Econometrics I 2020-06-08 0004-MXU

1. a) In order to test $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ vs $H_1: \neg H_0$

I will first calculate R^2.

$n = 18$

$k = 3$

$$R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{5078,71318}{85260,44444} = 0,9404329497$$

$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)} \sim F_{k,n-k-1} \text{ (if } H_0 \text{ is correct)}$$

$$F_{obs} = \frac{R^2/k}{(1-R^2)/(n-k-1)} = \frac{0,9404329497/3}{(1-0,9404329497)/(18-3-1)} = 73,67642139$$

We reject H_0 if $F_{obs} = 73,67642139 > F_{3,14} \Rightarrow c_{0,05} = 3,34$

Since $F_{obs} \approx 73 > F_{3,14} \Rightarrow c_{0,05} = 3,34$ we reject H_0 at 5% significance.

b) The test statistic will be:

$$F = \frac{(R^2_{ur} - R^2_r)/q}{(1-R^2_{ur})/(n-k-1)} \sim F_{q,n-k-1} \text{ (If } H_0 \text{ is correct)}$$

We have $R^2_r = 0,9404329497$ and SST is the same since no new variables were added.

$$R^2_{ur} = \frac{SSE_{ur}}{SST} = \frac{82707,77658195}{85260,44444} = 0,9700603501$$

$n = 18$

$k = 5$

$q = 2$

so $H_0: \beta_4 = \beta_5 = 0$ vs $H_1: \neg H_0$

$$F_{obs} = \frac{(0,9700603501 - 0,9404329497)/2}{(1-0,9700603501)/(18-5-2)} = 5,442638864$$

H_0 is rejected if $F_{obs} \approx 5,44 > F_{2,11} \Rightarrow c_{0,05} = 3,98$

Since $F_{obs} > F_{2,11}$ at 5% significance level H_0 is rejected.

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2. a) The dependent variable is total number of children ever born to a woman.
The interpretation of the coefficient on y82 is that the expected number of children ever born to a woman drops by 19,26076 % if she gave birth to a child in the year 1982.

b) The fertility of black women is estimated to be 36,03475 % higher then that of non black women holding other factors fixed.

c) Since the dispersion parameter $\sigma^2 = 1$ the standard deviation $\sigma = 1$.
This indicates that the Poisson variance assumption is fulfilled and does not point to under- or overdispersion.

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3. $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u$

In order to test $H_0: \beta_1 + \beta_2 = 1$ we first write it as $H_0: \beta_1 + \beta_2 - 1 = 0$ We then define $\theta_1 = \beta_1 + \beta_2 - 1$ and rewrite it as $\beta_1 = \theta - \beta_2 + 1$

Now we substitute this into the model which gives

$Y = \beta_0 + (\theta_1 - \beta_2 + 1)X_1 + \beta_2 X_2 + u \Rightarrow$

$\Rightarrow Y = \beta_0 + \theta_1 X_1 - \beta_2 X_1 + X_1 + \beta_2 X_2 + u \Rightarrow$

$\Rightarrow Y = \beta_0 + \theta_1 X_1 + \beta_2 (X_2 - X_1) + X_1 + u$

Since we have a X_1 without a parameter we subtract this on both sides which gives

$Y - X_1 = \beta_0 + \theta_1 X_1 + \beta_2 (X_2 - X_1) + u$

This model can be estimated and $H_0: \theta_1 = 0$ which is now equivalent to $H_0: \beta_1 + \beta_2 = 1$ can be tested using a t-test.

0004-MXD

4. since $\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})y_i}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$ (1)

and defining $\sum_{i=1}^{n}(x_i - \bar{x})^2$ as SST_x

we can rewrite (1) as:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(\beta_0 + \beta_1 x_i + u_i)}{SST_x} \Rightarrow$$

$$\Rightarrow \hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})\beta_0 + \sum_{i=1}^{n}(x_i - \bar{x})\beta_1 x_i + \sum_{i=1}^{n}(x_i - \bar{x})u_i}{SST_x}$$

since $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$ and $\sum_{i=1}^{n}(x_i - \bar{x})\beta_1 x_i = \beta_1 \cdot SST_x$

we now have $\hat{\beta}_1 = \beta_1 + \frac{\sum_{i=1}^{n}(x_i - \bar{x})u_i}{SST_x}$

So $E(\hat{\beta}_1) = \beta_1 + \frac{\sum_{i=1}^{n}[E(x_i - \bar{x})u_i]}{SST_x}$ since $E(\beta_1) = \beta_1$ and $E(SST_x) = SST_x$

Since $E(u_i|x_i) = 0$ we can write this as

$$E(\hat{\beta}_1) = \beta_1 + \frac{\sum_{i=1}^{n}(x_i - \bar{x})E(u_i)}{SST_x}$$

and since $E(u_i) = 0$ we are left with $E(\hat{\beta}_1) = \beta_1$