sidnr. 1 Tenta II ST223G kod: 0005 - DUD

1\. a) $H_0: \rho_k = 0$ vs: $H_1: \rho_k \neq 0$

b) Test statistic $Z_0 = r_k \sqrt{T} \sim N(0,1)$

$Z_0 = 5 \quad k = 1$

$r_1 = \hat{\rho}_1 = 0{,}454545$

$5 = 0{,}454545 \sqrt{T}$

$\sqrt{T} = \dfrac{5}{0{,}454545}$

$T = \left(\dfrac{5}{0{,}454545}\right)^2 = 121{,}000242$

A used 122 observations.

c) $\alpha = 0{,}05 \quad T = 122$

$Z_0 = 5$ Rejection Rule: Reject H_0 if $|Z_0| \geq Z_{\alpha/2}$

$Z_{\alpha/2} = Z_{0{,}025} = 1{,}96$

$|5| > 1{,}96$ We can reject H_0 on 5% significance level.

Sidnr. 2 Kod: 000S-DUD

2. a)

	Actual Sales	Predicted Sales	$e_t(1)$	$(e_t(1)/y_t)\cdot 100$
July	966	961	5	0,517598
August	970	975	-5	-0,515464
September	980	974	6	0,612245
October	944	949	-5	-0,529661
November	950	945	5	0,526316
December	978	975	3	0,306748

The forecast error in lead time $-\tau$ is given by

$e_t(\tau) = y_t - \hat{y}(t-\tau)$

We are interested in $\tau = 1$, so the foreast error is

$e_t(1) = y_t - \hat{y}(t-1)$. The forecast errors are displayed in the last column.

b) $MAD = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = \frac{1}{6}(|5| + |-5| + |6| + |-5| + |5| + |3|) = \frac{1}{6}(29)$

$= \frac{29}{6} \approx 4,83$

c) $MSE = \frac{1}{n}\sum_{t=1}^{n} [e_t(1)]^2 = \frac{1}{6}(5^2 + (-5)^2 + 6^2 + (-5)^2 + 5^2 + 3^2) = \frac{1}{6}(145)$

$= \frac{145}{6} \approx 24,17$

d) $MAPE = \frac{1}{n}\sum_{t=1}^{n} \left|\frac{e_t(1)}{y_t}\cdot 100\right| = \frac{1}{6}\left(\left|\frac{5}{966}\cdot 100\right| + \left|\frac{-5}{970}\cdot 100\right| + \right.$

$\left.\left|\frac{6}{980}\cdot 100\right| + \left|\frac{-5}{944}\cdot 100\right| + \left|\frac{5}{950}\cdot 100\right| + \left|\frac{3}{978}\cdot 100\right|\right)$

$= \frac{1}{6}(3,008032432) \approx 0,501$

3. a) MA(2) model

$$y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

b) $y_t = \varepsilon_t - 1,1\, \varepsilon_{t-1} - 0,9\, \varepsilon_{t-2}$

This gives us:

$\mu = 0 \quad \theta_1 = 1,1 \quad \theta_2 = 0,9$

c) An MA(q) is always stationary. It is invertible if it satisfy the following conditions:

1. $|\theta_2| < 1$
2. $|\theta_1| + \theta_2 < 1$

We have:

1. $|0,9| < 1$
2. $|1,1| + 0,9 = 2 > 1$

The model is stationary but not invertible

d) $y_t = \varepsilon_t - 1,1\, \varepsilon_{t-1} - 0,9\, \varepsilon_{t-2} = (1 - 1,1 B - 0,9 B^2)\, \varepsilon_t$

4. a) ARMA(1, 2) model

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q}$$

an ARMA(p, q) model on a general form

We have

$$y_t = 0{,}5\, y_{t-1} + \varepsilon_t - 1{,}3\, \varepsilon_{t-1} + 0{,}4\, \varepsilon_{t-2}$$

