STOCKHOLM UNIVERSITY
Department of Statistics
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WRITTEN RE-EXAMINATION, ECONOMETRICS II
2023-11-22

Time for examination: 08.00-13.00

Allowed tools: Pocket calculator, own formula sheet (1 double-sided A4 page), Course text-book (any edition): Wooldridge, J.M. *Introductory Econometrics - a Modern Approach (any edition)* and Montgomery, Jennings and Kulachi - *Introduction to Time Series and Forecasting*

Note that no formula sheet will be provided.

The exam consists of 6 independent problems. Well motivated and clear solutions are required for full scoring on a problem. Don't forget to state any necessary assumptions or conditions where needed.

Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description. Answers may be given in English or Swedish.

Good luck!

Problem 1. (15 points)
Without accounting for your reasoning, indicate which **single** alternative is correct for each of the following part questions, a through e. Note that providing more than one alternative results in 0 marks for the part question.

(a) Assume that the variable y_t follows an $I(3)$ process, then $(1-B)^2 y_t$

(i) is weakly dependent

(ii) has unit root

(iii) when B is the backshift operator, is trend stationary

(iv) is an independent process (white noise)

(b) If you use the *difference-in-difference* estimator to estimate the causal effect of a change in policy

(i) you do not require a treatment group

(ii) it can be estimated using a probability sample from a single point in time

(iii) the estimator is an estimator of the difference in th expected outcome in the treatment group before and after the policy chance, minus the difference in the expected outcome in a control group, before and after the policy change

(iv) you do not strictly speaking require a control group

(c) For longitudinal data, you use feasible generalised squares (FGLS) in order to

(i) Adjust the standard errors of the OLS estimator in the presence of collinearity

(ii) Adjust the standard errors of the OLS estimator in the presence of serially correlated errors

(iii) Eliminate serially correlated errors (under certain conditions)

(iv) Find a suitable instrument for endogenous variables

(d) Prais-Winsten estimation is

(i) An approach to obtain an unbiased OLS estimator under Assumption TS.3′

(ii) An approach to eliminate serially correlated AR(1) errors.

(iii) Used to test for heteroscedasticity.

(iv) Used to obtain an unbiased estimator in a time series regression

(e) In the time series regression

$$y_t = \alpha + \beta x_t + u_t,$$

which of the following is the *main* problem with serially correlated errors u_t

(i) The parameter α is estimated inconsistently because $Cov(u_t, \alpha) \neq 0$

(ii) They necessarily violate contemporaneuos exogeneity, i.e. $E(u_t \mid x_t) \neq 0$

(iii) The OLS estimator $\hat{\beta}$ is inconsistent, even if $E(u_t \mid x_t) = 0$

(iv) The test of H_0: $\beta = 0$ against H_1: $\beta \neq 0$ using the sampling distribution of the OLS estimator may give misleading results.

Problem 2. (10 points)

Write the following models using backshift operators. Clearly indicate the order of the different lag-polynomials, e.g. for an ARIMA(1,1,3),

$$(1 - \phi_1 B)(1 - B)y_t = \delta + (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)\epsilon_t.$$

(a) ARIMA$(0, 0, 3)$.

(b) ARIMA$(0, 1, 0) \times (2, 1, 2)$ with seasonal period $s = 12$.

(c) ARIMA$(2, 1, 1) \times (0, 1, 2)$ with seasonal period $s = 4$

Problem 3. (17 points)

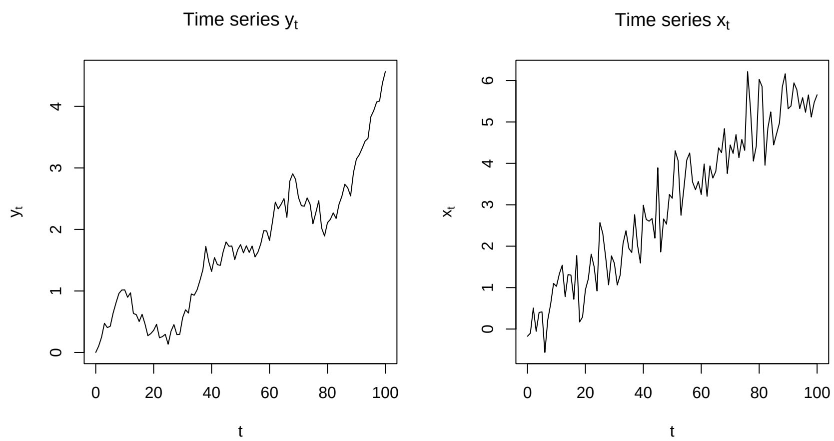

Figure 1: Two time series, y_t (left) and x_t (right) to be considered in Problem 3

Consider the data in Figure 1. In particular, reflect on stationarity, autocorrelation, and potential trends.

