



Stockholms universitet

OBS! Läs noga igenom anvisningarna i tentamen, t.ex. hur du ska skriva svaren. Det är ditt ansvar som student att följa de anvisningar som ges.

NOTE! Read the examination instructions carefully, e.g. how to write the answers. It is your responsibility as a student to follow the given instructions.

Skriv din anonymiseringskod och dagens datum på allt material du lämnar in.
(Enter your anonymization code and today's date on all submitted materials)

Anonymiseringskod (Anonymization code)	3	1	1	-	0	0	1	5	-	J	P	0
Datum (Date YYYY-MM-DD)	2022-01-13						Plats nr. (Seat No.)	21				

Kurs/Kurskod (Course/Course code)	ST223G
Kursmoment (Course component)	

Fylls i av tentamensvärd (To be filled in by invigilator)

Direkt i skrivning: (kryss)		Svarsblankett: (kryss)		Lösa svarsblad: (antal)	6
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Lämnat in blankt: (kryss)		Dator: (kryss)	
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Inlämningstid: 12 : 14

Signatur tentamensvärd:

Ulf Erik Jönsson

Fylls i av lärare/examinator (To be filled in by teacher/examinator)

Betyg:	B	Poäng:	78
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Signatur rättande lärare/examinator:

A. Kallp

Regler i skrivsalen

- Följ tentamensvärds anvisningar.
- Väskor och ytterkläder ska placeras på anvisad plats.
- Placera ID-handling väl synlig på bordet framför dig.
- Ingen student får lämna skrivsalen under de första 30 minuterna.
- Endast en student i taget får besöka toaletten. Vid toalettbesök skriv ditt namn och klockslag på avsedd lista. Efter toalettbesöket ska du åter ange klockslag på listan.
- Elektronisk utrustning som mobiltelefon eller Smartwatch ska vara avstängd och placerad på anvisad plats.
- Under tentamen gäller tystnad – det är förbjudet att prata, eller på annat sätt kommunicera, med andra studenter under pågående tentamen.
- Innan tentamenshandlingarna lämnas in; skriv sidnummer, anonymiseringskod och datum på alla inlämnade papper.

Om något är oklart – fråga gärna tentamensvärden. Lycka till!

Rules in the examination hall

- Follow the invigilator's instructions.
- Bags and outerwear must be placed at the designated place.
- Place your ID document clearly visible on the table in front of you.
- No student may leave the examination hall for the first 30 minutes.
- Only one student at a time may visit the toilet. Before visiting the toilet, write your name and time on the intended list. After the toilet visit, enter the time on the list again.
- Electronic equipment such as a mobile phone or Smartwatch must be switched off and placed at the designated place.
- During the exam, silence applies – you are not allowed to talk, or otherwise communicate, with other students during the exam.
- Before submitting the examination documents; remember to write the page number, anonymization code, and date on all papers.

Please do not hesitate to ask the invigilator if anything is unclear. Good luck!



Datum: 2022-01-13 (Date YYYY-MM-DD)	Kurs/Kurskod: ST2236 (Course/Course code)	Sidnr.: (Page no.)
Anonymiseringskod (Anonymization code)	3 1 1 - 0 0 1 5 - J P 0	1

- 1. (b) an ARMA (3,2) 2
- 2. (b) test if one time series has a causal (ceteris paribus) effect on another -
- 3. (c) independent of t -
- 4. (d) implies that adding more lagged values of either x or y will not improve the forecasting performance 2
- 5. (c) Random effects should be used when the unobserved variable is uncorrelated with the observed variables. 2

tot: 12/20

Uppg.nr.: (Task no.)	1
Lärarens kommentar: (Teacher's note)	
Poäng: (Points)	

Uppg.nr.:
(Task no.)

