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86.5p $\boxed{B}$

1 - d R

2 - b R ~~d~~ d

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3 - c R

4 - d R

5 - a R

6 - b R

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a) Simple exponential smoothing algorithm:

$$\tilde{y}_T = (1-\theta\, y_T) + \theta\, \tilde{y}_{T-1} \quad \text{or}$$

$$\tilde{y}_T = \lambda\, y_T + (1-\lambda)\, \tilde{y}_{T-1}$$

which in this case makes for

$$\tilde{y}_T = 0{,}3 \cdot y_T + 0{,}7 \cdot \tilde{y}_{T-1}$$

ok

Time, t	(0)	1	2	3	4	5	6	7	8	9	10
Data, y_t	(5)	5	7	8	12	14	14	17	19	18	20
Smoothed $\tilde{y}_t$		5	5,6	$\boxed{7,3}$	9,2	12,6	14	14,9	17,6	18,7	18,6

you get an error in the beginning such that the remaining numbers gets wrong.

Examples: $\tilde{y}_6 = 0{,}3 \cdot 14 + 0{,}7 \cdot 12{,}6 = 14$ $\quad \tilde{y}_8 = 0{,}3 \cdot 19 + 0{,}7 \cdot 14{,}9 = 17{,}6$

Predictions are done using $\hat{y}_{T+\tau}(T) = \tilde{y}_T$ for τ-steps ahead

ok

For $t=11$: $\hat{y}_{11}(10) = \hat{y}_{10+1} = \tilde{y}_{10} = 18{,}6$
$T=10 \quad \tau=1$

For $t=12$: $\hat{y}_{12}(10) = \hat{y}_{10+2} = \tilde{y}_{10} = 18{,}6$
$T=10 \quad \tau=2$

For $t=13$: $\hat{y}_{13}(10) = \hat{y}_{10+3} = \tilde{y}_{10} = 18{,}6$
$T=10 \quad \tau=3$

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2b) Holt's double exponential smoothing

Level at t formula:

$$L_t = \alpha y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

here; $L_t = 0{,}3 \cdot y_t + 0{,}7(L_{t-1} + T_{t-1})$ ok

Trend at t formula:

$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1}$$ ok

here; $T_t = 0{,}5(L_t - L_{t-1}) + 0{,}5\, T_{t-1}$

Time, t	0	1	2	3	4	5	6	7	8	9	10
Data, yt		5	7	8	12	14	14	17	19	18	20
Level, Lt	5	5,7	6,685	7,72175	9,689213	12,01305	13,93779	16,19443	18,49492	19,88102	21,16914
Trend, Tt	1	0,85	0,9175	0,977125	1,472294	1,898068	1,911399	2,084022	2,192254	1,789179	1,538649

R

Forecasts for τ-steps ahead

$$\hat{y}_{t+\tau}(t) = L_t + T_t \cdot \tau$$

For $t=11$, $\tau=1$: $\hat{y}_{11} = \hat{y}_{10+1}(10) = 21{,}169 + 1{,}539 \cdot 1 = 22{,}708$

For $t=12$, $\tau=2$: $\hat{y}_{12} = \hat{y}_{10+2}(10) = 21{,}169 + 1{,}539 \cdot 2 = 24{,}247$ R

For $t=13$, $\tau=3$: $\hat{y}_{13} = \hat{y}_{10+2}(10) = 21{,}169 + 1{,}539 \cdot 3 = 25{,}786$

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2,c)

Forecast error: $e_t(1) = y_t - \hat{y}_t(t-1)$

Mean error (ME): $\frac{1}{n}\sum_{t=1}^{n} e_t(1)$

Mean absolute error (MAD): $\frac{1}{n}\sum_{t=1}^{n} |e_t(1)|$

Mean squared error (MSE): $\frac{1}{n}\sum_{t=1}^{n} (e_t(1))^2$

Asked for RMSE

time, t	data, y_t	SES $\hat{y}_t(t-1)$	SES $e_t(1)$	Holt's $\hat{y}_t(t-1)$	Holt's $e_t(1)$
11	24	18,6	5,4	22,708	1,292
12	23	18,6	4,4	24,247	(-1,247)
13	25	18,6	6,4	25,786	-(0,786)

You get wrong results for SES since forecasts are wrong but that's ok.
But you should have calculated RMSE

$ME_{SES} = \frac{1}{3}\sum_{t=1}^{3} e_t(1)_{SES} = \frac{1}{3}(5,4+4,4+6,4) = 5,4$

