

## BASIC STATISTICS FOR ECONOMISTS, STE101. EXAM

Department of statistics Edgar Bueno 2024–10–11

Time: 08:00 — 13:00 Approved aid: Hand-held calculator with no stored text, data or formulas Provided aid: Formula Sheet and Probability Distribution Tables, returned after the exam.

# Problems 1 - 12: Multiple choice questions (max 60 points):

- A total of 12 multiple choice questions with five alternative answers per question one of which is the correct answer. Mark your answers on the attached **answer form**.
- Marking more than one alternative will result in zero points for that question.
- Each correct answer is worth 5 points.
- Written solutions should <u>not</u> be submitted; only your answers on the answer form will be considered in the assessment and final grading.

# Problems 13 — 14: Complete written solutions (max 40 points):

- Use only the provided answer sheets when submitting your solutions and answers.
- For full marks, clear, comprehensive and well-motivated solutions are required. Unclear and unexplained solutions will result in point deductions even if the final answer is correct.
- Check your calculations and solutions before submitting. Careless mistakes will result in unnecessary point deductions.

The maximum total number of points is 60 + 40 = 100. At least 50 points are required to pass (grades A-E). The grading scale is as follows:

**NOTE:** Fx and F are failing grades that require re-examination. Students who receive the grade Fx or F <u>cannot</u> supplement extra assignments for a higher grade.

## Part one. Multiple choice

- 1. The households' size in Sweden can be adequately modeled by a distribution with expectation 2.15 individuals. A consulting firm is going to draw a random sample of n households. Which of the following sentences is **correct**?
  - (a) the expectation of the sample mean is 2.15 individuals **only** if the distribution is symmetric;
  - (b) the expectation of the sample mean is 2.15 individuals **only** if the sample size n is large enough;
  - (c) the expectation of the sample mean is 2.15 individuals;
  - (d) the expectation of the sample mean is 2.15 individuals **only** if the distribution is the normal distribution;
  - (e) the expectation of the sample mean is **not** 2.15 individuals.
- 2. A real estate agent has estimated the regression that explains the closing price (in SEK) of the housing units in a region of interest with respect to their size (variable *size*, in  $m^2$ ), the number of rooms (variable *room*) and a dummy variable indicating whether the housing unit has a balcony (variable *balcony*). The output is shown below.

Regression Statistics						
Multiple R	0.2363					
R Square	0.0558					
Adjusted R Square	0.0495					
Standard Error	720474					
Observations	450					
ANOVA						
	df	SS	MS	F	Significance F	
Regression	3	1.369E+13	4.563E+12	8.791	1.129E-05	
Residual	446	2.315E+14	5.191E+11			
Total	449	2.452E+14				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	2000000	200000	10.00	0.0000	1606943	2393057
SIZE	-5000	3000	-1.67	0.0963	-10896	896
ROOM	400000	80000	5.00	0.0000	242777	557223
BALCONY	30000	70000	0.43	0.6684	-107570	167570

Considering a significance level of 1%, which of the following is **not correct**:

- (a) around 5% of the variability of the closing price is being explained by the three explanatory variables;
- (b) the combined effect of the three explanatory variables is significant for explaining the closing price;
- (c) for two housing units with the same size and the same number of rooms, it is expected that one with a balcony will be sold at around 30000 SEK more than one without a balcony;
- (d) the number of rooms has a significant effect on the closing price;
- (e) having a balcony has a significant effect on the closing price.

- 3. The owner of an electronic store wants to verify the hypothesis that 60%, 30% and 10% of the customers buy, respectively, cell phones of the brands A, B and C. Which of the following is an appropriate method to this end:
  - (a) time-series analysis;
  - (b) multiple linear regression;
  - (c) test of independence;
  - (d) simple linear regression;
  - (e) goodness-of-fit test.
- 4. Which of the following is **not** one of the components of a time series:
  - (a) Smooth component;
  - (b) Cyclical component;
  - (c) Trend component;
  - (d) Seasonal component;
  - (e) Irregular component.
- 5. A researcher has asked the thirteen married men in a small community about the brideprice they had to pay to the bride's family when they got married. The brideprice values (in USD) are

 $20000 \quad 3000 \quad 10000 \quad 20000 \quad 13000 \quad 0 \quad 31000 \quad 20000 \quad 63000 \quad 8000 \quad 3000 \quad 12000 \quad 4000$ 

What is the **median** of the brideprice?

- (a) 12000;
- (b) 15500;
- (c) 15923;
- (d) 20000;
- (e) 31000.
- 6. The probability that four students A, B, C and D get a passing grade in an exam of statistics is, respectively, 0.8, 0.7, 0.5 and 0.4. However, A and B have been studying together, and the probability that both of them pass the exam is 0.6. C and D have also been studying together, and the probability that both of them pass the exam is 0.3. Finally, A and B do not know C and D, so their grades can be considered to be independent. What is the probability that all four students pass the exam?
  - (a) 0.112;
  - (b) 0.18;
  - (c) 0.9;
  - (d) 1;
  - (e) 1.5.

