## STOCKHOLM UNIVERSITY

Department of Statistics Econometrics II, Time Series Analysis, ST223G Autumn Semester 2020

## Written Examination in Time Series Analysis

Date 2021-01-15 Hour: 9.00-14.00

Examiner: Jörgen Säve-Söderbergh

Allowed tools: 1) Textbook: Wooldridge, J.M. *Introductory* 

Econometrics: A Modern Approach,

Cengage, Boston.

2) Textbook: Montgomery, D.C., Jennings, C.L., and Kulachi, M.,

*Introduction to Time Series Analysis and Forecasting,* 

John Wiley & Sons, New Jersey.

3) Pocket calculator

4) Notes written in the text book are allowed.

- On problem 6 and 7 it is sufficient to just state which alternative you believe is true. Nothing further than that is required.
- Note that no formula sheet is provided.
- Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description.
- The maximum number of points for each problem is stated immediately after the question number. If not indicated otherwise, to obtain the maximum number of points on each problem, detailed and clear solutions are required. Answers may be given in English or Swedish.

For questions about the content of the exam, contact the course coordinator on jorgen.save-soderbergh@stat.su.se. Incoming e-mail questions are answered between 10.00 and 11.00 during the exam.

If the course coordinator needs to send out information to all students during the exam, this is done to your registered email address. Therefore, check your email during the exam.

Please note that practical help is only available during the first hour of the exam by email expedition@stat.su.se. Carefully read the enclosed instructions for exam submission. There you find all the necessary information about submission, anonymous code, etc. If you, despite the instructions have problems submitting the exam, email the exam to tenta@stat.su.se. However, this is only done in exceptional cases.

## Good luck!

- 1. (12 points) A climate researcher A has observed a time series over a certain number of weeks. The autocorrelation between observations one week apart was found to be 0.454545. You are invited as a time series expert and are posed a number of questions. However, the information from A is not presented in what we would regard as the correct order. A tells you that *correlation is important*, but I wish to test if it is zero or not. I remember that hypothesis testing is always regarded as a foundation nowadays.
  - (a) (3 points) Help A to formulate the relevant null hypothesis and alternative hypothesis from the information that A has provided.
  - (b) (6 points) The test statistic  $Z_0$  was found to be 5.00. A competing climate researcher B wondered how many observations that A had used in calculating  $Z_0$ . How many observations had been used?
  - (c) (3 points) Finally, perform the hypothesis test with significance level 5%.
- 2. (12 points) A government official was studying the effects of the pandemic on a certain sector in the Swedish economy. In order to evaluate a certain method for forecasting the following table presents predicted monthly sales and actual monthly sales for a company over the last six months of 2020 in this sector.

	Actual Sales	Predicted Sales
July	966	961
August	970	975
September	980	974
October	944	949
November	950	945
December	978	975

- (a) (2 points) Calculate the forecast error for each month.
- (b) (3 points) Calculate the MAD (mean absolute deviation).
- (c) (3 points) Calculate MSE (mean squared error).
- (d) (4 points) Calculate MAPE (mean absolute precent forecast error).
- 3. (12 points) Assume the model

$$y_t = \varepsilon_t - 1.1\varepsilon_{t-1} - 0.9\varepsilon_{t-2}$$

where  $E(\varepsilon_t) = 0$  and  $Var(\varepsilon_t) = \sigma^2$  and  $\varepsilon_t$  are independent random variables.

(a) (2 points) What model is this?

- (b) (2 points) What are the parameter values?
- (c) (6 points) Is the model stationary? Is it invertible?
- (d) (2 points) Rewrite the model using the backshift operator *B*.
- 4. (12 points) A climate researcher has found interest in the following model

$$y_t = 0.5y_{t-1} + \varepsilon_t - 1.3\varepsilon_{t-1} + 0.4\varepsilon_{t-2}$$

- (a) (2 points) What model is this? What are the parameter values?
- (b) (4 points) Analyze if the model is stationary by solving the associated polynomial equation  $m^p \phi_1 m^{p-1} \phi_2 m^{p-2} \cdots \phi_p = 0$  and interpret the roots that you recieve.
- (c) (6 points) Analyze if the model is invertible by solving the associated polynomial equation  $m^q \theta_1 m^{q-1} \theta_2 m^{q-2} \cdots \theta_p = 0$  and interpret the roots that you recieve.
- 5. (12 points) Rewrite the following ARIMA(0,1,0) model

$$(1-B) y_t = \varepsilon_t$$

in difference-equation form (that is, in a formula without the backshift operator that contains  $y_t$  and lagged values of  $y_t$  among other things).

- 6. (10 points) Adding a time trend can make an explanatory variable more significant if:
  - A. the dependent and independent variables have similar kinds of trends, but movement in the independent variable about its trend line causes movement in the dependent variable away from its trend line.
  - B. the dependent and independent variables have similar kinds of trends and movement in the independent variable about its trend line causes movement in the dependent variable towards its trend line.
  - C. the dependent and independent variables have different kinds of trends and movement in the independent variable about its trend line causes movement in the dependent variable towards its trend line.
  - D. the dependent and independent variables have different kinds of trends, but movement in the independent variable about its trend line causes movement in the dependent variable away from its trend line.

- 7. (10 points) Which of the following statements is true?
  - A. The calculated *t* statistic is valid and efficient in case of a spurious regression.
  - B. If an explanatory variable or a dependent variable is integrated of the order one, the OLS estimators are asymptotically normally distributed.
  - C. An error correction model can be used to study the short-run dynamics in the relationship between the dependent variable and the explanatory variables in a time series model.
  - D. The Dickey-Fuller test can be used to test for heteroskedasticity in the error terms.
- 8. (20 points) Assume that  $\{\varepsilon_t: t=1,2,\ldots\}$  is a sequence of independent random variables with  $E(\varepsilon_t)=0$  and

$$\gamma_{\varepsilon}(h) = \operatorname{Cov}(\varepsilon_{t}, \varepsilon_{t+h}) = \begin{cases} \sigma^{2} & h = 0\\ 0 & h \neq 0 \end{cases}$$

Define for any *t* 

$$y_t = \mu + \psi_0 \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \psi_3 \varepsilon_{t-3} + \psi_4 \varepsilon_{t-4} + \cdots$$

- (a) (10 points) Compute  $E(\varepsilon_{t-1}y_t)$ .
- (b) (10 points) Compute  $E(\varepsilon_t y_{t-1})$ .

*Hint* Use the expression for  $y_t$  and think of the infinite sum as a polynomial. Multiplying  $\varepsilon_{t-1}$  on the series works actually like a polynomial, so we get a new series. Now, calculate the expected value of each term.