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$ME_{Holt's} = \frac{1}{3}\sum_{t=1}^{3} e_t(1)_{Holt's} = \frac{1}{3}(1,292-1,247-0,786) = -0,247$

$MAD_{SES} = \frac{1}{3}\sum_{t=1}^{3} |e_t(1)_{SES}| = \frac{1}{3}(|5,4|+|4,4|+|6,4|) = 5,4$

$MAD_{Holt's} = \frac{1}{3}\sum_{t=1}^{3} |e_t(1)_{Holt's}| = \frac{1}{3}(|1,292|+|-1,247|+|-0,786|) \approx 1,1083$

$MSE_{SES} = \frac{1}{3}\sum_{t=1}^{3} (e_t(1)_{SES})^2 = \frac{1}{3}(5,4^2+4,4^2+6,4^2) \approx 29,827$

$MSE_{Holt's} = \frac{1}{3}\sum_{t=1}^{3} (e_t(1)_{Holt's})^2 = \frac{1}{3}(1,292^2+(-1,247)^2+(-0,786)^2) \approx 1,2807$

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MA model $E(y_t)$ and $V(y_t)$

3 a) $E(y_t) = E(-0{,}5 + a_t + 0{,}5a_{t-2} - 0{,}25a_{t-4}) =$

$= E(-0{,}5) + E(a_t) + 0{,}5\,E(a_{t-2}) - 0{,}25\,E(a_{t-4}) =$

$= (-0{,}5) + 0 + 0{,}5 \cdot 0 - 0{,}25 \cdot 0 = \underline{\underline{(-0{,}5)}}$

Since $a_t \sim N(0,1)$ then all $E(a_{t-2}) = 0$

All $Cor(a_t, a_s) = 0$ given in the problem

$Var(y_t) = Var(-0{,}5 + a_t + 0{,}5a_{t-2} - 0{,}25a_{t-4}) =$

The Variance of a constant is 0

$= Var(-0{,}5) + Var(a_t) + 0{,}5^2\,Var(a_t) + 0{,}25^2\,Var(a_t) =$

$= 0 + Var(a_t) + 0{,}5^2\,Var(a_t) + 0{,}25^2\,Var(a_t) =$

$Var(a_t) = 1$ given in the problem

$= Var(a_t) \cdot (1 + 0{,}5^2 + 0{,}25^2) = 1 + 0{,}5^2 + 0{,}25^2 = \underline{\underline{1{,}3125}}$

Answer: $E(y_t) = (-0{,}5)$, $Var(y_t) = 1{,}3125$

Note: There are shorter formulas for calculating the above values but the wording of the problem was unclear to me. Therefore I decided to show the whole process.

5p

3 b) Autocorrelation function

$$\rho_Y(k) = \frac{\gamma_Y(k)}{\gamma_Y(0)} = \begin{cases} \frac{-\theta_k + \theta_1\theta_{k+1} + \dots + \theta_{q-k}\theta_q}{1+\theta_1^2 + \dots + \theta_q^2}, & k=1,2,\dots q \\ 0, & k > q \end{cases}$$

Autocovariance function

$$\gamma_Y(k) = \begin{cases} \sigma^2(-\theta_k + \theta_1\theta_{k+1} + \dots + \theta_{q-k}\theta_q), & k=1,2,\dots,q \\ 0, & k > q \end{cases}$$

Variance

$$\gamma_Y(0) = Var(Y_t) = \sigma^2(1+\theta_1^2 + \dots + \theta_q^2)$$

In the model the problem gives the MA model uses lags 2 and 4 which could be re-written with weights that are $=0$ at lag 1 and 3

$$Y_t = -0,5 + a_t + 0 \cdot a_{t-1} + 0,5 a_{t-2} + 0 a_{t-3} - 0,25 a_{t-4}$$

weight defined as $-\theta$; $\theta_1 = 0$ $\theta_2 = -0.5$ $\theta_3 = 0$ $\theta_4 = 0,25$

The variance is previously known to be 1,3125, as calculated in problem a.