This gives the parameter values

$\delta = 0$ $\quad \phi_1 = 0{,}5 \quad \theta_1 = 1{,}3 \quad \theta_2 = -0{,}4$

b) $m^p - \phi_1 m^{p-1} - \phi_2 m^{p-2} - \ldots - \phi_p = 0$

We have $p = 1$ and $\phi_1 = 0{,}5$

$m^1 - 0{,}5\, m^{1-1} = 0$

$m^1 - 0{,}5 = 0$

$m^1 = 0{,}5$

$|0{,}5| = 0{,}5 < 1 \implies$ the model is stationary

c) $m^q - \theta_1 m^{q-1} - \theta_2 m^{q-2} - \ldots - \theta_q = 0$

We have $q = 2 \quad \theta_1 = 1{,}3 \quad \theta_2 = -0{,}4$

$m^2 - 1{,}3\, m^{2-1} + 0{,}4\, m^{2-2} = 0$

$m^2 - 1{,}3\, m + 0{,}4 = 0$

$m = \frac{1{,}3}{2} \pm \sqrt{\left(\frac{1{,}3}{2}\right)^2 - 0{,}4}$

$m = 0{,}65 \pm \sqrt{0{,}0225}$

$m_1 = 0{,}8$

$m_2 = 0{,}53$

$$\left.\begin{array}{l} |0{,}8| = 0{,}8 < 1 \\ |0{,}53| = 0{,}53 < 1 \end{array}\right\} \text{the model is invertible}$$

5. $(1-B)y_t = \varepsilon_t \iff$

$y_t - By_t = \varepsilon_t \iff$

$y_t - y_{t-1} = \varepsilon_t$

6 D

7. C

8 $Cov(\varepsilon_t, \varepsilon_{t+h}) = E(\varepsilon_t \varepsilon_{t+h}) - \underbrace{E(\varepsilon_t)E(\varepsilon_{t+h})}_{=0}$ since $E(\varepsilon_t) = 0$

When $h = 0$

$E(\varepsilon_t \varepsilon_t) = \sigma^2$ same index

When $h \neq 0$

$E(\varepsilon_t \varepsilon_{t+h}) = 0$ different index

a) $E(\varepsilon_{t-1} y_t) = E[\varepsilon_{t-1}(\mu + \psi_0 \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \dots)]$

$= E[\mu \varepsilon_{t-1} + \psi_0 \varepsilon_t \varepsilon_{t-1} + \psi_1 \varepsilon_{t-1} \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} \varepsilon_{t-1} + \dots]$

$= \mu E(\varepsilon_{t-1}) + \psi_0 E(\varepsilon_t \varepsilon_{t-1}) + \psi_1 E(\varepsilon_{t-1} \varepsilon_{t-1}) + \psi_2 E(\varepsilon_{t-2} \varepsilon_{t-1}) + \dots$

$= \psi_1 E(\varepsilon_{t-1} \varepsilon_{t-1}) = \psi_1 \sigma^2$

All ε with different index equals to zero, we only have one term where the index is equal

b) $E(\varepsilon_t y_{t-1}) = E[\varepsilon_t(\mu + \psi_0 \varepsilon_{t-1} + \psi_1 \varepsilon_{t-2} + \psi_2 \varepsilon_{t-3} + \dots)]$

$= E[\mu \varepsilon_t + \psi_0 \varepsilon_{t-1} \varepsilon_t + \psi_1 \varepsilon_{t-2} \varepsilon_t + \psi_2 \varepsilon_{t-3} \varepsilon_t + \dots]$

$= \mu E(\varepsilon_t) + \psi_0 E(\varepsilon_{t-1} \varepsilon_t) + \psi_1 E(\varepsilon_{t-2} \varepsilon_t) + \psi_2 E(\varepsilon_{t-3} \varepsilon_t) + \dots$

$= 0$ since none of the ε has the same index and $E(\varepsilon_t) = 0$ it all will sum to zero.