(a) For the time series in the right panel, x_t, propose and motivate a plausible model. Comment briefly on the characteristics of the time series

(b) For the time series in the left panel, y_t, propose and motivate a plausible model. Comment briefly on the characteristics of the time series

(c) For the data, suppose that you consider the time series regression model

$$y_t = \beta_0 + \beta_1 x_t + e_t$$

where e_t is a random variable with $E(e_t \mid x_t) = 0$. Further, assume that x_t follows your proposed process in (a), and that y_t follows the process that you propose in (b). Comment on how likely it is that Assumption TS.1$'$ is satisfied. Explain how you formally test this assumption. Use as much detail as necessary.

(d) Assume that you were to estimate α and β using OLS. What assumptions that usually hold for OLS would be violated and what would the consequences be? What could you do to remedy these issues (this may involve estimating a different model).

Problem 4. (18 points)

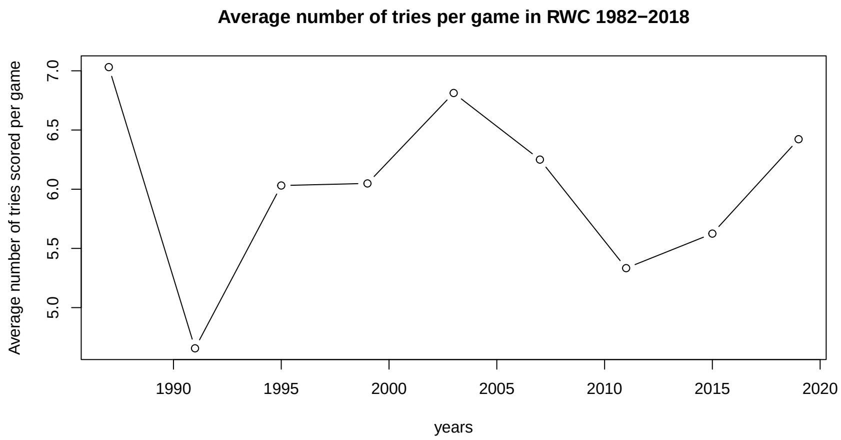

Figure 2: Average number of tries For problem 4

years	1987	1991	1995	1999	2003	2007	2011	2015	2019
tries	225	149	193	248	327	300	256	270	289
matches	32	32	32	41	48	48	48	48	45*

Table 1: Number of tries scored and matches played in Rugby World Cups, 1987-2019.
*Typhoon Hagibis cause 3 matches to be cancelled during the 2019 Rugby world cup

Scoring a try, grounding the ball behind the try-line, is the main form of scoring in rugby union. Table 1 provides the total number of tries scored in each of the nine Rugby World Cups (RWCs) prior to 2023. Before 1999, the competition encompassed 32 games, in 1999 41, and post 1999 48 matches. These changes reflect the enlargement of the competition in terms of the number of teams. The averagage number of tries scored is plotted in Figure 2.

Table 2 provides the averages y_T together with the smoothed values $\tilde{y}_T^{(1)}$ and $\tilde{y}_T^{(2)}$, obtained as

$$\tilde{y}_T^{(1)} = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}^{(1)},$$

$$\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)},$$

for $T = 1, \ldots, 9$, with $\tilde{y}_T^{(1)} = \tilde{y}_T^{(2)} = 7.03$, and λ chosen to minimise the sum of squares of the one-step ahead prediction errors (SSA) of a double exponential smoothing algorithm.

(a) The values tried for the algorithm were $\lambda = .05, .15, .25, .35, .45, .55, .65, .75$. Based on Figure 3, which value of λ was used to compute the smoothed values in Table 2?

(b) Consider the first four World Cups. Forecast the average number of tries scored per match for the 2003 RWC using DES and compute the forecast error. Note that you do not

Year	1987	1991	1995	1999	2003	2007	2011	2015	2019
y_T	7.03	4.66	6.03	6.05	6.81	6.25	5.33	5.62	6.42
$\tilde{y}_T^{(1)}$	7.03	6.67	6.58	6.50	6.55	6.50	6.33	6.22	6.25
$\tilde{y}_T^{(2)}$	7.03	6.98	6.92	6.86	6.81	6.76	6.70	6.63	6.57

Table 2: Average number of tries scored in Rugby World Cups, 1987-2019, and their smoothed values. To be used for Problem 4.