$$\gamma_Y(2) = 1 \cdot (-(-0,5) + 0 \cdot 0 + (-0,5) \cdot 0,25) = 0,375$$

$$\gamma_Y(3) = 1 \cdot (-0 + 0 \cdot 0,25) = 0$$

$$\gamma_Y(4) = 1 \cdot (-0,25) = -0,25$$

Answers

$$\rho_Y(2) = \frac{0,375}{1,3125} \approx \underline{0,2857}$$

$$\rho_Y(4) = \frac{(-0,25)}{1,3125} \approx \underline{(-0,1905)}$$

$$\rho_Y(3) = \frac{0}{1,3125} = \underline{0}$$

good

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3c) All the weights further off than lag 4 is zero. This means that if we rewrote the model with infinite lags it would look like

$$y_t = (-0{,}5) + \alpha_t + 0\,a_{t-1} + 0{,}5\,a_{t-2} + 0\,a_{t-3} - 0{,}25\,a_{t-4} + 0\,a_{t-5} + 0\,a_{t-6} + \dots + 0\,a_{t-k}$$

This means that θ_5 to θ_k or $\theta_{k>4}$ would all be zero.

The diviser $\gamma(0)$ in the autocorrelation function is uninteresting, it is enough if the dividend $\gamma(k) = 0$

$$\gamma(k) = -\theta_k + \theta_1\theta_{k+1} + \dots + \theta_{q-k}\theta_q$$

If we use a k-value higher than 4 then all our terms would include a factor of zero since $\theta_{k>4} = 0$ as already shown.
The last terms would include a $\theta_{i<0}$ which isn't either included, thus having the value 0 if the model where re-written to include them.

4p

Since we in every term has a factor 0 the value of $\gamma(k>4) = 0$ which implies that $\rho(k>4) = 0$

Good!

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3 d)

For an ARIMA process which the MA process is a part of the best forecast is the expected value given the past values.

$a_{T-3} = 1 \quad a_{T-2} = (-0{,}1) \quad a_{T-1} = (-0{,}75)$

$a_T = (-1{,}25)$

Given in the problem;
Expected value of forecast error is 0

$\hat{Y}_{T+1} = (-0{,}5) + 0 + 0{,}5 \cdot (-0.75) - 0{,}25 \cdot 1 = (-1{,}125)$

$\hat{Y}_{T+2} = (-0{,}5) + 0 + 0{,}5 \cdot (-1{,}25) - 0{,}25 \cdot (-0{,}1) = (-1{,}1)$

$\hat{Y}_{T+3} = (-0{,}5) + 0 + 0{,}5 \cdot 0 - 0{,}25 \cdot (-0{,}75) = (0{,}3125)$

$\hat{Y}_{T+4} = (-0{,}5) + 0 + 0{,}5 \cdot 0 - 0{,}25 \cdot (-1{,}25) = 0{,}3125$

I think you forgot -0.5

$\hat{Y}_{T+5} = (-0{,}5) + 0 + 0{,}5 \cdot 0 - 0{,}25 \cdot 0 = (-0{,}5)$

4p

Tot:19/20

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4 a)

(1) ARIMA (0,1,0) $(1-B)y_t = \varepsilon_t$ ✓
"The random walk" ✓

(2) ARCH(1) ✓

(3) AR(1) $(1+0{,}95B)y_t = 4+\varepsilon_t$ ✓

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back-shift operator?

(4) MA(2) ✓

(5) AR(2) ✓ $(1-0{,}95B+0{,}78B^2)y_t = 1+\varepsilon_t$ ✓

4 b)
(1) - b ✓
(2) - e ✓
(3) - c ✓
(4) - d ✓
(5) - a ✓

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4 c) (1)

(2)

(3) $E(y_t) = \mu = \dfrac{\delta}{1-\phi_1} = \dfrac{4}{1+0{,}95} = \dfrac{4}{1{,}95} \approx 2{,}05$ ✓

$Var(y_t) = \phi_1 \, \gamma_y(1) = (0{,}95) \cdot [Cov(y_t, y_{t+1}) = 0] =$

$= 0$ ✗

(4) $E(y_t) = \mu = 2$ ✓

$Var(y_t) = [\sigma^2 = 1](1 + 0{,}75^2 - 0{,}45^2) = 1{,}36$ +

(5) $E(y_t) = \mu = \dfrac{\delta}{1-\phi_1-\phi_2} = \dfrac{1}{1-0{,}95+0{,}78} \approx 1{,}2048$ ✓

3.5p

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3

4 d) Random walk aren't stationary so not (1) ✓

(2) Shifts in variance if we look at time series e

so what do you answer?

AR(1) and the MA(2) in (3) and (4) are stationary, they always are ✓

(5) Is also stationary since $0,95 - 0,78 < 1$ and $|-0,78| < 1$ and $-0,78 - 0,95 < 1$ ✓

Answer: 3, 4 and 5 are stationary

4p

tot: 20.5/30