



BASIC STATISTICS FOR ECONOMISTS, STE101. EXAM

Department of statistics
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Time: 08:00 — 13:00

Approved aid: Hand-held calculator with no stored text, data or formulas

Provided aid: Formula Sheet and Probability Distribution Tables, returned after the exam.

Problems 1 — 12: Multiple choice questions (max 60 points):

- A total of 12 multiple choice questions with five alternative answers per question one of which is the correct answer. Mark your answers on the attached **answer form**.
- Marking more than one alternative will result in zero points for that question.
- Each correct answer is worth 5 points.
- Written solutions should not be submitted; only your answers on the answer form will be considered in the assessment and final grading.

Problems 13 — 14: Complete written solutions (max 40 points):

- Use only the provided answer sheets when submitting your solutions and answers.
- For full marks, clear, comprehensive and well-motivated solutions are required. Unclear and unexplained solutions may result in point deductions even if the final answer is correct.
- Check your calculations and solutions before submitting. Careless mistakes may result in unnecessary point deductions.

The maximum total number of points is $60 + 40 = 100$. At least 50 points are required to pass (grades A-E). The grading scale is as follows:

Points	0—39	40—49	50—59	60—69	70—79	80—89	90—100
Grade	F	Fx	E	D	C	B	A

NOTE: Fx and F are failing grades that require re-examination. Students who receive the grade Fx or F cannot supplement extra assignments for a higher grade.

Part one. Multiple choice

A well-known investor is thinking about investing between two electric car companies ("Tesla" and "Polestar"). Both investments are considered as a fixed-income that will be paid in 5 years. There is a risk of bankruptcy associated with the above companies, where the investor will not get the money back. Let:

X be the return on investment in Polestar's company after five years

Y be the return on investment in Tesla's company after five years.

If the company does not go bankrupt the accumulated interest rate after five years is 10% for Polestar and 20% for Tesla. The risk that Polestar goes bankrupt is 3% and the corresponding risk for Tesla is 5%. The risk that both Polestar and Tesla go bankrupt during the 5 years is estimated to be 2%. The following contingency table includes the above information and it will be used in Exercises 1 to 6:

	$Y = -1$	$Y = 0.2$	
$X = -1$	0.02		0.03
$X = 0.1$			
	0.05		

1. What is the probability that the investor will get the accumulated return after five years from both investments (Polestar and Tesla)?(5p)
 - (a) 0.5;
 - (b) 0.7;
 - (c) 0.94;
 - (d) 0.95;
 - (e) 1;

2. What is the expected return on investment from Polestar and Tesla ($E(X)$ and $E(Y)$) ? (5p)
 - (a) $E(X) = 0.067$ and $E(Y) = 0.14$;
 - (b) $E(X) = 0.67$ and $E(Y) = 0.14$;
 - (c) $E(X) = 0.01$ and $E(Y) = 0.24$;
 - (d) $E(X) = -0.067$ and $E(Y) = 0.14$;
 - (e) $E(X) = 0.067$ and $E(Y) = -0.14$;

3. Calculate the covariance between the two investments. Choose the alternative closest to your answer. (5p)
 - (a) $Cov(X, Y) = 0.014$;
 - (b) $Cov(X, Y) = 0.024$;
 - (c) $Cov(X, Y) = 0.094$;
 - (d) $Cov(X, Y) = 0.24$;
 - (e) $Cov(X, Y) = 0.74$;

4. Calculate the correlation between the two investments. Choose the alternative closest to your answer.(5p)
- (a) $Corr(X, Y) = 0.15$;
 - (b) $Corr(X, Y) = 0.27$;
 - (c) $Corr(X, Y) = 0.40$;
 - (d) $Corr(X, Y) = 0.49$;
 - (e) $Corr(X, Y) = 0.85$;
5. If the investor has 1,000 SEK to invest in each company, what is the expected return from the portfolio? Hint: Let $W = 1,000X + 1,000Y$ be the return on the portfolio after 5 years(5p)
- (a) 107 ;
 - (b) 157 ;
 - (c) 207 ;
 - (d) 507 ;
 - (e) 1007 ;
6. If the investor has 1,000 SEK to invest in each company, what is the variance from the portfolio? Hint: Let $W = 1,000X + 1,000Y$ be the return on the portfolio after 5 years. Choose the alternative closest to your answer.(5p)
- (a) 100,000 ;
 - (b) 120,000 ;
 - (c) 151,000 ;
 - (d) 227,000 ;
 - (e) 351,000 ;

As part of a scientific study, a random sample of 10 overweight volunteers are given a course in nutrition by a nutritionist. The body weight of each participant is measured at the beginning of the study, and then again six months later. Assume that the body weights are normally distributed. You can find the weights in kilograms in the following table, that will be used in Exercises 7 and 8:

Weight before	80	95	78	105	90	90	91	79	115	85
Weight after	80	92	77	100	91	89	93	77	112	85

We want to test, at the 5% level of significance, the null hypothesis that the average weight change (after minus before) is equal to zero against the alternative that the average weight change is smaller than zero.