- 7. In a card game, the player has three possible outcomes: win, tie or lose. If the player wins (which happens with probability 0.19), he gets two dollars; if the player loses (which happens with probability 0.47), he loses one dollar; in the case of a tie, the player neither wins nor loses any money. What is the expected amount of money of the player at the end of one game?
  - (a) -0.39;
  - (b) -0.09;
  - (c) 0.00;
  - (d) 0.33;
  - (e) 1.00.
- 8. The amount of money spent on clothing by students on Stockholm University during 2024 can be modeled by a normal distribution with expected value of 1200 and variance of 40 000. The amount of money spent on course literature can be modeled by a normal distribution with expected value of 800 and variance of 18 000. The covariance between money spent on clothing and course literature is −24 000. What is the probability that one student chosen at random spent more than 2000 on clothing plus course literature?
  - (a) 0;
  - (b) 0.0967;
  - (c) 0.5;
  - (d) 0.25;
  - (e) 1.
- 9. A car rental company knows by experience that 10% of the customers rent a *sport utility vehicle* —suv— and that the customers' choice is independent of each other. What is the probability that, out of the next 100 customers, the number of customers renting a suv is larger than eight but at most eleven?
  - (a) 0.0300;
  - (b) 0.1322;
  - (c) 0.3000;
  - (d) 0.3822;
  - (e) 0.6178;
- 10. A teacher knows by experience that the number of points students get in the final exam can be adequately modeled by a normal distribution with variance equal to 441. At the end of one semester, a sample of nine students has been selected and their score in the exam has been measured. The sample mean is  $\bar{x}_s = 53.6$ . At the end of the next semester, a sample of nine students has been selected independently and their score in the exam has been measured. The sample mean is  $\bar{y}_s = 54.6$ . A 99% confidence interval for the difference of the expected number of points between the first and the second semester is:
  - (a) (-253.4, 251.4);
  - (b) (-229.0, 227.0);
  - (c) (-50.5, 48.5);
  - (d) (-26.5, 24.5);
  - (e) (-24.0, 22.0).

- 11. One week before the local elections of a city, a candidate, Mrs. A, believes that more than 30% of the voters support her. In order to verify her claim, the campaign has selected a sample of 100 voters. 37 out of the 100 voters in the sample claim that they will vote for Mrs. A. The value of the test statistic for testing the alternative that the proportion of voters for Mrs. A is larger than 30% is:
  - (a) 1.45;
  - (b) 1.66;
  - (c) 1.98;
  - (d) 14.50;
  - (e) 30.03;
- 12. Two teachers, Teacher 1 and Teacher 2, are in charge of grading 100 exams of statistics. 50 exams are randomly assigned to teacher 1 and the remaining 50 are assigned to teacher 2. The following table summarizes the results:

	Grade								
Teacher	A	В	С	D	Е	$\mathbf{F}\mathbf{x}$	F		
1	2	2	6	12	8	4	16		
2	2	6	7	8	7	4	16		

Having a significance level  $\alpha = 0.05$ , what is the critical value for the hypothesis that grade and teacher are independent:

- (a) 2.944;
- (b) 12.592;
- (c) 67.505;
- (d) 17.000;
- (e) none of the above.

#### Part two. Complete solution

On September 2024 a company that offers audio streaming services released a new logo. A random sample of six users was drawn and their time using the service before and after the release was measured. Let  $x_i =$  "time (in minutes) spent by the *i*th user using the service one week *before* the new logo was released" and  $y_i =$  "time (in minutes) spent by the *i*th user using the service one week *after* the new logo was released" ( $i = 1, \dots, 6$ ). It is known that the times using the service are adequately described by a normal distribution.

Some summary statistics of the collected data are shown below:

$$\sum_{s} x_i = 803, \qquad \sum_{s} y_i = 753, \qquad \sum_{s} x_i^2 = 139\,933, \qquad \sum_{s} y_i^2 = 110\,221, \qquad \sum_{s} x_i y_i = 121\,769$$

- 13. Calculate the following statistics: i. the mean of x,  $\bar{x}_s$ ; ii. the mean of y,  $\bar{y}_s$ ; iii. the variance of x,  $S^2_{x,s}$ ; iv. the variance of y,  $S^2_{y,s}$ ; and v. the covariance between x and y,  $S_{xy,s}$ . (10p.)
- 14. Confidence interval estimation:
  - (a) Find a 95% confidence interval for the expected change (*before* minus *after*) in time using the service. (**Hint:** Take into account that  $\bar{d}_s = \bar{x}_s \bar{y}_s$  and  $S_{d,s}^2 = S_{x,s}^2 2S_{xy,s} + S_{y,s}^2$ .) (5p.)
  - (b) Based on your results on 14a, has there been a significant change in the time using the service? Justify your answer. (5p.)
- 15. Simple linear regression:
  - (a) Estimate the intercept and the slope of a regression that explains the time using the service *after* the release of the new logo in terms of the time using the service *before* the release of the new logo. (5p.)
  - (b) Taking into account that  $\sum_{s} e_i^2 = 2145$ , estimate the variance of the intercept and the slope coefficients,  $\hat{\sigma}_{b_0}^2$  and  $\hat{\sigma}_{b_1}^2$ . (5p.)
  - (c) Find a 95% confidence interval for the intercept,  $\beta_0$ , and the slope,  $\beta_1$ . (5p.)
  - (d) **i.** Does the confidence interval for the intercept contain the value zero? **ii.** Does the confidence interval for the slope contain the value one? **iii.** In terms of the linear model, what does it mean for the intercept to be equal to zero and, at the same time, the slope to be equal to one in this particular problem? (5p.)