BASIC STATISTICS FOR ECONOMISTS, STE101. EXAM

Department of statistics
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Time: 08:00 — 13:00
Approved aid: Hand-held calculator with no stored text, data or formulas
Provided aid: Formula Sheet and Probability Distribution Tables, returned after the exam.

Problems 1 — 12: Multiple choice questions (max 60 points):

- A total of 12 multiple choice questions with five alternative answers per question one of which is the correct answer. Mark your answers on the attached **answer form**.
- Marking more than one alternative will result in zero points for that question.
- Each correct answer is worth 5 points.
- Written solutions should not be submitted; only your answers on the answer form will be considered in the assessment and final grading.

Problems 13 — 14: Complete written solutions (max 40 points):

- Use only the provided answer sheets when submitting your solutions and answers.
- For full marks, clear, comprehensive and well-motivated solutions are required. Unclear and unexplained solutions will result in point deductions even if the final answer is correct.
- Check your calculations and solutions before submitting. Careless mistakes will result in unnecessary point deductions.

The maximum total number of points is 60 + 40 = 100. At least 50 points are required to pass (grades A-E). The grading scale is as follows:

Points	0—39	40—49	50—59	60—69	70—79	80—89	90—100
Grade	F	Fx	E	D	C	B	A

NOTE: Fx and F are failing grades that require re-examination. Students who receive the grade Fx or F cannot supplement extra assignments for a higher grade.

Part one. Multiple choice

1. Let us consider a random experiment with sample space given by the months of the year, i.e. $S = \{jan, feb, mar, apr, may, jun, jul, aug, sep, oct, nov, dec\}$. Let $E_1 = \{jan, feb, mar\}$, $E_2 = \{apr, may, jun, jul\}$, $E_3 = \{jun, jul, aug, sep\}$ and $E_4 = \{oct, nov, dec\}$. Which of the following is **correct**:

   (a) E_1, E_2, E_3 and E_4 are disjoint;

   (b) E_1, E_2, E_3 and E_4 are a partition of S;

   (c) E_1, E_2, E_3 and E_4 are collectively exhaustive;

   (d) $E_1 \cup E_2$ is the complement of $E_3 \cup E_4$;

   (e) $E_1 \cap E_2$ is the complement of $E_3 \cap E_4$.

2. Which of the following is **correct** regarding the variance of a random variable:

   (a) it is only defined for continuous random variables, not for discrete random variables;

   (b) it is measured in the same units as the random variable itself;

   (c) it indicates the difference between the largest and the smallest outcome of the random variable;

   (d) if the expectation of the random variable is negative, the variance will be negative;

   (e) it indicates how spread are the outcomes of the random variable around its expectation.

3. Which of the following sentences is **not** correct regarding hypothesis testing:

   (a) the significance level is the probability of making a type I error;

   (b) the type II error is the probability of not rejecting the null hypothesis when it is false;

   (c) the test is constructed by assuming that the alternative hypothesis is true;

   (d) if the p-value is smaller than the significance level, the null hypothesis is rejected;

   (e) the type I error is the probability of rejecting the null hypothesis when it is true.

4. According to the latest census which took place in 2018, 41% of a country's households are of size one, 28% are of size two, 10% are of size three, 16% are of size four and the remaining 5% have a larger size. In order to test if the distribution has changed, a researcher draws a random sample of 100 households and measures their size. Using a statistical software, the researcher carries out a test of goodness–of–fit and obtains a p-value of 0.8185. Considering a significance level of 5%, which of the following is **correct** regarding the test that has been implemented:

   (a) the average size of the households has increased;

   (b) the average size of the households has decreased;

   (c) there is strong evidence for concluding that the distribution has changed;

   (d) there is no evidence for concluding that the distribution has changed;

   (e) the information provided is not enough for making a conclusion.

5. A researcher has asked the thirteen married men in a small community about the brideprice they had to pay to the bride's family when they got married. The brideprice values (in USD) are

   20000 3000 10000 20000 13000 0 31000 20000 63000 8000 3000 12000 4000

   What is the **mode** of the brideprice?

   (a) 12000;
   (b) 15500;
   (c) 15923;
   (d) 20000;
   (e) 31000.

6. A car rental company knows by experience that 10% of the customers rent a *sport utility vehicle* —suv— and that the customers' choice is independent of each other. What is the probability that out of the next ten customers, exactly one will rent a suv?

   (a) 0.00;
   (b) 0.10;
   (c) 0.40;
   (d) 0.75;
   (e) 1.00;

7. The amount of money spent on clothing by students on Stockholm University during 2024 can be modeled by a normal distribution with expected value of 1200 and variance of 40 000. The amount of money spent on course literature can be modeled by a normal distribution with expected value of 800 and variance of 18 000. The covariance between money spent on clothing and course literature is $-24\,000$. What is the probability that one student chosen at random spends more than 2000 on clothing plus course literature?