7. Choose the hypotheses and the corresponding decision rule of the test (5p)

- (a) $H_0 : \mu_{after} - \mu_{before} = 0$ vs $H_A : \mu_{after} - \mu_{before} \neq 0$
Reject H_0 if $|t_{obs}| > 2.262$;
- (b) $H_0 : \mu_{after} - \mu_{before} = 0$ vs $H_A : \mu_{after} - \mu_{before} < 0$
Reject H_0 if $t_{obs} < -1.833$;
- (c) $H_0 : \mu_{after} - \mu_{before} = 0$ vs $H_A : \mu_{after} - \mu_{before} < 0$
Reject H_0 if $t_{obs} > 1.833$;
- (d) $H_0 : \mu_{after} - \mu_{before} = 0$ vs $H_A : \mu_{after} - \mu_{before} > 0$
Reject H_0 if $t_{obs} > 1.833$;
- (e) $H_0 : \mu_{after} - \mu_{before} < 0$ vs $H_A : \mu_{after} - \mu_{before} \geq 0$
Reject H_0 if $t_{obs} < -2.262$;

8. Find the observed value of the statistic. Choose the alternative closest to your answer.(5p)

- (a) 1.8;
- (b) 2.2;
- (c) 2.5;
- (d) -2.2;
- (e) -1.8;

The teacher of a course in statistics wants to explain the score of students in the final exam (variable *exam*) in terms of the score in a previous home assignment (variable *assignment*) through a linear regression of the form:

$$exam = \beta_0 + \beta_1 assignment + \epsilon$$

The following tables, that will be used in Exercises 9 and 10, show the output of the linear regression analysis:

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	1	914,64	914,64	3,45	0,11
Residual	6	1592,86	265,48		
Total	7	2507,50			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	-25,17	43,17	-0,58	0,58	-130,81	80,47
assignment	1,51	0,81	1,86	0,11	-0,48	3,49

9. Calculate a 95% confidence interval for the mean score on the exam given that the score in the assignment is 55 ($\mu_{Y|X=55}$) and taking into account that the mean value of the variable assignment is 52.75 and the variance is 57.76.

Choose the alternative closest to your answer. (5p)

- (a) (0.5 , 20.5)
- (b) (10.7, 25.9)
- (c) (20.7, 56.1)
- (d) (43.1, 72.7)
- (e) (84.5, 104.3)

10. Calculate a 95% prediction interval for the mean score on the exam given that the score in the assignment is 55 ($\mu_{Y|X=55}$) and taking into account that the mean value of the variable assignment is 52.75 and the variance is 57.76.

Choose the alternative closest to your answer. (5p)

- (a) (-15.4, 100.4)
- (b) (15.4, 100.4)
- (c) (25.4 , 75.4)
- (d) (-25.4 , 75.4)
- (e) (-100.4, 100.4)

The following estimated multiple regression model will be used in Exercises 11 and 12 and shows the expected log return price of the Tesla stock \hat{y}_t explained by four dummy variables:

$$\hat{y}_t = 0.027 - 0.047D_{1t} - 0.009D_{2t} - 0.012D_{3t} - 0.022D_{4t}$$

where

D_{1t} is a dummy variable which takes the value 1 when it is Tuesday and 0 otherwise,
 D_{2t} is a dummy variable which takes the value 1 when it is Wednesday and 0 otherwise,
 D_{3t} is a dummy variable which takes the value 1 when it is Thursday and 0 otherwise,
 D_{4t} is a dummy variable which takes the value 1 when it is Friday and 0 otherwise,
and Monday is the base category.

11. Which of the following statements is correct regarding the expected log return on Friday: (5p)

- (a) The expected log return on Friday is equal to 0.022 ;
- (b) The expected log return on Friday is equal to - 0.02 ;
- (c) The expected log return on Friday is equal to 0.018 ;
- (d) The expected log return on Friday is equal to 0.015 ;
- (e) The expected log return on Friday is equal to 0.005 ;

12. Which of the following statements is correct regarding the expected log return on Monday:
(5p)
- (a) The expected log return on Monday is equal to 0.027 ;
 - (b) The expected log return on Monday is equal to - 0.047 ;
 - (c) The expected log return on Monday is equal to - 0.009 ;
 - (d) The expected log return on Monday is equal to - 0.012 ;
 - (e) The expected log return on Monday is equal to - 0.022 ;

Part two. Complete solution

13. Romme Alpin is the largest ski resort in southern Sweden. Snowboarders can rent their ski equipment at the resort during their visit. There are five different snowboard sizes and riders choose their size based on their height. The five available sizes are size1, size2, size3, size4 and size5. The resort claims that each size is equally frequent in renting during the season. Two students from the Stockholm university visited the resort during the sport's break, known as "Sportlov". They collected a random sample of 200 snowboarders and asked them the size of the board they rented. In the table below you can find the details:

size1	size2	size3	size4	size5
30	40	50	50	30

Test at the 5% level of significance whether resort's claim is correct, i.e. determine if the five sizes are equally likely in preference or if they differ.

- (a) State your hypotheses and the test statistic. (5p)
 - (b) State the critical value and decision rule. (5p)
 - (c) Calculate the observed value of the test statistic. (5p)
 - (d) State your conclusion and give a verbal interpretation. (5p)
14. It is known that 50% of the students in the course Basic Statistics for Economists pass the exam.
- (a) A random sample of $n = 10$ students was drawn and their grade was reported after the examination on June 3rd. What is the probability that more than half of the 10 students passed the exam? (5p)
 - (b) A random sample of $n = 225$ students is available, what is the (approximated) probability that more than half of the students passed the exam? (5p)
 - (c) Consider the event $A =$ "the number of students passing the exam is between $112.5 - \alpha$ and $112.5 + \alpha$ ". Find the value α if the probability of A is equal to 0.9 and $n = 225$. (5p)
 - (d) Consider the event $B =$ "the average number of students passing the exam is between $112.5 - \beta$ and $112.5 + \beta$ ". Find the value β if the probability of B is equal to 0.9 and $n = 225$. (5p)