(1)

a) $\ln(\text{Yield}) = \beta_0 + \beta_1 \text{ P-E ratio} + \beta_2 \text{ 1990 rank} + \varepsilon$

$H_0: \beta_1 = \beta_2 = 0$ corresponds to testing the overall significance of the regression. The restricted model in this case is just
$\ln(\text{Yield}) = \beta_0 + \varepsilon$, i.e. knowing "P-E ratio" and "1990 rank" does not explain $\ln(\text{Yield})$.

From Wooldridge p. 147 we have that:

$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)} \sim F_{k,n-k-1} \text{ under } H_0.$$

$$R^2 = \frac{SSE}{SST} = 0.5953$$

$$k = 2$$

$$n = 20$$

$\Rightarrow$

$$F = \frac{0.5953/2}{(1-0.5953)/(20-2-1)} = 12.5$$

From Wooldridge p. 788 we have that

$$F_{\alpha=5\%, 2, 17} = 3.59$$

<u>Conclusion</u>: We can reject H_0 in favor of H_1, and conclude that P-E ratio and 1990 rank are jointly significant.

forts. 1

b) To test this hypothesis in the "traditional way" we would compare the ~~SSR~~ SSR (or R^2) of the restricted and unrestricted model and do an F-test:

Unrestricted: $\ln(\text{Yield}) = \beta_0 + \beta_1 \text{ P-E ratio} + \beta_2 \text{ 1990 rank} + \beta_3 \text{ 1989 rank} + \varepsilon$

Restricted $(\beta_2 = \beta_3 = 0)$: $\ln(\text{Yield}) = \beta_0 + \beta_1 \text{ P-E ratio} + \varepsilon$

Using $F = \dfrac{(R^2_{UR} - R^2_R)/k}{(1-R^2_{UR})/(n-k-1)} \sim F_{k,n-k-1}$

But we don't have R^2_R (~~...~~ nor SSE_R nor SSR_R), or at least I don't see how they can be calculated from the given information. We have:

TSS ; ~~...~~ $ESS_{PE,1990}$ and $ESS_{PE,1990,1989}$

So we can calculate "marginal improvements" in fit for:

- going from no expl. var. to joint test of PE & 1990 (a)
- going from no expl. var. to joint tst of PE, 1990 & 1989
- going from PE & 1990 to PE, 1990 and 1989, thus testing whether β_3 is 0.

⇒ Can't isolate effect of P-E ratio (?).

See next page...

... b) So let's try some other logic...

- From a) we know that β_1 and β_2 are jointly significant. In other words $\beta_1 \neq 0$ and/or $\beta_2 \neq 0$ ~~[illegible]~~ ~~[illegible]~~ (at least.)

- We can test the overall significance of the b) regression. using same method as in a).

$$R_b^2 = \frac{ESS_b}{TSS} = \frac{29.97726}{48.0803} = 0.6235$$

still the same

$$F = \frac{R_b^2/k}{(1-R_b^2)/(20-3-1)} = 8.83 > 3.24 = F^{crit}_{\alpha=5\%,3,16}$$

So now we also know that

$\beta_1 \neq 0$ and/or $\beta_2 \neq 0$ and/or $\beta_3 \neq 0$.

- We can test whether $\beta_3 = 0$ (single restriction) using

$$F = \frac{(R_b^2 - R_a^2)/q}{(1-R_b^2)/(n-k-1)} \sim F_{q,n-k-1} \text{ under } H_0.$$

$$= \frac{(0.6235-0.5953)/1}{(1-0.6235)/(20-3-1)} = 1.198 < 4.49 = F^{crit}_{\alpha=5\%,1,16}$$

squared ~~[illegible]~~ t-value of same t-test of $\beta_3 = 0$.

So, we can't reject $H_0: \beta_3 = 0$.

Conclusion: β_3 not significant.
$\beta_1, \beta_2, \beta_3$ jointly significant
β_1, β_2 jointly sign.
$\Rightarrow$ not very conclusive on $\beta_2 = \beta_3 = 0$... //

(2) a) The described procedure is the Breusch-Pagan test for heteroskedasticity. The regression the financial analyst has run is the "auxiliary regression":

$$\hat{u}^2 = \delta_0 + \delta_1 \text{PE ratio} + \delta_2 \text{1990 rank} + \delta_3 \text{1989 rank} + \underbrace{v}_{\text{error term}}$$

$H_0: \delta_1 = \delta_2 = \delta_3 = 0$

$H_1: \neg H_0$, i.e. at least one of $\delta_1, \delta_2, \delta_3 \neq 0$

Intuition: If the error term u in the regression model from 1b is homoskedastic, that means that $E(u^2 \mid \text{P-E ratio}, \text{1990 rank}, \text{1989 rank}) =$
$= E(u^2) = \sigma^2$, i.e. constant.