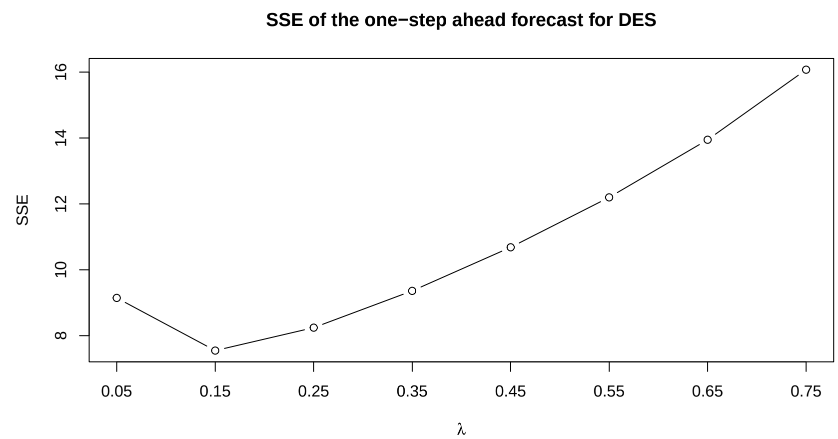

Figure 3: Sum of squares of the one-step ahead prediction errors (SSE) for double exponential smoothing (DES) as a function of λ. To be used for problem 4

need to recompute the smoothed values in Table 2.

(c) Using the available data, forecast the average number of tries scored per game in the 2023 RWC with *simple exponential smoothing.*

(d) Using the available data, forecast the average number of tries scored per game in the 2023 RWC with *double exponential smoothing.*

(e) In the 2023 World cup (which South Africa won by beating New Zeeland in the final) 325 tries were scored across 48 matches. Compute the forecast errors of the methods in (c) an (d). In addition, compute the relative forecasting errors in percentage form. Based on this measure, which one of the forecast methods in (c) and (d) performs the best?

Problem 5. (20 points)
Suppose that

$$y_t = z + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_3 \epsilon_{t-3},$$

where z is a stochastic variable with $E(z) = 1$ and $Var(\epsilon) = \sigma_z^2$, and the errors e_t are i.i.d for all t, with $E(\epsilon_t) = 0$ and $E(\epsilon_t^2) = \sigma^2$. Assume that ϵ_t and z are *independent* for all t.

(a) Is y_t an MA process?

(b) Compute the expected value of y_t

(c) Compute the variance of y_t

(d) Compute the covariance $Cov(y_t, y_{t+3})$

(e) Is the process for y_t weakly dependent?

(f) Assume that $z \sim \mathcal{N}(1, 1)$ and $\epsilon \sim \mathcal{N}(0, 0.25)$, and let $\theta_1 = -0.1$ and $\theta_3 = -0.2$. At time $t = 10$, what is the conditional probability that y_{11} is greater than 2, conditional on $\epsilon_{10} = .19$, $\epsilon_9 = .10$, and $\epsilon_8 = -0.12$?

Problem 6. (20 points)
Suppose that $\{(x_i, y_i) : t = 1, 2, \ldots\}$ are two stationary and weakly dependent time series. Consider the model

$$y_t = \delta_0 + \delta_1 x_t + \epsilon_t,$$

where $E(\epsilon_t \mid x_t, x_{t-1}, \ldots) = 0$, and $E(\epsilon_t^2 \mid x_t, x_{t-1}, \ldots) = \sigma^2$. Further, assume that for x_t itself, we have

$$x_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \nu_t,$$

where $E(\nu_t \mid x_{t-1}, \ldots, y_{t-1}, y_{t-2}, \ldots, u_{t-1}, \ldots) = 0$.

(a) Assume that $\gamma_1 = 0$ and $\gamma_2 = 0$ and compute $Cov(\epsilon_t, x_{t-1})$

(b) Assume that $\gamma_1 = 0.25$ and $\gamma_2 = -0.5$ and compute $Cov(\epsilon_t, x_{t-1})$

(c) For conditions in (b), is the OLS estimator of δ_1 unbiased for finite samples?

(d) Suppose instead that y_t and x_t are I(1) processes (integrated of order 1) and you want to use OLS to estimate a new regression

$$y_t = \alpha + \beta x_t + u_t.$$

What assumption is violated and what are the potential consequences?