1.

Uppgift 1

0003-THZ

Test statistic:

$$F = \frac{(SSR_p - (SSR_1 + SSR_2))}{SSR_1 + SSR_2} \cdot \frac{n - 2(k+1)}{k+1}$$

We can write each groups model as: $y = \beta_{a,0} + \beta_{a,1}x_1 + \beta_{a,2}x_2 + \cdots + \beta_{a,k}x_k + u$

a shows that which group the model belongs to

$H_0: \beta_{1,0} = \beta_{2,0},\ \beta_{1,1} = \beta_{2,1}\ \beta_{1,2} = \beta_{2,2} \cdots, \beta_{1,k} = \beta_{2,k}$

$n = 40,\ k = 5,\ SSR_p = 7304.498,\ SSR_1 = 5014.321,\ SSR_2 = 1289.055.$

We get all of these from the problem.

$$F = \frac{(7304.498 - (5014.321 + 1289.055))}{5014.321 + 1289.055} \cdot \frac{(40 - 2(5+1))}{5+1}$$

$$= (7304.498 - 6303.376)/6303.376 \cdot \frac{28}{6}$$

$$\approx 0.7412$$

$k+1 = 6 \quad n - 2(k+1) = 28$. We use $\alpha = 0.05$. $F_{6,28} = 2.45$

The critical region is $\{F : F \geq 2.45\}$

$0.7412 < 2.45$, so we can not reject the H_0.

2.

0003-THZ

Uppgift 2

a)

We have binary response, so we use probit or logist models. We can drive the models from a latent (unobserved) variable model.

$y^* = x\beta + e$ (unobserved)

$y = \begin{cases} 0 & \text{if } y^* \leq 0 \\ 1 & \text{if } y^* > 0 \end{cases}$ $y = 1$ if y^* happen

$P(y=1|x) = P(y^* > 0|x) = P(e > -x\beta|x)$
$= 1 - G(-x\beta) = G(x\beta)$

From the problem, we know the x_j is a continuous explanatory variable, so $\frac{dP(y=1|x)}{dx_j} = g(x\beta)\beta_j$

b) Temp, wind and precip. Their p-values are < 0.05.

c) On average, the probability of having low so2 increases with 5.12 percentage points if temp changes by 1 unit.
On average, the probability of having low so2 increases with 13.20 percentage points if wind changes by 1 unit.
On average, the probability of having low so2 decreases with 1.38 percentage points if precip changes 1 unit.
So, the probability of having low so2 increases if temp and wind change 1 unit, decreases if precip changes by 1 unit.

3.

Uppgift 3 0003-THZ

Let $\theta_1 = \beta_1 - \beta_2$. Then $\beta_1 = \theta_1 + \beta_2$

Put this into the model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$

$$y = \beta_0 + (\theta_1 + \beta_2)x_1 + \beta_2 x_2 + u$$
$$= \beta_0 + \theta_1 x_1 + \beta_2 x_1 + \beta_2 x_2 + u$$
$$= \beta_0 + \theta_1 x_1 + \beta_2 (x_1 + x_2) + u$$

Rewrite the model: $y - x_1 = \beta_0 + \theta_1 x_1 + \beta_2 (x_1 + x_2) + u$

We can estimate the model and get test statistic $H_0: \theta = 0$ (t-test) by refining the vatiables as in the Rewrited model.

4.

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In equation (2.52) in the book 'Introductory Econometrics: A Modern Approach, Cengage', we know that $\hat{\beta}_1 = \beta_1 + (1/SST_x) \sum_{i=1}^{n} d_i u_i$, in this equation, the β_1 is a constant. Our condition is x_i, so SST_x and $d_i = x_i - \bar{x}$ are not random. Because of the SLR 2 (Random Sampling), u_i contains the unobserved variables that affect y_i, so the varians of the sum is the sum of the variances.

$Var(u_i) = \sigma^2$

$$Var(\hat{\beta}_1) = (1/SST_x)^2 Var\left(\sum_{i=1}^{n} d_i u_i\right) = (1/SST_x)^2 \left(\sum_{i=1}^{n} d_i^2 Var(u_i)\right)$$

$$= (1/SST_x)^2 \left(\sum_{i=1}^{n} d_i^2 \sigma^2\right) = \sigma^2 (1/SST_x)^2 \left(\sum_{i=1}^{n} d_i^2\right) \quad \left(SST_x = \sum_{i=1}^{n} d_i^2\right)$$

$$= \sigma^2 (1/SST_x)^2 SST_x = \sigma^2 \cdot \frac{1}{SST_x^{\cancel{2}}} \cdot \cancel{SST_x} = \sigma^2 / SST_x$$

$\left(SST_x = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2\right.$, so $\sigma^2 / SST_x = \sigma^2 / \sum_{i=1}^{n} (x_i - \bar{x})^2$