We estimate u^2 with $\hat{u}^2$ and then see if this can be explained by the regressors. If it can, then at least one of $\delta_1, \delta_2, \delta_3$ will be ~~[illegible]~~ different from 0, and we can then reject the null of homoskedasticity.

forts (2)

b) $$F = \frac{R^2_{\hat{u}^2}/k}{(1-R^2_{\hat{u}^2})/(n-k-1)} \sim F_{k,n-k-1} \text{ under } H_0$$

$$F = \frac{0.07472829/3}{(1-0.07472829)/(20-3-1)} = 0.43$$

$$F^{crit}_{\alpha=5\%,3,16} = 3,24$$

Thus, we cannot reject the H_0 of homoskedasticity at the 5% level, using the F-test.

c) $$LM = n \cdot R^2_{\hat{u}^2} \sim \chi^2_k \text{ under } H_0.$$

$$LM = 20 \cdot 0.07472829 = 1.49$$

$$LM^{crit}_{\alpha=5\%,3} = 7.81$$

Thus, we cannot reject the H_0 of homoskedasticity at the 5% level, using the LM test.

d) We can't find evidence for heteroskedasticity, so the conclusion is that our OLS estimator for the 1b regression model should be efficient (BLUE). At least we have found no evidence of the contrary.

(3)

We use the hint and insert

$$\theta = \beta_1 + a\beta_2 \iff \beta_1 = \theta - a\beta_2$$ into
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$ which gives

$$y = \beta_0 + (\theta - a\beta_2)x_1 + \beta_2 x_2 + u$$
$$= \beta_0 + \theta x_1 - a\beta_2 x_1 + \beta_2 x_2 + u$$
$$= \beta_0 + \theta x_1 + \beta_2(x_2 - a x_1) + u$$

$$= \beta_0 + \theta x_1 + \beta_2 x_3 + u$$ where we have defined a new variable $x_3 \equiv x_2 - a x_1$.

Under H_0 $\theta = 0$ and we can test this by estimating the model

$$\hat{y} = \hat{\beta}_0 + \hat{\theta} x_1 + \hat{\beta}_2 x_3$$ and do a t-test for the $\hat{\theta}$ coefficient (when we have chosen a value for a). In practice this means we first have to calculate $x_3 = x_2 - a x_1$ from our sample data.

//

(4)

$$\overset{\text{SLR 1}}{\downarrow}$$

$$y = \beta_0 + \beta_1 x + u$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})y_i}{SST_x} \quad \text{where } SST_x \equiv \sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= \frac{1}{SST_x}\left[\sum_{i=1}^{n}(x_i - \bar{x})(\beta_0 + \beta_1 x_i + u_i)\right]$$

$$= \frac{1}{SST_x}\left[\beta_0 \sum_{i=1}^{n}(x_i - \bar{x}) + \beta_1 \sum_{i=1}^{n}(x_i - \bar{x})x_i + \sum_{i=1}^{n}(x_i - \bar{x})u_i\right]$$

It can be shown that $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$ and that $\sum_{i=1}^{n}(x_i - \bar{x})x_i = \sum_{i=1}^{n}(x_i - \bar{x})^2 = SST_x$ so :

$$\hat{\beta}_1 = \frac{1}{SST_x}\left[\beta_1 \cdot SST_x + \sum_{i=1}^{n}(x_i - \bar{x})u_i\right]$$

$$= \beta_1 + \frac{1}{SST_x}\sum_{i=1}^{n}(x_i - \bar{x})u_i \qquad (1)$$

Taking expectations over (1) we have that

$$E(\hat{\beta}_1) = E\left[\beta_1 + \frac{1}{SST_x}\sum_{i=1}^{n}(x_i - \bar{x})u_i\right]$$

$$= \beta_1 + E\left[\frac{1}{SST_x}\sum_{i=1}^{n}(x_i - \bar{x})u_i\right]$$

SLR4

$$= \beta_1 + \frac{1}{SST_x}\sum_{i=1}^{n}(x_i - \bar{x})E(u_i)$$

$$= \beta_1 + \frac{1}{SST_x}\sum_{i=1}^{n}(x_i - \bar{x}) \cdot 0 = \beta_1$$

//.