   (a) 0;
   (b) 0.0967;
   (c) 0.5;
   (d) 0.25;
   (e) 1.

8. It is known that the lifetime of the light bulbs produced by a company has an expected value of 40 000 hours and a variance of 25 000 000. A random sample of 250 bulbs has been selected. What is the (approximated) probability that the average lifetime of the bulbs in the sample is less than 39 500 hours?

   (a) 0.0569;
   (b) 0.1056;
   (c) 0.4372;
   (d) 0.4980;
   (e) 0.4999;

9. It is known that the weight in grams of the boxes produced in a packaging line follows a normal distribution with variance equal to 25. A sample of nine boxes has been selected and its weight has been measured:

497.9	493.8	483.8	500.9	506.1	498.5	495.9	487.8	509.2

A 95% confidence interval for the expected weight of the boxes in this packaging line is:

(a) $(487.3\,, 506.9)$;

(b) $(488.9\,, 505.3)$;

(c) $(491.7\,, 502.5)$;

(d) $(493.8\,, 500.4)$;

(e) $(494.4\,, 499.8)$.

10. One week before the local elections of a city, a candidate, Mrs. A, believes that more than 30% of the voters support her. In order to verify her claim, the campaign has selected a sample of 100 voters. 45 out of the 100 voters in the sample claim that they will vote for Mrs. A. With a significance level of 1%, which of the following is **correct**. (**Hint:** Use the alternative $P > 0.3$):

(a) the critical value is 2.36 and the test statistic is 3.02, therefore the null hypothesis is rejected;

(b) the critical value is 3.02 and the test statistic is 2.36, therefore the null hypothesis is rejected;

(c) the critical value is 2.36 and the test statistic is 3.02, therefore the null hypothesis is not rejected;

(d) the critical value is 3.02 and the test statistic is 2.36, therefore the null hypothesis is not rejected;

(e) the critical value is 2.33 and the test statistic is 60.61, therefore the null hypothesis is rejected.

11. The following table shows the scores in a home assignment (variable *assignment*) and the final exam (variable *exam*) of the eight students in a course in statistics:

Assignment	42	48	50	50	51	55	59	67
Exam	38	43	57	33	81	50	48	84

The teacher of the course wants to explain the score in the final exam in terms of the score in the home assignment through a linear regression of the form:

$$exam_i = \beta_0 + \beta_1\, assignment_i + \epsilon_i \qquad \text{for } i = 1, \cdots, 8$$

where $\epsilon_1, \cdots, \epsilon_8$ are a random sample from a normal distribution with expectation $\mu_\epsilon = 0$ and variance σ_ϵ^2. Fitting the desired regression yields an intercept $b_0 = -25.2$ and a slope $b_1 = 1.5$. The model variance σ_ϵ^2 is estimated as:

(a) 0;

(b) 109.3;

(c) 265.7;

(d) 313.5;

(e) 418.

12. Table 1 summarizes the scores of 170 students in an exam of statistics:

Points	$[0\,,40)$	$[40\,,50)$	$[50\,,60)$	$[60\,,70)$	$[70\,,80)$	$[80\,,90)$	$[90\,,100)$
Frequency	51	17	22	34	21	17	8

Table 1: Scores of 170 students in an exam of statistics

The teacher of the course wants to test whether the scores in Table 1 can be considered as a random sample from a (truncated) normal distribution. If it was, the probability in each class would be as given in the following table. (**Hint:** This is a goodness of fit test.)

Points	$[0\,,40)$	$[40\,,50)$	$[50\,,60)$	$[60\,,70)$	$[70\,,80)$	$[80\,,90)$	$[90\,,100)$
Probability	0.33	0.17	0.17	0.14	0.10	0.06	0.03

What is the value of the test statistic:

(a) 1.97;

(b) 12.59;

(c) 14.07;

(d) 18.51;

(e) 389.92.

Part two. Complete solution

13. Table 2 shows the number of passengers by class and survival status (Alive or Dead) after the sinking of the Titanic on April 14, 1912. We want to investigate whether class and survival status are independent or not.

		Survival status	
		Alive	Dead
Class	First class	201	123
	Second class	119	166
	Third	180	530
	Crew	212	677

Table 2: Passengers on board the Titanic by class and survival status

(a) State the hypothesis of interest. (5p.)