Lärares
kommentar:
(Teacher's
note)

Poäng:
(Points)



a) $y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 x_{t-2} + u_t$ is a finite distributed lag model (FDL) 3

x_t measured in thousands

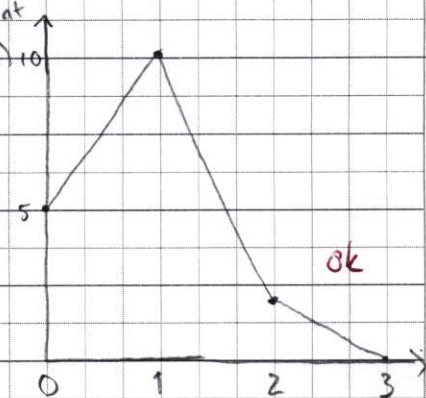
coefficient (β_i) 10

b) 1st year: $\hat{y}_1 = 10 + 5 \cdot 1000 + 10 \cdot 0 + 2 \cdot 0 = 5010$

$\hat{y}_2 = 10 + 5 \cdot 0 + 10 \cdot 1000 + 2 \cdot 0 = 10010$

$\hat{y}_3 = 10 + 5 \cdot 0 + 10 \cdot 0 + 2 \cdot 1000 = 2010$

$\hat{y}_4 = 10 + 5 \cdot 0 + 10 \cdot 0 + 2 \cdot 0 = 10$



After 3 lags the shock will disappear and have no further effect

c) $y_t = \beta_0 + \beta_1(c+1) + \beta_2 c + \beta_3 c$

$y_{t+1} = \beta_0 + \beta_1(c+1) + \beta_2(c+1) + \beta_3 c$

$y_{t+2} = \beta_0 + \beta_1(c+1) + \beta_2(c+1) + \beta_3(c+1)$ giving LRP = $\beta_1 + \beta_2 + \beta_3 = 5 + 10 + 2 = 17$ ok

So over time the amount of games would have increased with $17 \cdot 1000 = 17000$ games

x_t measured in thousands

d) LRP = $\beta_1 + \beta_2 + \beta_3$ hur testa!

e) TS 1: linear in all parameters

TS 2: No perfect collinearity

TS 3: Zero conditional mean

TS 4: Homoskedasticity

TS 5: No serial correlation

TS 6: Normality most important to be able to use t -statistics

↑
other assumptions are at least as important.

tot: 14/20

Uppg.nr.:
(Task no.)

2

Lärarens kommentar:
(Teacher's note)

Poäng:
(Points)

Uppg.nr.:
(Task no.)

Lärarens
kommentar:
(Teacher's
note)

Poäng:
(Points)



a) Weak stationarity occurs when $E(X_t^2) < \infty$ and is covariance stationary if $E(X_t)$ is constant, $Var(X_t)$ is constant, for any $t, h > 1$, $cov(X_t, X_{t+h})$ depends only on h and not on t

Uppg.nr.:
(Task no.)

3

Autocorrelation is when there is correlation between the errors in different time periods. *Not necessarily errors just observations*

Lärarens kommentar:
(Teacher's note)

White noise occurs when a time series consists of uncorrelated observations and has constant variance. *ok*

4

b) 1. $y_t = \alpha + y_{t-1} + \epsilon_t$ or $(1-B)y_t = \epsilon_t + \alpha$ *ok*

2. $y_t = \sigma + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \theta_1 \epsilon_{t-1} + \epsilon_t$

or $(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)y_t = \sigma + \epsilon_t(1 + \theta_1 B)$ *ok*

3. $y_t = \sigma + \phi_1 y_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1}$ or $\phi(B)(1 - \phi_1 B)y_t = \sigma + \epsilon_t(1 + \theta_1 B)$
not the same

4

c) 1. Is a ARIMA(0,0,1) also MA(1) which is always stationary and is invertible since $|\theta_1| < 1$ $\theta_1 = 0.8$ *ok*

2. Is a ARIMA(1,1,0) and is not stationary hence $I(1)$ although since it is a AR(1) when differenced it is invertible. *due to $|\phi_1| < 1$ is not correct $\phi_1 = 1$ stationary even if not differenced*

3. It is not stationary, it is a AR(2) when differenced and then invertible. However I'm unsure how many times it need to be differenced but 1 seems good if so it's a ARIMA(2,1,0)

4

even if not differenced $\phi_1 + \phi_2 < 1$ not accurate $0.6 + 0.5 = 1.1 > 1$ not stationary

Uppg.nr.:
(Task no.)

