). (Guessing on multiple figures yields 0 points).

(c) Calculate the mean and variance of each time series.

(d) Which of the time series are stationary? Motivate your answer.

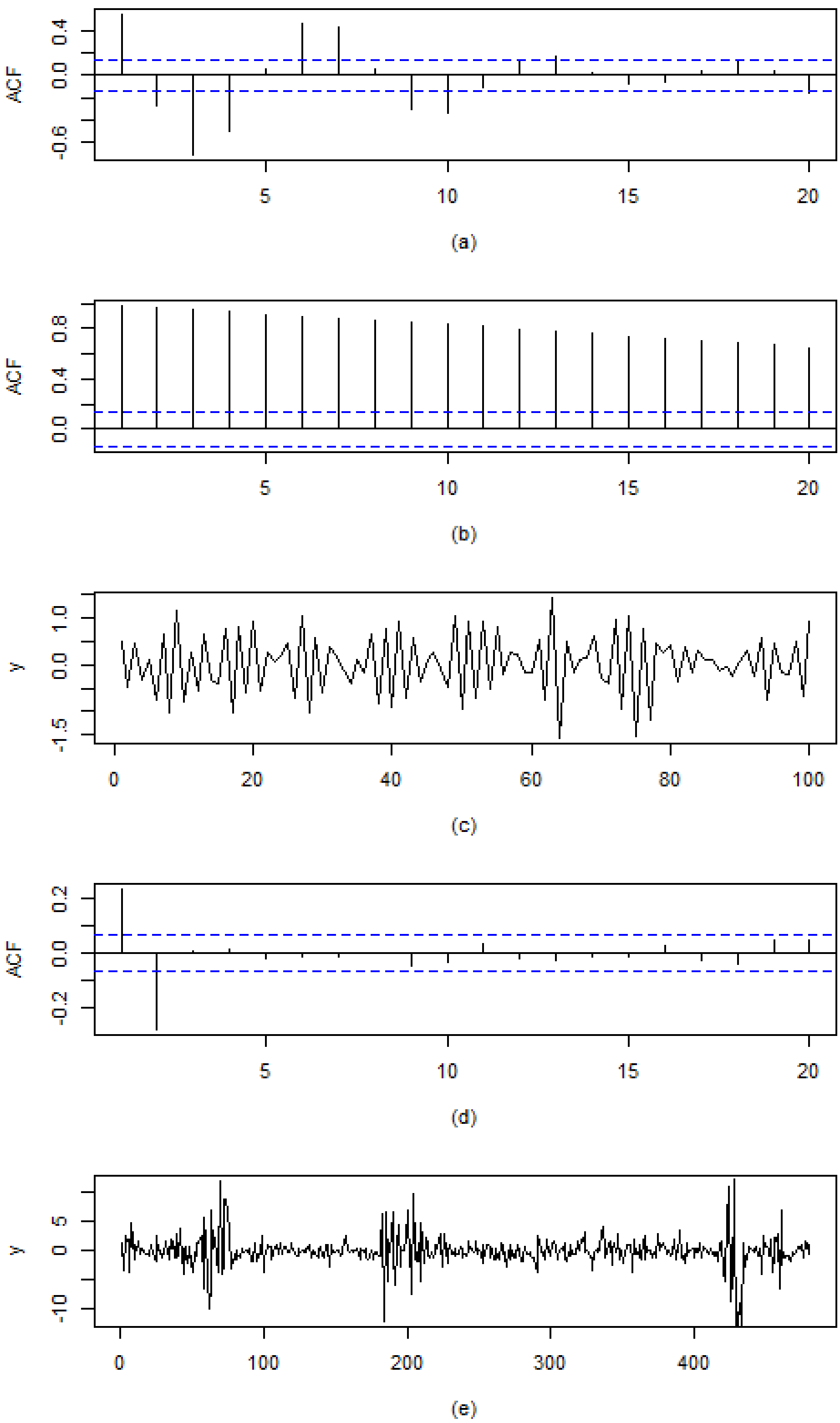

Figure 1: Time series and autocorrelation plots for Problem 4