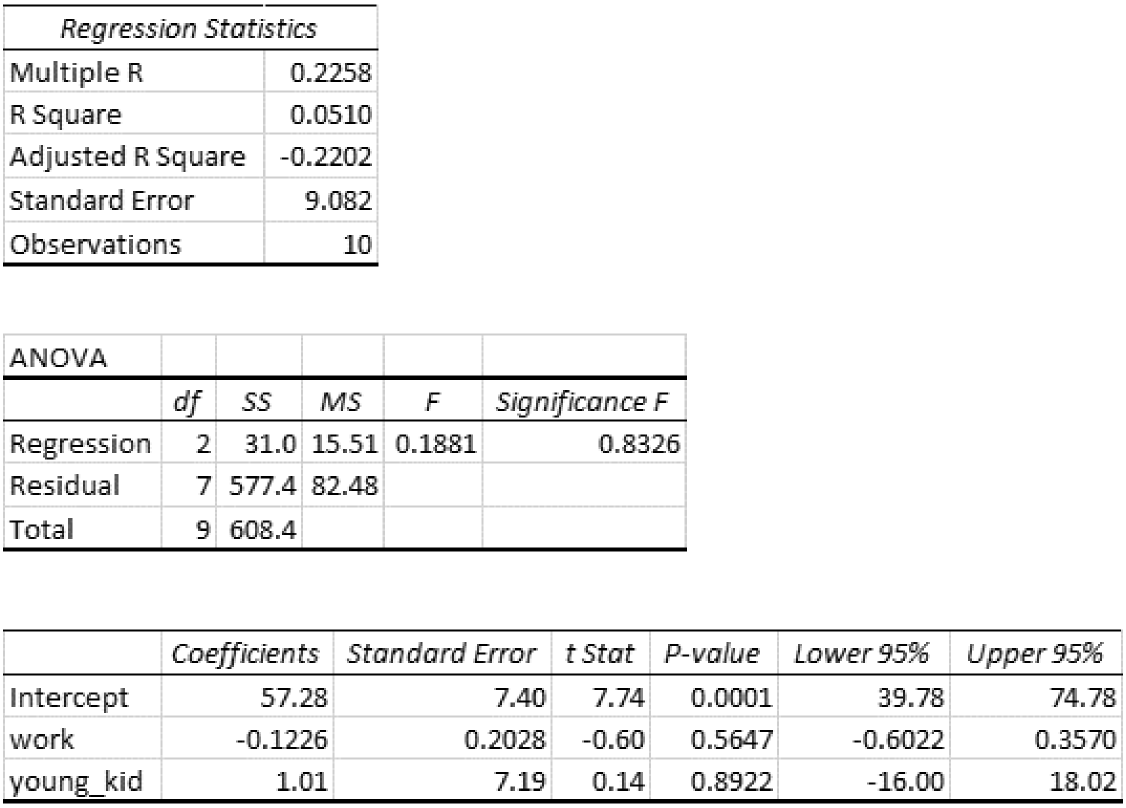
(b) Compute the test statistic and the critical value (using a significance level of 5%) (10p.)

(c) What is the conclusion regarding the hypothesis? (5p.)

14. A multiple linear regression that explains "sleeping time" (variable *sleep*, in hours per week) in terms of "working time" (variable *work*, in hours per week) and a dummy variable indicating whether the individual has kids younger than three

$$young\_kid = \begin{cases} 1 & \text{if the individual has kids younger than 3} \\ 0 & \text{otherwise} \end{cases}$$

has been fitted and the Excel output is shown in Figure 1.



Regression Statistics	
Multiple R	0.2258
R Square	0.0510
Adjusted R Square	-0.2202
Standard Error	9.082
Observations	10

ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	31.0	15.51	0.1881	0.8326
Residual	7	577.4	82.48		
Total	9	608.4			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	57.28	7.40	7.74	0.0001	39.78	74.78
work	-0.1226	0.2028	-0.60	0.5647	-0.6022	0.3570
young_kid	1.01	7.19	0.14	0.8922	-16.00	18.02

Figure 1: Excel output for the regression in Exercise 14

(a) **i.** How many individuals were included in the analysis? **ii.** What is the value of the coefficient of determination? **iii.** What is the value of the sum of squares error? **iv.** What is the 95% confidence interval for the coefficient associated to *work*? **v.** What is the p-value for the test that all coefficients in the model are equal to zero? (5p.)

(b) Predict the number of hours sleeping for a person working twenty hours per week who has two kids younger than three. (5p.)

(c) Using a significance level of 5%, is the hypothesis that all coefficients are equal to zero rejected or not? What does that mean in this particular problem? (5p.)

(d) Is the following sentence correct or incorrect?: "a person who works one more hour per week than another one, sleeps 0.12 hours less". (Justify your answer.) (5p.)