Lärarens
kommentar:
(Teacher's
note)

Poäng:
(Points)



1. $\phi_1 = 1.2$ $\phi_2 = -0.4$ For stationarity: $\phi_1 + \phi_2 < 1 = 0.8 < 1$
 $\phi_2 - \phi_1 < 1 = -1.6 < 1$
 $|\phi_2| < 1 = 0.4 < 1$

Uppg.nr.:
(Task no.)

4

y_t is stationary 5

Lärens kommentar:
(Teacher's note)

2. $E[y_t] = \mu = \frac{\sigma}{1 - \phi_1 - \phi_2} = \frac{4}{1 - 1.2 + 0.4} = \frac{4}{0.2} = 20$ 5

for stationary processes

3. $\rho_Y(1) = \frac{\text{Cov}(y_t, y_{t-1})}{\text{Var}(y_t)}$ OK $\text{Var}(y_t)$ is calculated by Yule Walker equations

$$\text{Var}(y_t) = \frac{(1 - \phi_2) \sigma_\varepsilon^2}{(1 + \phi_2)(1 - \phi_1 - \phi_2)(1 + \phi_1 - \phi_2)}$$

$$\text{Cov}(y_t, y_{t-1}) = \text{Cov}(\phi_1 y_{t-1} - \phi_2 y_{t-2} + \phi_1 y_{t-2} - \phi_2 y_{t-3})$$

$$\text{Cov}(\phi_2 y_{t-2}, \phi_1 y_{t-2}) = -\phi_1 \phi_2 \text{Cov}(y_{t-2}, y_{t-2})$$

$$\text{Var}(y_t) = \frac{2.8}{0.312} \approx 8.974...$$

$$\text{Cov}(y_t, y_{t-2}) = \text{Cov}(y_t, y_{t-2}) - \text{Cov}(y_t, y_{t-3}) - \text{Cov}(y_{t-1}, y_{t-2}) + \text{Cov}(y_{t-1}, y_{t-3})$$

every pair have different indices therefore $\text{Cov}(y_t, y_{t-2}) = 0$
this is not correct

$\text{Cov}(y_{t-2}, y_{t-2}) = \text{Var}(y_{t-2})$ stationaritet ger $\rightarrow \text{Var}(y_{t-2}) = \text{Var}(y_t)$

1

no $\rho_Y(1) = -(\phi_1 \cdot \phi_2) = 1.2 \cdot 0.4 = 0.48$

$\rho_Y(2) = \frac{\text{Cov}(y_t, y_{t-2})}{\text{Var}(y_t)} = 0$

Answer: $\rho_Y(1) = 0.48$ $\rho_Y(2) = 0$

4. $\hat{y}_{100} = 4 + 1.2 y_{99} - 0.4 y_{98} = 15.2$

$\hat{y}_{101} = 4 + 1.2 \hat{y}_{100} - 0.4 y_{99} = 15.44$ 5

$\hat{y}_{102} = 4 + 1.2 \hat{y}_{101} - 0.4 \hat{y}_{100} = 16.448$

Uppg.nr.:
(Task no.)

