STOCKHOLM UNIVERSITY
Department of Statistics
Econometrics II, Time Series Analysis, ST223G
Spring semester 2020

Written Examination in Time Series Analysis

Date	2020-06-02
Hour:	15.00-20.00
Examiner:	Jörgen Säve-Söderbergh
Allowed tools:	1) Textbook: Wooldridge, J.M. *Introductory Econometrics: A Modern Approach*, Cengage, Boston. 2) Textbook: Montgomery, D.C., Jennings, C.L., and Kulachi, M., *Introduction to Time Series Analysis and Forecasting*, John Wiley & Sons, New Jersey. 3) Pocket calculator 4) Notes written in the text book are allowed.

- On problem 6 and 7 it is sufficient to just state which alternative you believe is true. Nothing further than that is required.
- Note that no formula sheet is provided.
- Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description.
- The maximum number of points for each problem is stated immediately after the question number. If not indicated otherwise, to obtain the maximum number of points on each problem, detailed and clear solutions are required. Answers may be given in English or Swedish.

Good luck!

1. (12 points) Analyze the following data with the simple exponential algorithm

$$\tilde{y}_T = (1-\theta)\, y_T + \theta \tilde{y}_{T-1},$$

using $\theta = 0.2$ and as starting value $\tilde{y}_0 = 3$

Time	
1	4
2	3
3	4

Make a forecast for time period five.

2. (12 points) The following table presents predicted monthly sales and actual monthly sales for a company over the first six months of 2019.

	Actual Sales	Predicted Sales
January	270	265
February	263	268
March	275	269
April	262	267
May	250	245
June	278	275

   (a) (2 points) Calculate the forecast error for each month.

   (b) (3 points) Calculate the MAD (mean absolute deviation).

   (c) (3 points) Calculate MSE (mean squared error).

   (d) (4 points) Calculate MAPE (mean absolute precent forecast error).

3. (12 points) A stochastic process (or model) is given by

$$y_t = 0.75 y_{t-1} - 0.50 y_{t-2} + \varepsilon_t$$

where ε_t is independent and normally distributed with expected value 0 and with known variance $\sigma_\varepsilon^2 = 1$.

   (a) (2 points) What model is this?

   (b) (2 points) What are the parameter values?

   (c) (4 points) Is the model stationary? Is it invertible?

   (d) (4 points) Rewrite the model using the backshift operator B.

4. (12 points) Assume the model

$$y_t = \varepsilon_t - 0.50\varepsilon_{t-1}$$

where $E(\varepsilon_t) = 0$ and $\text{Var}(\varepsilon_t) = \sigma^2$ and ε_t are independent random variables.

(a) (4 points) What kind of model is this? Is it invertible?

(b) (4 points) Compute $E(y_t)$.

(c) (4 points) Compute $\text{Var}(y_t)$.

5. (12 points) Rewrite the following ARIMA(1,1,1) model

$$(1 - \phi_1 B)(1 - B)\, y_t = (1 - \theta_1 B)\, \varepsilon_t$$

in difference-equation form (that is, in a formula without the backshift operator that contains y_t and lagged values of y_t among other things).

6. (10 points) What does *asymptotically uncorrelated* mean?

A. That a covariance stationary stochastic process x_t has an autocorrelation function $\text{Corr}(x_t, x_{t+h})$ that converges to zero as the distance between any two points goes to infinity.

B. That a covariance stationary stochastic process x_t has an autocorrelation function $\text{Corr}(x_t, x_{t+h})$ that converges to one as the distance between any two points goes to infinity.

C. That a covariance stationary stochastic process x_t has an autocorrelation function $\text{Corr}(x_t, x_{t+h})$ that converges to zero as the distance between any two points goes to zero.

D. That a covariance stationary stochastic process x_t has an autocorrelation function $\text{Corr}(x_t, x_{t+h})$ that converges to infinity as the distance between any two points goes to zero.

7. (10 points) What does *weak dependence* for a stochastic process $\{x_t : \ t = 1, 2, \ldots\}$ mean?

   A. A stochastic process $\{x_t : \ t = 1, 2, \ldots\}$ where x_t are independent and identically distributed for all values of h.

   B. A stationary stochastic process $\{x_t : \ t = 1, 2, \ldots\}$ where x_t and x_{t+h} are "almost independent" as h goes to infinity.

   C. A stochastic process $\{x_t : \ t = 1, 2, \ldots\}$ where the expected value does not depend on time t and the autocovariance function $\text{Cov}\,(x_t, x_{t+h})$ for any lag h is only a function of h and not time t.

8. (20 points) Let $\{y_t : \ t = 1, 2, \ldots\}$ follow a random walk

$$y_t = y_{t-1} + e_t$$

where $y_0 = 0$. We assume that $\{e_t : \ t = 1, 2, \ldots\}$ is independent and identically distributed with mean zero and variance σ_e^2. We also assume that the initial value y_0 is independent of e_t for all $t \geq 1$. Show that

$$\text{Corr}\,(y_t, y_{t+h}) = \sqrt{\frac{t}{t+h}} \quad \text{for } t \geq 1, \quad h > 0$$