Lärarens
kommentar:
(Teacher's
note)

Poäng:
(Points)



$\lambda = 0,2$ $Y_0 = 3$ simple exponential smoothing: $\tilde{y}_t = \lambda y_t + (1-\lambda)\tilde{y}_{t-1}$

double exponential smoothing: $\tilde{y}_t^{(2)} = \lambda \tilde{y}_t^{(1)} + (1-\lambda)\tilde{y}_{t-1}^{(2)}$

Y_t	y_t	$\tilde{y}_t^{(1)}$	$\tilde{y}_t^{(2)}$	\tilde{y}_t
2003	3	3	3	3
2004	16	5,6	3,52	7,68
2005	7	5,88	3,992	7,768
2006	15	7,704	4,7344	10,6736
2007	17	9,5632	5,70016	13,42624
2008	25	12,65056	7,09024	18,21088

$\tilde{y}_3 = 0,2 \cdot 3 + 0,8 \cdot 3 = 3$
 $\tilde{y}_4 = 0,2 \cdot 16 + 0,8 \cdot 3 = 5,6$
 \vdots
 $\tilde{y}_3^{(2)} = 0,2 \cdot 3 + 0,8 \cdot 3 = 3$
 $\tilde{y}_4^{(2)} = 0,2 \cdot 5,6 + 0,8 \cdot 3 = 3,52$

good! 6

Uppg.nr.:
(Task no.)

5

Lärarens kommentar:
(Teacher's note)

Need to clear out the Bias by $\tilde{y}_t = 2\tilde{y}_t^{(1)} - \tilde{y}_t^{(2)}$ double exponential smoothing = \tilde{y}_t

(des)

b) Of the two methods I think that double exponential smoothing is more suitable since there is a clear trend and des takes the trend more into account when smoothing an extra time, and taking away bias

Simple exponential smoothing is more suitable when there is no trend

4

(SES)

c) Forecast for simple exponential smoothing is constant $Y_T = \mu + E_T$ last smoothed value

	SES	DES	Y_t	
2009	12,65	19,4998	16	DES: $\hat{y}_{T+\tau}^{(2)} = \left(2 + \frac{\tau}{1-\lambda}\right) \tilde{y}_T^{(1)} - \left(1 + \frac{\tau}{1-\lambda}\right) \tilde{y}_T^{(2)}$
2010	12,65	20,99104	14	
2011	12,65	22,38112	28	
2012	12,65	23,7712	30	

mindre närläggat.
small calculation error

Ex: $\hat{y}_{2009}^{(1)} = 2,25 \cdot \tilde{y}_{2008}^{(1)} - 1,25 \cdot \tilde{y}_{2008}^{(2)}$
 $\hat{y}_{2010}^{(2)} = 2,5 \cdot \tilde{y}_{2008}^{(1)} - 1,5 \cdot \tilde{y}_{2008}^{(2)}$

5

d) $ME = \frac{1}{n} \sum_{t=1}^n (y_t - \tilde{y}_t)$

SES: $ME = \frac{1}{4} \sum_{t=1}^4 (y_t - \tilde{y}_t) = \frac{37,39776}{4} = 9,34944$ OK

DES: $ME = \frac{1}{4} \sum_{t=1}^4 (y_t - \hat{y}_t) = \frac{1,35684}{4} = 0,33921$ OK

This was expected since in DES forecast we overestimate the first two years which gave us negative errors that's why ME is so close to zero and why SES is much larger since we always underestimated every year.

5

Poäng:
(Points)

Uppg.nr.:
(Task no.)

Lärarens
kommentar:
(Teacher's
note)

Poäng:
(Points)



$$e) MAE = \frac{1}{n} \sum_{t=1}^n |e_t|$$

$$RMSE = \sqrt{\frac{1}{T} \sum_{i=1}^T (y_{T+i} - \hat{y}_{T+i}(T))^2}$$

Uppg.nr.:
(Task no.)

5

	MAE	RMSE
SES	9,34944 P	13,1191... -
DES	5,58463	5,7335..

✓

4

Small errors
due to (E)

Answer: The double exponential smoother is more accurate since the MAE and RMSE are both smaller than for simple exponential smoother

Lärarens kommentar:
(Teacher's note)

Poäng:
(Points)

24/25

Uppg.nr.:
(Task no.)

Lärarens
kommentar:
(Teacher's
note)

Poäng:
(Points)